

## Particle acceleration in rotating and shearing AGN jets

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**Abstract.** Steady state solutions of the kinetic equations describing the transport of energetic particles in a collisionless, rotating background flow are discussed for conditions assumed to prevail in the jets of active galactic nuclei. By considering velocity profiles from rigid to keplerian rotation, the centrifugal acceleration and the shear acceleration of particles scattered by magnetic inhomogeneities are distinguished. In the case where shear effects dominate, it is confirmed that power-law particle momentum solutions exist if the mean scattering time is an increasing function of momentum. If there is a more complex interplay between shear and centrifugal acceleration, a flattening of the momentum spectra with increasing azimuthal velocity is observed. The relevance of shear acceleration in addition to Fermi-type particle acceleration in AGN jets is pointed out with reference to recent observations in 3C273.

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### 1 Introduction

The observation of jets from radio-loud AGN are among the most interesting phenomena relevant to astrophysics. Today there is convincing evidence that the central engine in these AGN is a rotating, supermassive black hole surrounded by a geometrically thin accretion disk which gives rise to the formation of a pair of relativistic jets. If jets and disks are indeed symbiotic features (e.g. Falcke and Biermann, 1995), the presence of jets may however be related to a much wider class of objects.

The observations of superluminal motion and theoretical opacity arguments indicate that the plasma in these jets moves at relativistic speeds along the axis. In real jets one also expects there to be a significant velocity shear perpendicular to the jet axis. We know indeed of several observational evidence pointing to intrinsic rotation in AGN jets, e.g. in the case of NGC 4258 (Cecil et al., 1992), M87 (Biretta, 1993) and of the blazar 3C345 (Schramm et al., 1993). From a theoretical

point of view, intrinsic jet rotation is also expected in magnetohydrodynamical models for the formation and collimation of astrophysical jets (e.g. Begelman, 1994). In such jet models intrinsic rotation with speeds up to some fraction of the velocity of light is a natural consequence of the assumption that the flow is centrifugally accelerated from the accretion disk. It should be noted however, that the rotation profile in the jet does not need to be necessarily disk-like, i.e. the set of available jet rotation profiles could be much wider and might include, for example, rigid, flat and keplerian profiles (e.g. Hanasz et al., 2000). In particular, rigid rotation inside a well-defined light cylinder might be related to foot points of the magnetic field lines near the last inner stable orbit (e.g. Camenzind, 1996; Fendt, 1997a), while more general differential rotation might be intuitively expected if there is an intrinsic connection between jet motion and the rotating disk (cf. also Fendt, 1997b; Lery and Frank, 2000).

Particle acceleration by shear flows has been investigated so far by several authors. A pioneer approach has been given by the kinetic analysis of Berezhko (1981; 1982) and also Berezhko & Krymskii (1981) showing that the steady state particle distribution might follow a powerlaw spectrum if the mean interval between scattering events increases with momentum according to a power-law.

Independently, particle acceleration in the diffusion approximation at a gradual shear transition for non-relativistic flows has been analysed by Earl et al. (1988). They derived Parker's equation, i.e. the transport equation including the well-known effects of convection, diffusion and adiabatic energy changes, but augmented by new terms describing the viscous momentum transfer and the effects of inertial drifts. Jokipii and Morfill (1990) have used a microscopic treatment to analyse the non-relativistic particle transport for a moving, scattering fluid which undergoes a step-function velocity change in the direction normal to the flow, showing that particles may gain energy at a rate proportional to the square of the magnitude of the velocity change. Based on a Monte Carlo simulation method, Ostrowski (1990), has studied the acceleration at a

sharp tangential velocity discontinuity involving relativistic speeds. He finds that only relativistic flows can provide conditions for efficient acceleration resulting in a very flat particle energy spectra which depends only weakly on the scattering conditions (cf. also Ostrowski, 1999).

