Proceedings of ICRC 2001: 212 © Copernicus Gesellschaft 2001

ICRC 2001

Fractal Study of extensive air shower time series

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Abstract. Arrival time intervals of air showers were studied using a fractal analysis method (Taken's method). The data were obtained by an air shower array, located at Mitsuishi ($34.8°$N, $134.3°$E) in Japan, from January 1989 to April 1999, with an event rate of approximately one per minute and a mean energy of 1.1×10^{15} eV. We found 54 chaotic events with fractal dimensions of less than 5.0 by grouping 256 air showers in a bucket. In the right ascension distribution of the air showers in these events, we observed a peak at 9 hour with a 2.2 sigma. The longest chaotic event lasted for four hours, founded by a time-development analysis of the chaotic events conducted by shifting (in 10 min steps) the starting time of the buckets.

1 Introduction

Fractal dimension analysis has received much attention in the last several years in the study of the origins and propagation mechanisms of cosmic rays. We applied the Takens' method to the arrival time series of air showers collected by the Mitsuishi Air Shower Array ($34.8°$N, $134.3°$E) in Japan.

we made an initial report of the chaotic features found in the 128 or 256 time series with fractal dimensions of $2.5 \sim 4.1$, using data collected from January 1989 to December 1994 (Y. Katayose *et al.*, 1995). In this paper, we present the results of the analysis, by grouping the 256 time series, using the data from January 1989 to April 1999 (except the year 1997). In addition, we studied the time development of each chaotic event by shifting (in 10 min steps) the boundaries of the time series.

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2 Data Analysis and Results

The number of air showers used was 4,549,186, collected by the Mitsuishi Air Shower Array, with an average event rate of $1 \sim 2$ event/min. The air showers were selected using the air-shower analysis method mentioned previously (Y. Katayose *et al.*, 1998). We required muon(s) with an energy of $\geq$ 6GeV to accompany the air showers. The mean energy of the air showers was 1.1×10^{15}eV. After selecting the air showers by the zenith angle $\cos\theta \geq 0.7$, the Takens' method (F. Takens, 1981) was applied to the arrival time sequences. Time intervals are defined as (Y. Katayose *et al.*, 1995),

$$x(i) = t_i - t_{i-1} \tag{1}$$

where $x(i)$ denotes the i-th time interval and t_i the arrival time of the i-th air shower. The time interval series with n data points are embedded in the m-dimensional pseudo-phase space by constructing a vector $V_m(i)$ as

$$V_m(i) = [x(i), x(i+1\tau), x(i+2\tau), ..., x(i+(m-1)\tau)], \tag{2}$$

where m is the embedding dimension and τ is the delay time. The correlation dimension, D_m, is defined by

$$D_m = \frac{d\log C_m(r)}{d\log r} \tag{3}$$

where C_m is the number of vector point pairs, $V_m(i)$ and $V_m(j)$, for which the mutual distance in the m-dimensional phase space is less than r. This definition of D_m was introduced by Takens and the range of its applicability for noisy data was studied by Ohara *et al* (S. Ohara *et al.*, 1992) using the synthesized time series. In our analysis, air showers were divided into buckets with 256 time intervals, which corresponds to a period of 4 hours. We applied the Takens' method with τ = 1 for the 256 time series of air showers in each bucket (n=256). If the arrival time is random, the

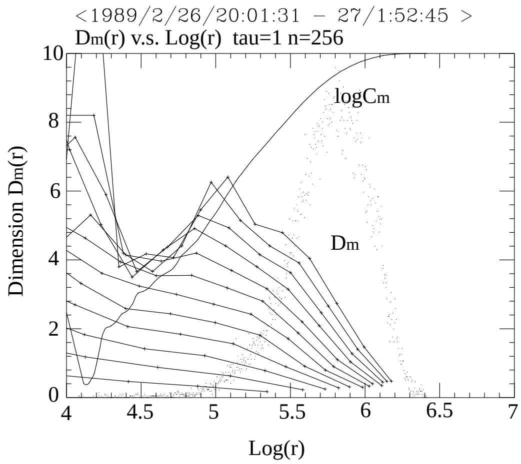

Fig. 1. Correlation between Dm and log(r); Feb. 26th 1989.

