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Application of the - Forecasting - pronostic method to the evaluation of tot pp at very high energies

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Abstract. The ability of a statistical method to reproduce data of any physical quantity with high precision gives the pattern for the prediction of out of the range data. Prediction procedures of significant physical quantities represent a useful tool in drawing inferences about the behavior of the out-of- the range data and so, about the generator events. Theoretical predictions out of the range of a data set involve a certain degree of uncertainty. With the aim of evaluating the confidence of such predictions it is convenient to determine the uncertainty associated to the predictions of the data. In the context of p-p cross sections at very high energies a great deal of work has been done out of the energy range of accelerators using different models (single-pomeron, dipole pomeron, multiple-diffraction, QCD and so on) to extrapolate accelerator data: predictions are usually compared to cosmic ray data producing a disagreement which explanation has also been widely discussed in the literature. We claim that such comparison requires of a highly confident band of uncertainty for any parametrization model. Here, we present a statistical method that allows to determine the relevant uncertainty. Our preliminary study shows that extrapolations without a trusful determination of error bands may agree with most of the results of Cosmic ray experiments, because their reported experimental errors are very large, but as soon as such a confident determination is made the predicted energy dependence of σ_{pp}^{tot} with its error band delimitates the range of agreement between any prediction model and cosmic ray results.

1 INTRODUCTION

Elastic proton-proton scattering is the most simple process in high-energy hadronic interactions. In this context a wide variety of methos have been developed for its study, ranging from phenomenological approaches up to QCD formal treatements. One of the main tasks in these studies is to determine the total proton cross section σ_{pp}^{tot} in order to reproduce experimental data. Total cross sections are known from accelerator experiments since the 70's, in the energy range $\sqrt{s} \leq 1.8$ TeV (Amaldi et al., 1973), (Amendiolea et al., 1973), within the range $\sqrt{s} = 6 - 40$ TeV from Extensive Air Shower (Gaisser et al., 1987), (Honda et al., 1993) and from Fly's Eye Experiments (Glauber et al., 1970), (Baltrusaitis et al., 1985) at $\sqrt{s} = 30, 40$ TeV. In order to know the σ_{pp}^{tot} energy behavior within the accelerator data range and beyond, it is generally proceeded to fit the available set data and also to predict data by extrapolation to high energies. The pioneer work with this aim was given in (Amaldi et al., 1977) with very good results. Actually the extrapolation is based in purely theoretical, empirical or semi-empirical methods widely accepted (Velasco et al., 1999), because it is a useful tool to draw inferences about the σ_{pp}^{tot} energy behavior. However, if we analize the diverse works existing in the literature about the extrapolation problem, we find in some of them that the uncertainty associated to the σ_{pp}^{tot} prediction points is relatively large or even in other cases (Martini et al., 1977) the corresponding uncertainties associated to their predictions are not reported. Besides, the disagreement existing between the extrapolated data to high energies from accelerator data and cosmic data, widely discussed in the literature, could be better studied, if prediction methods would offer a confident error interval. The main goal of different methods is to minimize the involved errors to obtain highly precise predictions. In this context , popular methods are those derivated from the so called χ^2 technique, based on the minimization of the quadratic sum of data deviations with respect to the employed mathematical model of prediction. In this work we present an alternative prediction method that allows to determine a confident statistical error interval around each of the σ_{pp}^{tot} predicted points. Predictions are developed on the basis of the multiple-diffraction model to estimate σ_{pp}^{tot} in the center of mass range $10 - 40$ TeV ($10^{17} - 10^{18}$ eV $in\ lab$) which covers both LHC and the highest cosmic ray energies.

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2 THE FORECASTING PRONOSTIC METHOD

The validity of any statistical method to predict a given physical quantity out of the range values (extrapolation) depends on its precision to reproduce the employed data (interpolation, namely fitting). A fundamental task of of any prediction method is to minimize the error band of the predicted set of values. In the specific case of σ_{pp}^{tot} what is searched is to obtain a prediction beyond the energy range of the employed data with the minimum of dispersion. Among the several statistical methods to determine a confident interval around each predicted value, we use here the *Forecasting technique* (Mendenhall et al., 1993). This method is based on the *multiple linear regresion* theory and consists in determining a prediction equation for a quantity y (dependent variable), that in turns depends on k independent variables (x_i), that is

$$E(y) = \sum_{i=1}^{k} \beta_i f_i(x_i) \qquad (with\ x_o = 1) \quad (1)$$

Where f_i are arbitrary functions of x_i, and β_i are the fitting constants. In the generalized version the variable x_i may depend on other parameters, i.e., $x_i = x_i(s, t, ..)$