The work on (gradual) shear acceleration by Earl et al. (1988) has been extended to the relativistic regime by Webb (1989; 1991). Assuming the scattering to be strong enough to keep the distribution function almost isotropic in the comoving frame, i.e. the diffusion approximation to apply, he derived the relativistic diffusive particle transport equation for rotating and shearing flows employing the co-moving particle momentum  $p'$ . Our present approach utilizes a simple version of the relativistic transport equation derived by Webb (cf. also Webb et al., 1994) and examines the effect of different intrinsic flow rotation profiles on the acceleration of energetic particles in a basic jet model. If intrinsic jet rotation is considered, it should be noted that particle energization may in general be a consequence of both, centrifugal and shear effects.

## 2 Model background

Starting from the relativistic Boltzmann equation and using the differential moment equations, Webb (1989; 1992) has derived the relativistic version of the particle transport equation. The scattering of high energetic particles has been modelled using a simple BKG time relaxation approximation. In the underlying physical picture, scattering of high energy particles by small-scale magnetic field irregularities carried in a collisionless plasma background flow is assumed to occur. In each scattering event the particle momentum is randomized in direction but its magnitude  $p'$  is assumed to be conserved in the (local) comoving flow frame where the electric field vanishes. Since the rest frame of the scattering centres is regarded to be essentially that of the background flow, particles would not gain energy or momentum merely by virtue of the scattering if there is no shear or rotation present. However if shear in the background is present, the particle momentum relative to the flow changes while travelling across the shear flow. Since the particle momentum in the local flow frame is preserved in the subsequent scattering event, a net increase in particle momentum may occur (cf. Jokipii and Morfill, 1990). Thus, if rotation and shear is present, high energy particles which do not corotate with the flow will sample the shear flow and may be accelerated by the centrifugal and shear effects (e.g. Webb et al, 1994).

For the present application, we consider an idealized (hollow) cylindrical jet model where the plasma moves along the  $z$ -axis at constant (relativistic)  $v_z$  while the velocity component in the plane perpendicular to the jet axis is purely azimuthal and characterized by the angular frequency  $\Omega$ . Using cylindrical coordinates for the position four vector, i.e.  $x^\alpha = (ct, r, \phi, z)$ , the metric tensor becomes coordinate-dependent (i.e.  $(g_{\alpha\beta}) = \text{diag}\{-1, 1, r^2, 1\}$ ) and for the chosen holonomic basis the considered flow four velocity could

be written in shortened notation as

$$u^\alpha = \gamma_f (1, 0, \Omega/c, v_z/c), \quad (1)$$

$$u_\alpha = \gamma_f (-1, 0, \Omega r^2/c, v_z/c), \quad (2)$$

where the normalization  $\gamma_f = 1/\sqrt{1 - \Omega^2 r^2/c^2 - v_z^2/c^2}$  denotes the Lorentz factor of the flow and where the angular velocity may be selected to be a function of the radial coordinate, i.e.  $\Omega = \Omega(r)$ . As suggested by Webb et al. (1994), the resulting transport equation may be cast in a quite suitable form by introducing the variable  $\Phi = \ln(H)$  replacing the comoving momentum variable  $p'$ , where  $H$  is given by

$$H = p'^0 c \exp\left(-\int^r dr' \frac{\gamma_f^2 \Omega^2 r'}{c^2}\right). \quad (3)$$

In the case of highly relativistic particles (with  $p'^0 \simeq p'$ ) we may finally arrive at the steady state transport equation (cf. Rieger and Mannheim, 2001, in preparation) for the (isotropic) phase-space distribution function  $f(r, z, p')$