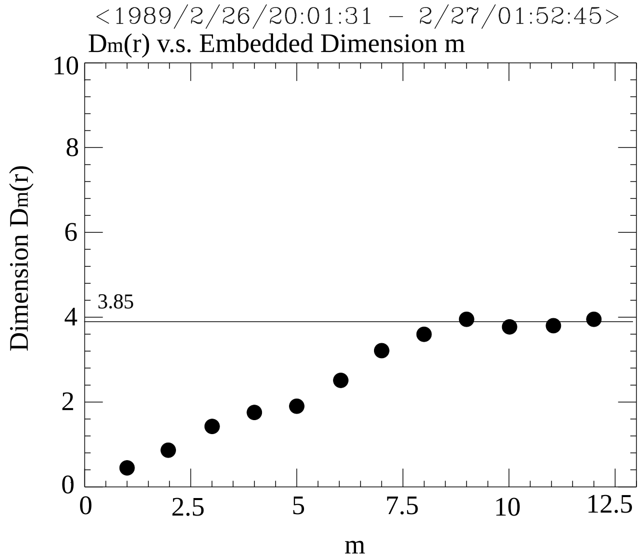

Fig. 2. Embedded dimension vs Fractal dimension; Feb. 26th 1989.

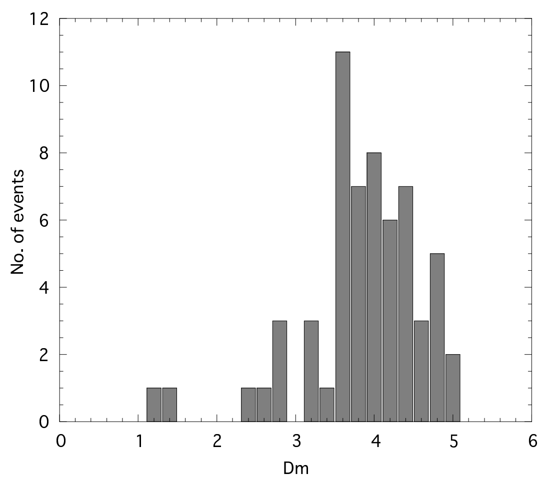

Fig. 3. Dimension distribution of the chaotic events.

correlation dimension should have no flat region in the D_m-log(r) plot. Existence of a flat region in the D_m-log(r) plot indicates the existence of fractal features. We used this characteristic to select the events. Figure 1 shows a typical event with n=256, whose period was approximately 6 hours, observed on 2/26/20h $\sim$ 2/27/02h in 1989. Figure 2 shows the fractal dimension as a function of the embedding dimension for this event. As seen in Fig. 2, the D_m of the flat region increases with the embedding dimension m, showing a plateau at 3.85 for $m \geq 8$. The existence of the plateau indicates that the air showers in this fractal event have a chaotic feature. We have found 56 events with the chaotic feature, as listed in Table 1. In Table 1, the starting time of the chaotic event is coded in the format year/month/day/hour. The events, 1, 2, 17, 23, 26 and 54 have chaotic features in two consecutive buckets. Figure 3 shows the frequency distribution of the fractal dimension D_m with the mean value of 3.83.

To study the time development of the chaotic features of these events, we changed the bucket boundaries by shifting the starting time of the bucket (n=256) by ten minutes, and measured the fractal dimensions using the data collected in 1996. Figure 4 shows the time variation of the fractal dimension for a typical chaotic event. As seen in this example, a characteristic pattern was observed in the time development of the fractal dimension. The fractal dimension appears at the start of the event with high value. Then the dimension decreases and stays at a minimum during the event. It then increases again and disappears at the end. The event in Fig. 4 has a minimum D_m = 3.6.

We found that the incident directions of air showers in the chaotic events are not uniform. The right ascension distribution of all the air showers in all the chaotic events peaks at 9 hour. The observational period (acceptance) in the right ascension is approximately uniform; its variation is smaller than 1 %. The direction of the chaotic event was measured by the averaged right ascensions of the air showers in the chaotic event with an accuracy of 1 hour. Figure 5 shows the directions of the chaotic events as a function of the right ascension. The distribution has a peak at 9 hour with a 2.2 σ. Figure 6 shows the directions of the chaotic events plotted in the galactic coordinate. The events are clustered in the galactic latitude $15^\circ \sim 80^\circ$. The observable region was $-10^\circ <$ declination $< 80^\circ$. The large closed loop, shown in Fig. 6, shows the trajectory of the vertical direction at Mitsuishi. There was no difference in the size and zenith angle distributions between the air showers in the chaotic events and normal air showers. Also, no difference was found in the distribution of the number of accompanying muons.