To solve the prediction problem involved in equation (1), we use the matrix formalism. Denoting with Y the matrix of $(n \times 1)$-dimension of the dependent variables and with X the matrix of $[(k+1) \times n]$-dimension of the k independent variables, so that the row $x_{11}x_{12}.....x_{1k}$ determines the value y_1 of the dependent variable, the row $x_{21}x_{22}.....x_{2k}$ determines the value y_2 and so on. The studied variables contained in the matrixes X,Y can be related by the matrix equation Y = BX, then the B matrix of the β_i fitting constant are determined through $B = (X^{'}X)^{-1}XY$, where $X^{'}$ denotes the transposed matrix of X and $(X^{'}X)^{-1}$ denotes the inverse matrix of $X^{'}X$. The $[(k+1) \times 1]$-dimension matrix B contains the values of the constants β_i needed to write in explicit form the prediction equation (1) Mendenhall et al. (1993) . Essentially, this equation minimizes the quadratic sum of the deviations of points (x_{ij}, y_j) with respect to the prediction equation proposed through equation (1). With those matrixes, we determine several statistical estimators,such as the Sum of Square Errors, $SSE = YY^{'} - B^{'}(XY^{'})$ and the Mean Square Errors(s^2), given as $s^2 = SSE/[n-(k+1)]$, where the denominator defines the number of degrees of freedom for errors, given by the amount of β_i - parameters considered in equation (1). Based on these estimators we can then evaluate the uncertainty band with a $100(1-\alpha)\%$ of precision degree as follows: for fitting (prediction within the range of data) by means of

$$INTB = y \pm t_{\alpha/2}^{n-p} \left\{ s^2 A^{'} (X^{'}X)^{-1} \right\} \quad (2)$$

for extrapolation with

$$EXTB = y \pm t_{\alpha/2}^{n-p} \left\{ s^2 \left[1 + A^{'} \left(X^{'}X \right)^{-1} A \right]^{1/2} \right\} \quad (3)$$

Here y denotes the central prediction corresponding to the set data included in $X - matrix$, $t_{\alpha/2}^{n-p}$ denotes the so called *the student's* **t** for the n values of independent variables with p degrees of freedom, and $\alpha/2$ denotes the degree of precision. $INTB(+)$, $EXTB(+)$ and $INTB(-)$, $EXTB(-)$ denote the corresponding Upper and Lower bounds respectively. In our estimations we have used $\alpha/2 = 0.125$ which correspond to 95% of precision. The matrix A denotes the column-matrix of $1 \times (k+1)$ dimension, which elements $\{1, x_1, x_2, \ldots, x_k\}$ correspond to the numerical values of the β_i appearing in equation (1). $A^{'}$ is the transposed matrix of A.

3 THE PARAMETRIZATION MODEL

In order to illustrate the use of our statistical prediction method within the context of p-p interactions, we must determine σ_{pp}^{tot}. There are several alternatives to do it, one of them through the Glauber's multiple difraction theory (Glauber et al., 1959), under the particular approach given in (Martini et al., 1977), which has the advantage that it uses only five parameters: two of them (a^2 and β^2) associated with the form factors G_A and G_B, and three ($C(s)$, $\alpha^2(s)$ and $\lambda(s)$) with the elementary amplitude. Within this frame the so called opacity function $\Omega(b,s)$ is determined through the equation

$$\Omega(b,s) = \int_0^{\infty} qdq\, G^2 Imf(q,s) J_0(q,b) \quad (4)$$

Which explicit expression is

$$\begin{aligned}\Omega(b,s) \quad = \quad & C\{E_1 k_0(\alpha b) + E_2 k_0(\beta b) + E_3 k_{ei}(ab) + \\ & E_4 k_{er}(ab) + b\left[E_5 k_1(\alpha b) + E_6 k_1(\beta b)\right]\} \quad (5)\end{aligned}$$

Where k_0, k_1, k_{ei}, and k_{er} are the modified Bessel functions, and E_1 to E_6 are functions of the free parameters. Therefore, σ_{pp}^{tot} is determined with the following expression:

$$\sigma_{pp}^{tot} = 4\pi \int_0^{\infty} bdb \left\{ 1 - e^{-\Omega(b,s)} cos\left[\lambda \Omega(b,s)\right] \right\} J_0(qb) \quad (6)$$

Where b is the impact parameter, $q^2 = -t$ the four-momentum transfer squared, Jo is the zero-order Bessel function and λ is the undimensional energy-dependent parameter mentioned above. This equation was numerically evaluated, and details are described in (Pérez-Peraza et al., 2000).