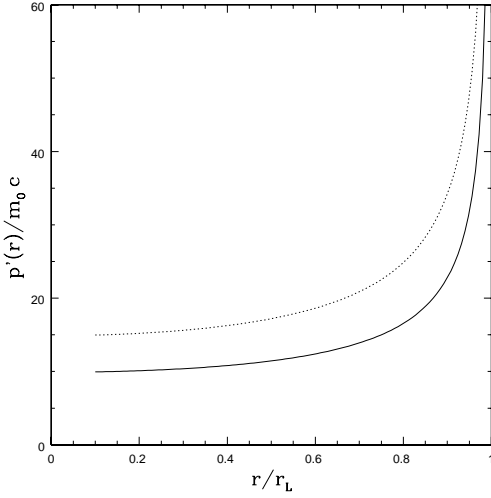
$$\begin{aligned} \frac{\partial^2 f}{\partial r^2} + \left(\frac{1+\beta}{r} + [3+\alpha] \frac{\gamma_f^2 \Omega^2 r}{c^2}\right) \frac{\partial f}{\partial r} \\ + \frac{\gamma_f^4 r^2}{5c^2} (1 - v_z^2/c^2) \left(\frac{d\Omega}{dr}\right)^2 \left([3+\alpha] \frac{\partial f}{\partial \Phi} + \frac{\partial^2 f}{\partial \Phi^2}\right) \\ - \frac{\gamma_f v_z}{\kappa} \frac{\partial f}{\partial z} + (1 + \gamma_f^2 v_z^2/c^2) \frac{\partial^2 f}{\partial z^2} = -\frac{Q}{\kappa}, \end{aligned} \quad (4)$$

using an (isotropic) diffusion coefficient of the form  $\kappa = \kappa_0 p'^\alpha r^\beta$ . The general solution of Eq. 4 is quite complicated. However, we are especially interested in an analysis of the azimuthal effects of particle acceleration in rotating jet flows and may thus be content with the  $z$ -independent solution of the transport equation, i.e. with an investigation of the so-called one-dimensional Green's function which preserves much of the physics involved. The corresponding source term could be written as  $Q = q_0/p'_s \times \delta(r-r_s) \delta(\Phi - \Phi_s)$ , describing mono-energetic injection of particles with momentum  $p' = p'_s$  from a cylindrical surface at  $r = r_s$ . The constant  $q_0$  is defined by  $q_0 = N_s/(8\pi^2 p_s'^2 r_s)$ . In order to solve the  $z$ -independent transport equation we apply Fourier techniques and consider Green's solution satisfying homogeneous, i.e. zero Dirichlet boundary conditions.

## 3 Applications

### 3.1 Rigid rotation - no shear

We have recently considered the acceleration of charged test particles at the base of a rigidly rotating jet magnetosphere showing that there exists an upper limit of the maximum attainable Lorentz factor given by the breakdown of the bead-on-the-wire approximation which occurs in the vicinity of the light cylinder (Rieger and Mannheim, 2000). If we consider relativistic particle transport in a rigidly rotating background flow (i.e.  $\Omega = \Omega_0 = \text{const.}$ ) using the model above, shearing is absent (i.e.  $d\Omega/dr = 0$ ) and particle energization only

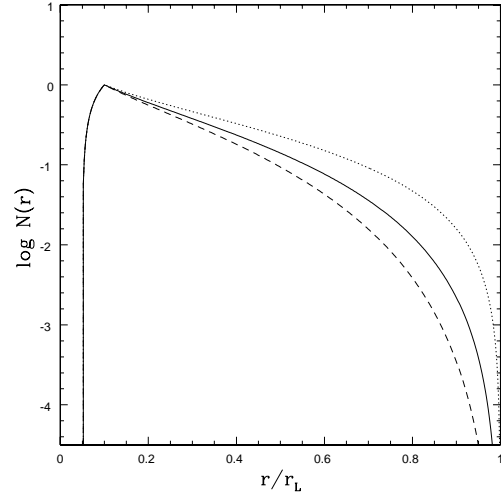


**Fig. 1.** Particle momentum  $p'$  as a function of the radial coordinate for particles injected at  $r_s = 0.1 r_L$  with initial Lorentz factor  $\gamma_s = 10$  (solid) and 15 (dashed).

occurs as a consequence of centrifugal acceleration. Eq. 3 is then analogous to the Hamiltonian for a bead on a rigidly rotating wire (cf. also Webb et al., 1994), i.e.  $H = p'^0 c/\gamma$  while the transport equation becomes purely spatial. Using Noether's theorem  $H$  could be shown to be a constant of motion and thus the comoving particle momentum  $p'$  could be simply expressed as a function of the radial coordinate  $r$ , i.e.  $p' = p'(r)$ . The transport equation could be easily solved analytically applying homogeneous boundary conditions at the jet inner radius  $r_{in}$  and its relevant outer radius  $r_{out}$ . Results are shown in Fig. 1, 2 indicating that the efficiency might be reduced if the time between two collisions increases with momentum.