Table 1. Chaotic events, DATE/TIME shows the starting time of the chaotic event in year/month/day/hour. The events 1, 2, 17, 23, 26 and 54 have chaotic features in two consecutive buckets.

No.	DATE/TIME (U.T.)	D_m	No.	DATE/TIME (U.T)	D_m	No.	DATE/TIME (U.T)	D_m
1	1989/2/26/20	3.85	18	1992/7/30/0	4.30	36	1996/3/23/10	4.20
	1989/2/26/22	3.55	19	1992/8/14/8	4.00	37	1996/4/7/0	3.70
2	1989/6/16/22	3.55	20	1992/8/19/16	3.60	38	1996/5/16/10	4.40
	1989/6/17/0	3.65	21	1992/9/12/8	2.85	39	1996/6/23/0	4.80
3	1989/8/13/4	4.30	22	1993/6/17/14	2.85	40	1996/10/13/6	4.60
4	1989/9/14/12	4.30	23	1994/5/24/12	3.20	41	1996/10/20/6	4.85
5	1989/9/16/8	3.20		1994/5/24/14	3.95	42	1996/12/12/12	3.55
6	1989/9/24/22	4.70	24	1994/5/27/0	3.85	43	1996/12/15/6	4.00
7	1989/10/8/14	4.05	25	1994/10/8/22	3.60	44	1996/12/16/0	2.50
8	1989/10/17/8	4.20	26	1994/10/26/6	1.45	45	1998/5/2/2	3.60
9	1989/12/2/16	3.45		1994/10/26/8	1.20	46	1998/5/15/0	4.40
10	1990/2/10/10	4.00	27	1994/11/14/16	4.40	47	1998/6/16/6	3.20
11	1990/4/25/2	2.45	28	1995/2/18/14	3.50	48	1998/8/1/12	4.24
12	1990/5/2/20	2.75	29	1995/2/27/22	5.00	49	1998/8/3/4	4.64
13	1990/5/15/18	3.75	30	1995/3/13/12	3.80	50	1998/10/5/8	3.60
14	1990/5/30/2	3.50	31	1995/8/3/10	3.95	51	1999/2/3/10	3.80
15	1991/5/12/22	3.65	32	1995/8/16/10	4.55	52	1999/2/25/16	4.70
16	1991/6/5/12	3.95	33	1995/8/20/16	3.90	53	1999/4/9/22	4.20
17	1992/2/17/18	3.80	34	1995/10/12/4	4.35	54	1999/4/11/16	4.10
	1992/2/17/20	4.15	35	1996/1/10/4	4.90		1999/4/11/18	4.80

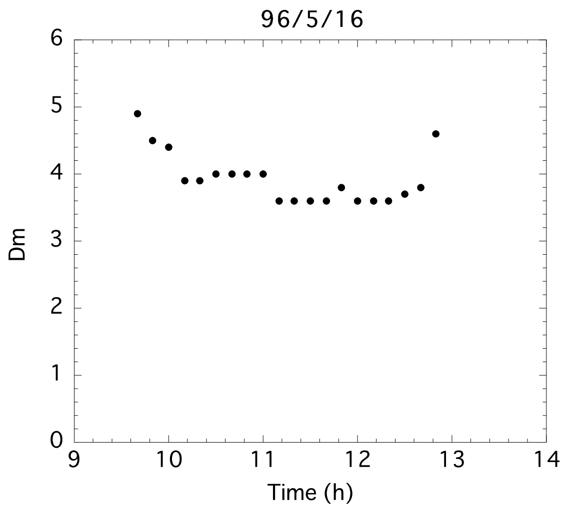

Fig. 4. Variation of dimension with time; May 16th 1996.

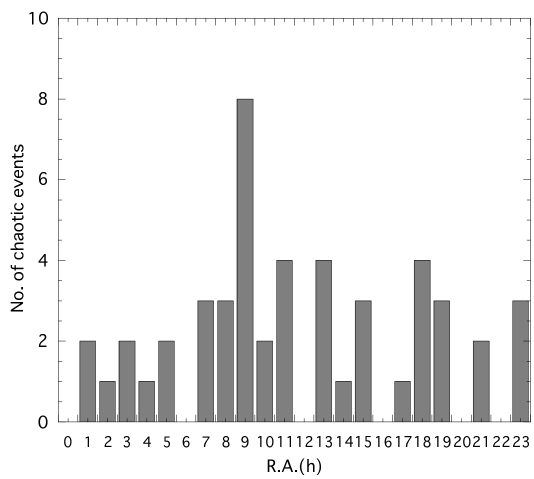

Fig. 5. Right ascensions distribution of the chaotic events.

