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Electromagnetic energy loss for muons and taus at high energy

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Abstract. We present a new evaluation of charged lepton energy loss via photonuclear interactions. The evaluation relies on HERA results for real and virtual photon interactions with nucleons and a phenomenological treatment of nuclear shadowing. Implications for high energy μ's and τ's are discussed. We extend our results to more massive charged particles and discuss the case of relativistic magnetic monopoles.

1 Introduction

Neutrino telescopes have the potential for detecting distant sources of high energy neutrinos, for example, from Active Galactic Nuclei and Gamma Ray Bursters(Gaisser, Halzen and Stanev, 1995). Muons produced by neutrino charged current interactions with nuclei are the main signal of muon neutrinos. In addition, muons are produced in cosmic ray induced extensive air showers. Measuring the muon multiplicity can help determine the composition of the primary cosmic ray. At high energies, the muon flux should reflect the onset of charm production in the atmosphere. Thus, a good understanding of muon energy loss at high energies is an essential ingredient for neutrino astronomy and high energy cosmic ray physics.

Recent SuperK measurements of atmospheric neutrinos strongly suggest $\nu_\mu \rightarrow \nu_\tau$ oscillations with large mixing angle(Fukuda et al., 1998-2000). For astrophysical sources, the same oscillation parameters result in conversion of about half of all muon neutrinos into tau neutrinos, so production and propagation of τ leptons is an inescapable component of neutrino astronomy. The τ decay length at high energy, $l_\tau = 50 E_\tau/\mathrm{PeV}$ m, is long enough that dE/dX for τ's must also be considered.

The energy loss by muons is often written as

$$\frac{dE}{dX} = \alpha + \beta E, \tag{1}$$

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where α and β are slowly varying functions of energy, α accounts for ionization, and β includes contributions from bremstrahlung β_b, pair production β_p, and photonuclear reactions β_γ. All three are discussed in the standard work by Lohman, Kopp and Voss (1985). The brehmstrahlung and pair production contributions are on solid footing, but the photonuclear term β_γ relies on phenomenological extensions to laboratory data(Bezrukov and Bugaev, 1981). Given the improvements in understanding hadronic physics in general and the wealth of new experimental data on photonuclear reactions from HERA, it is appropriate to revisit β_γ. In Section 2 we present a summary of our recent calculation(Dutta et al., 2001), but first we discuss some more general considerations of photonuclear reactions.

In general, contributions to β from a lepton scattering process σ_l may be written as an integral over the fractional energy lost per scattering event,

$$\beta = \int y \frac{d\sigma_l}{dy} dy. \tag{2}$$

where, $y = \frac{\Delta E}{E}$. The effective photon approximation is useful for estimating the cross section of such reactions when the process is mediated by virtual photons. This technique relies on the observation that a boosted Coulomb field is primarily transverse and may be approximated by an equivalent photon distribution, $n_{eff}(\omega) \sim Z^2\alpha/\omega$, where $\alpha = 1/137$ and Z is the charge of the incident particle. The cross-section for reactions induced by relativistic charged particles can then be approximated as

$$\frac{d\sigma_l}{d\omega} \approx n_{eff}(\omega)\sigma_\gamma(\omega), \tag{3}$$

where σ_γ is the cross-section for the equivalent process involving real photons of energy ω. Since the equivalent photon energy is transferred to the target, one may use $\omega = yE$, and in this approximation

$$\beta_\gamma \approx Z^2\alpha \int^{y_{max}} \sigma_\gamma(yE) dy. \tag{4}$$

The raw upper limit is $y_{max} = 1 - m_l/E \approx 1$, but one must also understand that the treatment of the virtual photon as "real" is only valid if $\omega < \Lambda\gamma$, where Λ is the mass scale describing the reaction products and $\gamma = E/m_l$. Crudely, these conditions may be summarized by $y_{max} = \mathrm{Min}(1, \Lambda/m_l)$. For the case of photoproduction, $\Lambda \sim \Lambda_{QCD}$, and $\sigma_{\gamma N} \approx 100\mu\mathrm{b}$ and grows slowly with energy.