### 3.2 Keplerian rotation - shear dominance

In the case of keplerian rotation ( $\Omega \propto r^{-1.5}$ ) both shear and centrifugal effects are present. Analytical solutions for the Fourier transformed transport equation in terms of the confluent hypergeometrical functions may be found for the case of non-relativistic rotation eventually being appropriate for the outer jet solutions. For the idealized conditions, the relative strength of the contribution by shear to centrifugal energization scales as  $\sqrt{r}$ , i.e. shear eventually dominates over centrifugal acceleration. This is illustrated in Fig. 3 where we have plotted the logarithmic of the (normalized) particle distribution function  $f$  above  $p'/p'_s \geq 5$  for  $\beta = 0$  (i.e. no spatial dependence in the diffusion coefficient) and various momentum dependence of  $\kappa$ . Obviously, powerlaw-type momentum spectra are recovered, i.e.  $f \propto p'^{-(3+\alpha)}$ . This agrees very well with the results derived by Berezhko & Krymskii (1981). For a collisionless plasma with shear flow  $U(y) e_x$  and a simple BGK term, they found the shear acceleration to give rise to a power-law momentum



**Fig. 2.** Spatial distribution  $N(r)/N(r_s)$  for a different energy dependence of the diffusion coefficients, i.e. for  $\alpha = \beta = 0$  (solid line),  $\alpha = -2, \beta = -0.01$  (dotted line) and  $\alpha = 2, \beta = -0.01$  (dashed line). Boundary conditions  $r_{in} = 0.05 r_L, r_s = 0.1 r_L$  and  $r_{out} = 0.999 r_L$  has been used for the calculations.  $r_L$  is defined by  $r_L = c(1 - v_z^2/c^2)^{0.5}/\Omega_0$ .

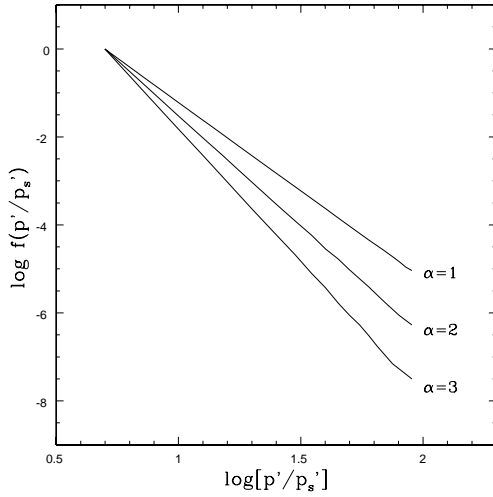
spectrum for the steady state comoving particle distribution  $n(r, p') \propto p'^2 f \propto p'^{-(1+\alpha)}$  if the collision time  $\tau_c$  depends on momentum as  $\tau_c \propto p'^\alpha$  with  $\alpha > 0$ . If the momentum index  $\alpha$  however, is smaller than zero, i.e.  $\alpha < 0$ , an exponential spectrum could be developed.