3 Summary

Fractal analysis was performed to air showers collected by the Mitsuishi Air Shower Array over a period of ten years (January 1989 to April 1999). The average event rate was $1 \sim 2$ event/min. The mean size of the air showers was $\mathrm{N}_e \simeq 1.1\times10^5$. The air showers were required to have accompanying muon(s) with $E_\mu \geq$6 GeV. We found 54 chaotic events with fractal dimensions of $1.2 \sim 5.0$, using the Takens' method, applied to the arrival time of the air showers in the 256 series, which corresponds to approximately four hours. The longest chaotic event lasted for four hours. The arrival directions of the chaotic events in the right ascension peak at 9 hour with a 2.2 σ. In the galactic coordinate, the chaotic events are clustered in the galactic latitude $15^\circ \sim 80^\circ$. The size and zenith angle of the air showers in the chaotic events have no significant difference from the normal air showers.

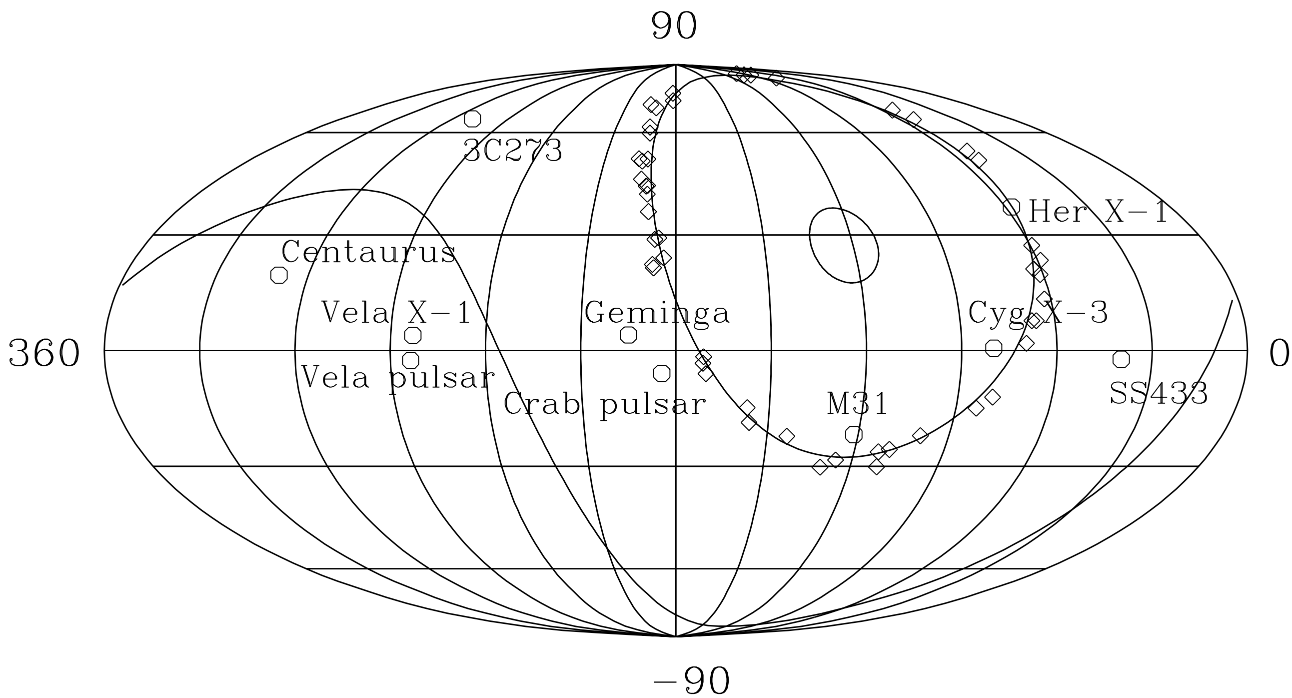

Fig. 6. Arrival directions of the chaotic events (open diamonds) in the galactic coordinate.

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