Table 1. $C(s)$, $\alpha^{-2}(s)$ and $\lambda(s)$.

$\sqrt{s}$	$C(s)$ (GeV^{-2})	$\alpha^{-2}(s)$ (GeV^{-2})	$\lambda(s)$
13.8	9.970	2.092	-0.094
19.4	10.050	2.128	0.024
23.5	10.250	2.174	0.025
30.7	10.370	2.222	0.053
44.7	10.890	2.299	0.079
52.8	11.150	2.370	0.099
62.5	11.500	2.439	0.121
546	18.500	3.540	0.182
630	19.210	3.590	0.184
1800	27.890	4.170	0.197

4 RESULTS

Table 1 contains the energy $\sqrt{s}$, and the model parameters $C(s)$, $\alpha^{-2}(s)$ and $\lambda(s)$, obtained through the method described in (Pérez-Peraza et al., 2000). The first seven values correspond to the same energies employed in (Martini et al., 1977). A second-order fitting of the values $C(s)$, $\alpha^2(s)$ of Table 1 and a linear fitting of the λ values have been obtained from the next expresions:

$$C(s) = 19.24521 - 2.86114\, ln(s) + 0.22616\, ln(s)^2 \quad (7)$$

$$\alpha^{-2} = 1.8956 - 0.03937\, ln(s) + 0.01301\, ln(s)^2 \quad (8)$$

$$\lambda(s) = 0.01686 + 0.00125\left(1 - e^{-ln(s/400)/0.18549}\right) + 0.19775\left(1 - e^{-ln(s/400)/3.74642}\right) \quad (9)$$

Using the procedure described through equations (1) - (3),

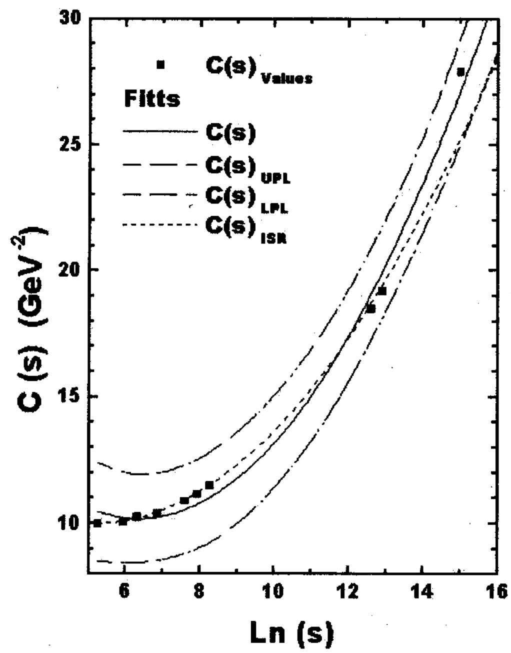

Fig. 1. Fitting of $C(s)$ with upper and lower bounds, including all available accelerator data (black squares) and employing only the ISR accelerator data (dashed curve).

we have determined the Upper and Lower bounds for $C(s)$, $\alpha^{-2}(s)$ and $\lambda(s)$ at each one of the energy values. The results are shown through figures (1) - (3). The specific determination of the uncertainty associated with σ_{pp}^{tot} values was done by the previous evaluation of the corresponding Upper and Lower bounds of the free parameters (C, α, λ). The obtained results are shown through figures (4) and (5).