For the muon we may take $y_{max} = 1$ and estimate $\beta_\gamma \approx 1\mu\mathrm{b}$ per nucleon. For significantly heavier particles, e.g. τ, we expect a suppresion by Λ_{QCD}/m_l. Of course, this is all very rough. The kinematic constraint on y occurs at the edge of where virtual photons may be treated as real. However, a sharp cutoff is not appropriate since this region dominates the y integral. To obtain an accurate result requires a more detailed formalism and that is the content of the next section.

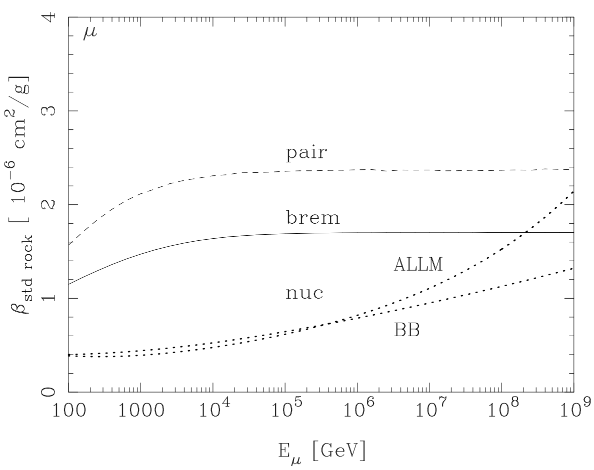

Fig. 1. Contributions to Muon energy loss.

2 Photonuclear formalism

Photonuclear $lN \to lX$ processes contribute to β through virtual photon exchange. As noted above, calculations of β emphasize the cross-section at large y, a region of phase space which includes virtual photons with a range of Q^2. As such, we treat $lN \to lX$ using the deep-inelastic scattering formalism and a nucleon structure function F_2 consistent with data over the full range of Q^2. In this approach, both soft physics at low Q^2 and hard perturbative physics at high Q^2 are incorporated.

The cross section of interest is $d\sigma/dy$, which we write as

$$\frac{d\sigma}{dy} = \int -\frac{x}{y}\frac{d\sigma(x,Q^2)}{dQ^2 dx} dQ^2, \tag{5}$$

and make use of(Badelek and Kwieciński, 1996)

$$\begin{aligned}\frac{d\sigma(x,Q^2)}{dQ^2dx} &= \frac{4\pi\alpha^2}{Q^4}\frac{F_2(x,Q^2)}{x}\left[1 - y - \frac{Mxy}{2E}\right. \\ &\quad \left. + \left(1 - \frac{2m_l^2}{Q^2}\right)\frac{y^2(1+4M^2x^2/Q^2)}{2(1+R(x,Q^2))}\right] .\end{aligned} \tag{6}$$

In these expressions, m_l is the lepton mass, M is the nucleon mass, and the standard kinematic variables for $l(k)N(p) \to l(k')X$ scattering are: $q^2 = (k-k')^2 = -Q^2$, $x = \frac{Q^2}{2p\cdot q}$, and $y = \frac{p\cdot q}{p\cdot k}$. We use the following limits of integration:

$$Q^2_{\min} \leq Q^2 \leq 2MEy - ((M+m_\pi)^2 - M^2) \tag{7}$$
$$y_{\min} \leq y \leq 1 - m_l/E \,, \tag{8}$$

where $Q^2_{\min} \simeq m_l^2 y^2/(1-y)$ and $y_{\min} \simeq ((M+m_\pi)^2 - M^2)/(2ME)$.

The quantity R in Eq. 6 is defined by F_L and F_1,

$$R(x,Q^2) = \frac{F_L(x,Q^2)}{2xF_1(x,Q^2)} \,. \tag{9}$$

The structure function $F_L(x,Q^2)$ is proportional to the longitudinal photon-nucleon cross section. In the $Q^2 \to 0$ limit, $F_L \sim Q^4$ while $F_1 \to Q^2$, so $R \to 0$. We have used $R(x,Q^2)$ modeled by Badelek, Kwieciński and Staśto (1997) for $10^{-7} < x < 0.1$ and 0.01 GeV^2 $< Q^2 <$ 50 GeV^2. For $x > 0.1$, the parameterization of Whitlow et al. (1990) is used. There is no evidence for target dependence of R. The value of β_γ for muons evaluated with these R differs by only a few percent from that calculated with $R = 0$. Consequently, in what follows, we explicitly set $R = 0$.