### 3.3 Flat rotation - interplay between shear and centrifugal effects

In the case of flat rotation (i.e.  $\Omega = \Omega_0 r_0/r = v_{\phi f}/r$ ) a more complex interplay between shear and centrifugal effects might be investigated. The solutions of the fourier-transformed equation could be easily cast in analytical terms and inverse Fourier integration might be done for homogeneous boundary conditions at  $r_{in}$  and  $r_{out}$ . Here we have considered, for example, typical jet flows with relativistic  $v_z/c = 0.95$  and different azimuthal velocities  $v_{\phi f}$  (i.e. for a range of  $\gamma_f \sim (3-4)$ ). Results are plotted in Fig. 4 for a constant diffusion coefficient (i.e.  $\alpha = \beta = 0$ ). The calculated distribution functions reveal a (slightly curved) powerlaw-type momentum dependence with momentum exponent near by  $-5.2$  (solid curve),  $-3.7$  (dashed curve) and  $-2.8$  (dotted curve), i.e. a flattening is observed with increasing azimuthal velocity.

## 4 Conclusion

The present results may serve as an instructive example revealing the power of shear and centrifugal acceleration in jets of AGN. Since intrinsic jet rotation is expected in the AGN setting, the analysed mechanism could operate in a quite nat-



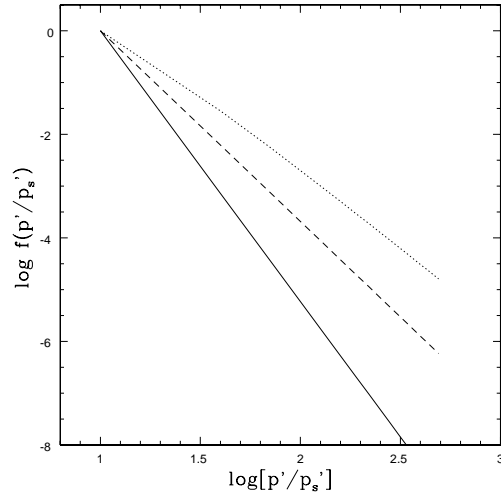
**Fig. 3.** The momentum-dependence of the (normalized) distribution function  $f$  for Keplerian rotation using  $\beta = 0$ , calculated for  $\alpha = 1, 2, 3$  at fixed  $r = 40 r_{\text{ms}}$ . Boundary and injection conditions have been specified as  $r_{\text{in}} = 10 r_{\text{ms}}$ ,  $r_{\text{out}} = 1000 r_{\text{ms}}$ ,  $r_s = 20.0 r_{\text{ms}}$ , where  $r_{\text{ms}}$  has been defined as  $r_{\text{ms}} = GM/(c^2 - v_z^2)$ .

ural manner over a large range. This suggests that particle acceleration in a rotating and shearing background flow could be of particular relevance for an explanation of the continuous optical emission observed from the jets of several AGN jets (e.g. for 3C273; M87; PKS 0521-36; cf. Meisenheimer et al., 1997; Jester et al., 2001). These observations do not show evidence for localized particle acceleration or emission zones, as expected if shock-type acceleration occurs, but rather point to an extended so-called "jet-like" acceleration mechanism which seems to operate over many kpc. The range of the (two-point) spectral flux indices, derived from the observations, starts from  $-0.6$  and reaches values up to  $-1.5$ . If in the context of shear and centrifugal acceleration, the particle energy is locally dissipated by synchrotron radiation, the spectral emissivity  $j_\nu$  for a power-law particle number density distribution  $n \propto p'^2 f = p'^{-x}$  is given by  $j_\nu \propto \nu^{-y}$  with  $y = (x - 1)/2$ . Thus, for  $y = (0.6 - 1.5)$  we require  $x = (2.2 - 4)$  which, for example, eventually might be met in the case of a constant diffusion coefficient and flat rotation or in the case of a simple shear flow with (strong) momentum-dependent diffusion.

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**Fig. 4.** The momentum-dependence of the (normalized) distribution function  $f(r, p')$  for flat rotation, calculated for  $r_{\text{in}}/r_{\text{out}} = 0.02$ ,  $r_s/r_{\text{out}} = 0.04$  at position  $r/r_{\text{out}} = 0.2$ . Chosen azimuthal velocities are  $v_{\phi f}/c = 0.05$  (solid),  $0.1$  (dashed),  $0.2$  (dotted).

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