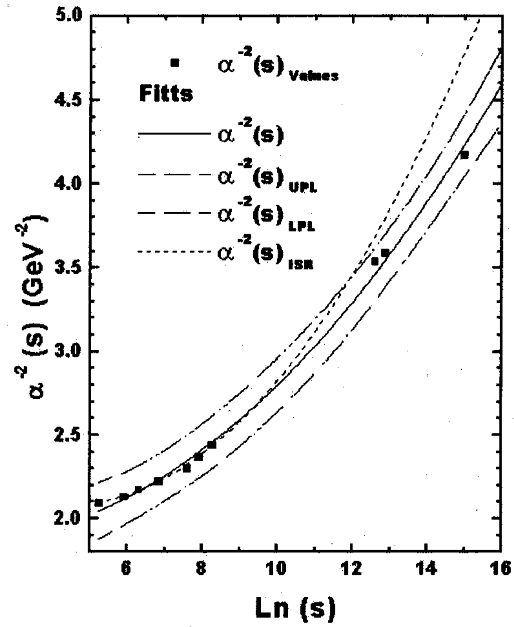

Fig. 2. Fitting of $\alpha^{-2}(s)$ with upper and lower bounds including all available accelerator data (black squares) and employing only the ISR accelerator data (dashed curve).

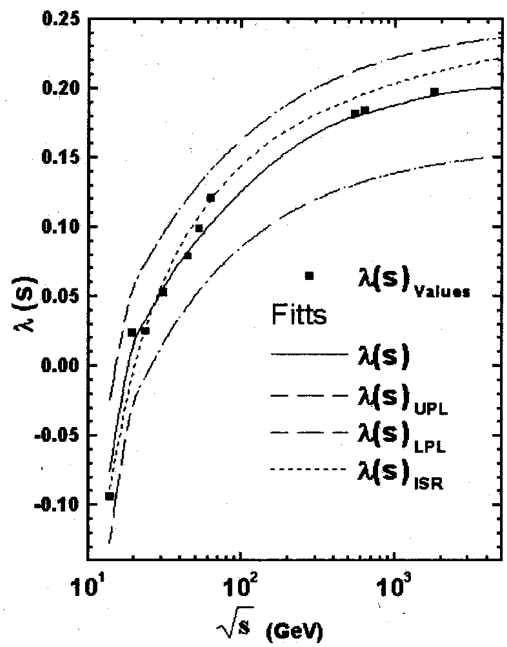

Fig. 3. Fitting of $\lambda(s)$ with upper and lower bounds including all available accelerator data (black squares) and employing only the ISR accelerator data (dashed curve).

5 DISCUSSION AND CONCLUSIONS

The values of the free parameters of the model and their corresponding fittings obtained in this preliminary approach are not yet the best ones, however, the analysis of figures (1) – (5) shows that as data becomes more scarse, as it happens as energy increases, the uncertainty interval becomes wider, which indicates an information lost in the predicted values; obviously, this lost is reflected on the σ_{pp}^{tot} predicted

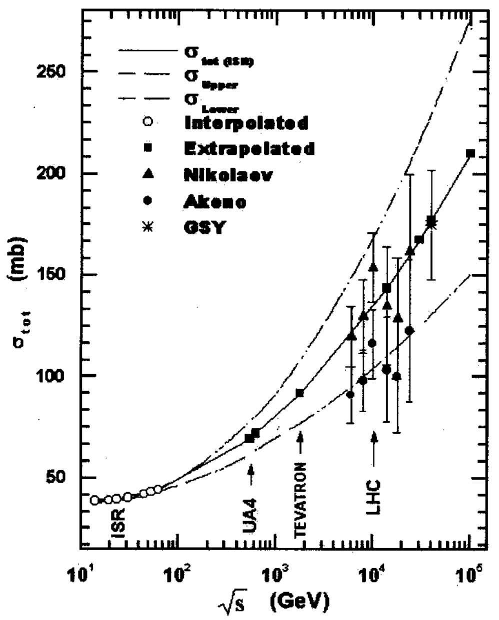

Fig. 4. Interpolation and extrapolation of σ_{tot}^{pp} values (black line) with upper and lower bounds (dashed curves), employing only the ISR accelerator data.