The nuclear structure function F_2^A depends on the particular target, denoted by mass number A. Data from experiments at CERN (NMC collaboration, 1991-1995) and Fermilab(E665 Collaboration, 1992-1995) show a systematic reduction of nuclear structure function $F_2^A(x,Q^2)$ with respect to that from A free nucleons $AF_2^N(x,Q^2)$. We define the shadowing ratio by

$$a(A,x,Q^2) = \frac{F_2^A(x,Q^2)}{AF_2^N(x,Q^2)} \,. \tag{10}$$

Theory and data suggest that the suppression is weakly dependent on Q^2 in the range of interest for the photonuclear cross section. Accordingly, we take a Q^2 independent function of x and A consistent with the Fermilab E665:

$$a(A,x) = \begin{cases} A^{-0.1} & x < 0.0014 \,, \\ A^{0.069\log_{10}x + 0.097} & 0.0014 < x < 0.04 \,, \\ 1 & 0.04 < x \,. \end{cases} \tag{11}$$

The structure function F_2^A is then approximated by

$$\begin{aligned} F_2^A &= a(A,x)\frac{A}{2}(F_2^p + F_2^n) \\ &= a(A,x)\frac{A}{2}(1+P(x))F_2^p \,, \end{aligned} \tag{12}$$

assuming $Z = A/2$. Here $P(x) = 1 - 1.85x + 2.45x^2 - 2.35x^3 + x^4$ describes the ratio F_2^n/F_2^p, parameterized by the BCDMS experiment(Benvenuti et al., 1990).

The quantity $F_2^p(x,Q^2)$ is extracted in a variety of experiments in a range of $0 < Q^2 < 5000$ GeV^2 and $5\times10^{-6} < x < 1$, though kinematic limits restrict the range of Q^2 and x in any given experiment. The differential cross section

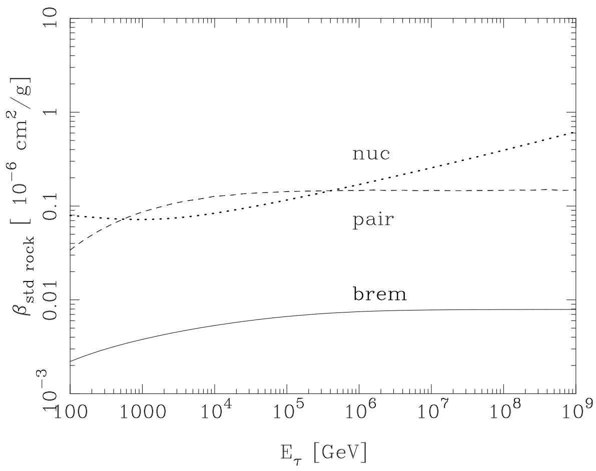

Fig. 2. Contributions to Tau energy loss.

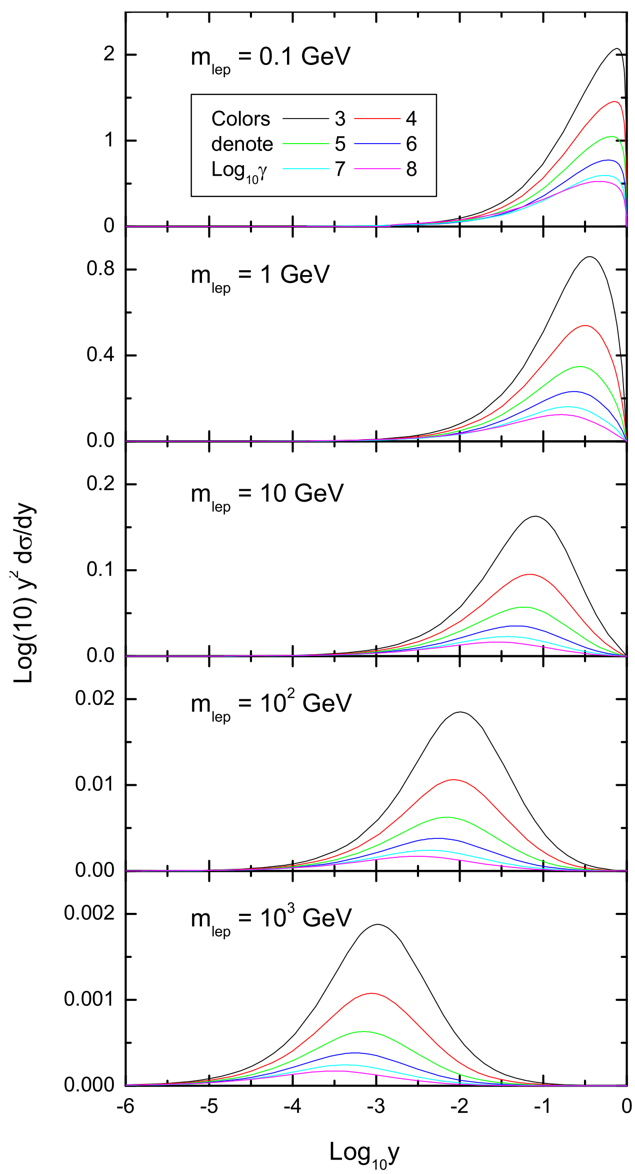

Fig. 3. Contributions to β_γ as a function of the fraction of energy transferred to hadrons y. Panels are labeled by lepton mass. Note that the vertical scale is different for different panels. For $m_l \gg \Lambda_{QCD}$, β_γ scales as $1/m_l$ as does the value of y which makes the dominant contribution to β_γ. Within each panel the individual curves are for leptons with different velocities, i.e. relativistic γ factors of $10^3 - 10^8$. As the lepton energy increases, the equivalent photon energy for a given y also increases. The contribution to σ increases as well, reflecting the rise in $\sigma_{\gamma N}$

must be integrated from $Q^2 = 0$, where the perturbative QCD description of F_2 is not valid, to values of Q^2 where QCD is valid. Consequently, a parameterization of F_2^p consistent with all the data is most useful for our purposes. The parameterization of F_2^p used here is the one by Abramowicz, Levin, Levy and Maor (ALLM)(Abramowicz and Levy, 1997) based on data available from the pre-HERA era as well as H1 and ZEUS data published through 1997.

3 Results

The result of applying the formalism of the previous section to a muon propagating through rock is illustrated in Fig. 1. β_γ is shown as the dotted line labeled ALLM. We also show the contributions to β from bremstrahlung and pair production, as well as the photonuclear contribution from LKV based on the Bezrukov-Bugaev treatment. Although our result exceeds that from LKV at high energies, even at 10^{18} eV our treatment yields only a 10-15% correction to the total β. At higher energy, though, it is clear that photonuclear reactions will start to dominate dE/dX for muons. For lighter materials, e.g. water/ice/air, photonuclear reactions will be somewhat more important.

Our result for τ's is shown in Fig. 2. There is no previous β_γ result to compare to. Shown on a log-log plot, photonuclear processes dominate dE/dX for $E > 10^{15}$ eV. Bremstrahlung, which scales as $1/m_l^2$, is quite unimportant. Pair production, however, scales as just one power of $1/m_l$; as does photoproduction in the limit of large lepton mass. It is a bit puzzling, then, that β_γ is more important for τ leptons, whereas β_p wins out for μ's.

Indeed, pair production can be treated in the effective photon approximation similar to the discussion in Section 1 with the result that $\beta_p \sim \int^{y_p} \sigma_p$, with $y_p \approx m_e/m_l$ and $\sigma_p \sim Z^2\alpha^3/m_e^2$. The key is to notice that both μ and τ are heavier than the electron, so the ratio of $y_{p,\tau}/y_{p,\mu}$ is truly m_μ/m_τ. On the other hand for photonuclear reactions, m_μ is in fact less than Λ_{QCD}, so the relevant y_{max} is $y_{\gamma,\mu} = 1$. It follows that $\beta_{\gamma,\tau}/\beta_{\gamma,\mu} \approx y_{\gamma,\tau}/y_{\gamma,\mu} \approx \Lambda_{QCD}/m_\tau$. In going from m_μ to m_τ photonuclear gains relative to pair production by roughly Λ_{QCD}/m_μ, and hence the dominant contribution to β switches.

Another aspect of the comparison between β_p and β_γ is their high energy behavior. β_γ continues to grow with energy, reflecting the underlying growth of $\sigma_{\gamma N}$. β_p also grows, reflecting the logarithmic growth in the underlying $\gamma N \to$

Ne^+e^- cross section, until the effective photon energy is large enough that the target nuclear field is screened by atomic electrons, i.e. at $\omega = m_e/\alpha^2$. For pair production by μ's, $\omega_{max} \sim m_e(E/m_\mu)$, so β_p flattens out at an energy $E \approx m_\mu/\alpha^2$. The result is correspondingly higher for τ's.