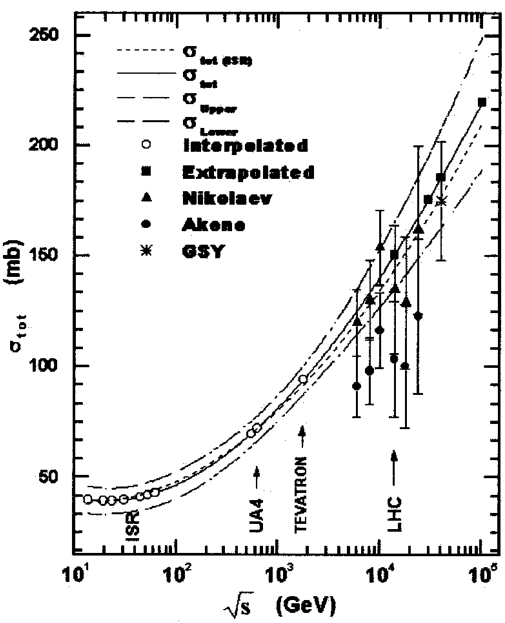

Fig. 5. Interpolation and extrapolation of σ_{tot}^{pp} values (black line) with upper and lower bounds (dashed curves), employing all available accelerator data. Dot-dashed line is the central line of Fig. (4).

values. We can observe in figures $(1) - (3)$ that the difference between both curves, that limited to the ISR accelerator data given in (Martini et al., 1977)(hereafter M & M) and that including 546, 630 and 1800 GeV data is increased as the energy is increased. Nevertheless, the uncertainty band of our study gives consistency to both works. In figure (4) we can note that as energy is increased the uncertainty associated to the predicted σ_{pp}^{tot} values also is increased; no difference is seen between the predicted values of the interpolation of M &M and the prediction of our interpolation (the solid line), because we are dealing with the same seven values of the ISR accelerator data. It is also notorious in figure (4) that the uncertainty is very large at high energies. However, if more points are added to the prediction process (e.g. at energies 0.546, 0.630 and 1.8 TeV), the uncertainty at high energies is decreased as it can be seen in figure (5), where it is shown the interpolation employing all available accelerator data (white boxes) and extrapolations (black squares); the dashed line corresponds to the central curve of figure (4) when only the ISR data is considered.

Though this is a preliminary study and fittings of the free parameters can be improved, we conclude that the employement of a trusful statistical method to delimitate highly confident uncertainty bands leads to obtain highly precise extrapolation results. The use of the *Forecasting method* in the specific case of σ_{pp}^{tot}, limitates the high energy values of σ_{pp}^{tot} to a narrow band which seems to be consistent only with the analysis of (Nikolaev, 1993), obtained from Cosmic Ray experiments. A relevant discussion about the implication of trusty σ_{pp}^{tot}, is given in (Pérez-Peraza et al., 2000).

References

Amaldi, U. et al. Phys. Lett. B 44, 11 ,1973.

Amaldi, U. et al. Phys. Lett. B 66, 390, 1977. Amendiolea, S.R. et al. Phys. Lett. B 44,119, 1973.

Baltrusaitis, R.M. et al. Proc. 19^{th} ICRC, La Jolla, 1985.

Gaisser, T.K., Sukhadme, U.P. and Yodt, G.B. Phys. Rev. D 36, 1350, 1987.

Glauber, R.J. In Lectures in Theoretical Physics, edited by W.E. Briten et al. Interscience, New York, 1959

Glauber, R.J. and Matthiae, G. Nucl. Phys. B 21, 135, 1970.

Honda, M. et al. Phys. Rev. Lett. 70, 525, 1993.

Martini, A.F. and Menon, M.J. Phys. Rev. D 56, 4338, 1997.

Mendenhall, W. and Cincich, T. A second Course in Statistic, Prentice Hall, 1993.

Nikolaev, N.N., Phys. Rev. D48, R1904, 1993.

Pérez-Peraza, J. Velasco, J., Gallegos-Cruz, A. Alvarez-Madrigal, M., Faus-Golfe, A. and Sánchez-Hertz, A. Parametrization of Hadronic Cross-section in the range $10^{-2} - 10^{2}$ TeV, in AIP Conf. Proceedings No. 566 (International Workshop on Observing Ultra High Energy Cosmic Rays from Space and Earth), p. 326-334, 2000. Also in Preprint arXiv:hep-ph/0011167 of Los Alamos National Laboratory, Nov. 13, 2000.

Velasco, J., Pérez-Peraza, J., Gallegos-Cruz, A., Alvarez-Madrigal, M., Faus-Golfe, A. and Sánchez-Hertz, A., Proc. 26^{th} ICRC, Utha, 1, 198, 1999.