These arguments are illustrated in Fig. 3, which shows contributions to β_γ as a function of y for a range of hypothetical lepton masses. For $m_l = 100$ MeV ($< \Lambda$), one can see clearly that the contributing region of y is scrunched up against $y = 1$. For large masses, the $1/m_l$ scaling in y_{max} is apparent: the peak of the contributions closely follows Λ/m_l, with an effective value of $\Lambda \approx 1$ GeV. With a mass of 1.777 GeV the τ lepton is in the transition region. If β_γ scaled simply with mass one would expect $\beta_{\gamma,\mu}/\beta_{\gamma,\tau} \approx 17$, instead it is only about 3. One can also see the growth in β_γ with the energy of the incident lepton.

3.1 Monte Carlo estimates of lepton range

Fig. 3 also provides some insight into the practical problem of developing monte carlo programs to model dE/dX. For muon propagation, it is common practice to treat pair production as a continuous process, but divide bremstrahlung into hard and soft regimes divided by some cutoff y_{cut}(Lipari and Stanev, 1991). The soft regime is treated as a continuous process, but for $y > y_{cut}$ the interactions are treated stochastically. From Fig. 3 we see that $\beta_{\gamma,\mu}$ is dominated by $y \sim 1$. As such, a stochastic treatment is required for monte carlo studies of μ range. Similarly, although $\beta_{\gamma,\tau}$ is slightly softer it is still dominated by $y > 0.1$ and a stochastic approach is advised. Fig. 3 also serves to justify the soft treatment of pair production by muons, for which case $y_{max} \approx m_e/m_\mu$. This is a value similar to that shown in the $m_l = 100$ GeV panel for contributions to β_γ, where it is easily seen that hard events are not a significant component. For τ's, pair production would be even softer.

3.2 Magnetic monopoles

Integrating under the curves of Fig. 3 yields $\beta_\gamma(E)$ for different hypothetical lepton masses, as depicted in Fig. 4. One may apply these results to any charged particle by a simple change in Z^2. For example, relativistic low mass magnetic monopoles are a possible component of the cosmic ray spectrum. Their detectibility depends on dE/dX both to penetrate through the Earth, but also to leave a trail in potential detectors(Wick, Kephart and Weiler, 2001). From Figs. 3 and 4 one may approximate

$$\beta_{\gamma,M}(E) \approx 4700\frac{15m_\mu}{m_M}\beta_{\gamma,\mu}(E'), \tag{13}$$

where $E' = Em_\mu/m_M$. The numerical factors come from our calculation of β_γ for large lepton mass(15) and from the large effective charge of the monopole(4700). The scaling of E' causes $\beta_{\gamma,\mu}$ to be evaluated at the same velocity as for the monopole, and thus the same equivalent photon energy. Eq. 13 must be considered as approximate since the photon field associated with the monopole has the opposite polarization to that used in the photonuclear cross-section calculation described above. We are not aware of a formalism that properly allows us to implement a monopole photon vertex in the QED sense.

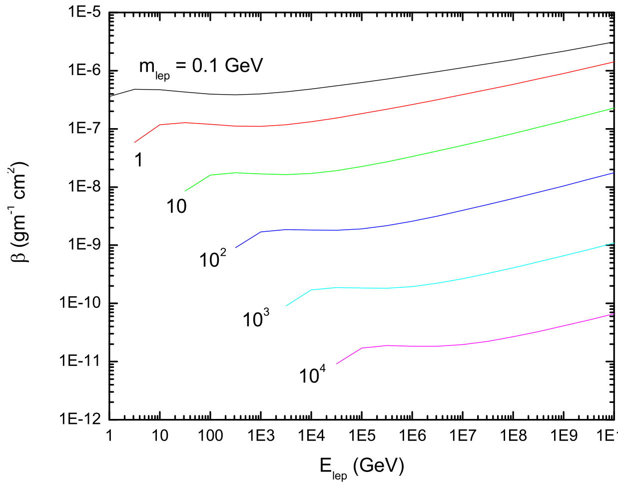

Fig. 4. β_γ for different mass leptons, as a function of lepton energy. Curves are labled by lepton mass in GeV.

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