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An easy way to calculate γ-ray line emission

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Abstract. The main difficulty in the calculation of γ-ray line emission and the corresponding data interpretation is that the spectrum of the energetic particles (EPs) which actually interact in the ISM is not the same as the source spectrum, due to energy dependent energy losses and escape times. Here, we show that it is possible to simplify considerably the calculations by working out the propagation of the particles once and for all, using a standard propagation model. This is achieved through a mathematical transformation which introduces the total γ-ray yields of individual particles as a function of their initial energy. We provide these quantities which enable anyone to calculate the γ-ray production induced by EPs with any spectrum and any composition, without having to take particle transport into account and calculate the propagated spectrum oneself.

1 Introduction

While γ-ray line astronomy is currently experiencing a considerable development, data interpretation in this field remains rather tricky, even at the phenomenological level, because the production of a γ-ray line through nuclear deexcitation is an indirect process. First, one has to identify an *accelerator*, where particles are brought to energies above the nuclear excitation thresholds with a given energy spectrum; then one has to work out the interactions of the distribution of energetic particles (EPs) with the ambient medium, according to our knowledge of the nuclear excitation cross-sections. Usually people working on particle acceleration are not the same as those working on γ-ray line astronomy or phenomenology. The latter use the output of the former's calculations, namely the distribution and fluxes of the EPs just leaving the accelerator, as an input for γ-ray line modelling.

Unfortunately, the particles producing the γ-rays through nuclear excitation do not have the same energy spectrum as those who leave the acceleration site, because some time elapses in-between. The particles are the same, of course, but as they travel from their source to their target, they experience some energy losses, mostly through Coulombian interactions. Since the energy losses experienced by a given particle depend on its nuclear species (proton, carbon or iron nucleus) and of its energy, both the EP composition and the shape of the EP spectrum change as the particles propagate through the interstellar medium (ISM). In order to calculate accurately the γ-ray line production induced by a given population of EPs, one has to take these effects into account and integrate the excitation cross-sections over the *propagated spectrum* rather than over the *source spectrum*, coming out of the astrophysical accelerator.

In this paper, we propose to make easier the calculation of γ-ray line emission induced by a given EP distribution by working out *once and for all* the integrated effect of energy losses on individual particles injected at any energy in the ISM. We will justify the fact that the 'propagation step' in a standard γ-ray line emission calculation can be 'factorized out' and calculated separately, independently of the EP source spectrum and composition. As a result, we shall obtain the absolute γ-ray yields of energetic nuclei as a function of their initial energy, from which the γ-ray line emission induced by EPs of any spectrum and composition can be straightforwardly calculated. In addition to making γ-ray line calculations much easier, these absolute yields (or particle efficiencies for γ-ray line production) considerably help phenomenological interpretation of the observational data, as these yields only need to be convolved with the EP *source* spectra, rather than *propagated* ones.

2 Gamma-ray line emission rate

The γ-ray line emission rate, in $\mathrm{ph/cm^3/s}$, induced in the ISM is obtained by integrating the nuclear excitation cross-sections over the local flux of EPs, and summing the contributions to each γ-ray line of all the nuclear reactions $i + j \rightarrow k$, where i represents the projectile, j the target nucleus, and

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k the excited nucleus produced, or equivalently the 'photon species' emitted through nuclear de-excitation:

$$\frac{\mathrm{d}N_k}{\mathrm{d}t} = \sum_{i,j} \int_0^{+\infty} N_i(E)\,[n_j \sigma_{i,j;k}(E) v(E)]\,\mathrm{d}E. \tag{1}$$

Here $N_i(E)$ is the spectral density of the projectiles i, in $\mathrm{cm}^{-3}(\mathrm{MeV/n})^{-1}$, $v(E)$ is the velocity of the projectiles (independent of the nuclear species, i, if the energy E is expressed in MeV/n), n_j is the number density of the target nuclear species j, and $\sigma_{i,j;k}$ is the cross section for the reaction $i + j \rightarrow k$. For instance, k might represent photons from the ^{12}C de-excitation line at 4.44 MeV, produced by the reaction $p + {}^{12}\mathrm{C} \longrightarrow {}^{12}\mathrm{C}^*$ or $\alpha + {}^{16}\mathrm{O} \longrightarrow {}^{12}\mathrm{C}^*$.

As explained in Section 1, the EP distribution function, $N_i(E)$, to be used in the above integral is *not* the source function, but is derived from it by solving the following propagation equation (steady state, one zone model):

$$\frac{\partial}{\partial E}(\dot{E}_i(E) N_i(E)) = Q_i(E) - \frac{N_i(E)}{\tau_i^{\mathrm{tot}}(E)}, \tag{2}$$

where the injection function, $Q_i(E)$, gives the number of particles of nuclear species i injected in the ISM at energy E (in $\mathrm{cm}^{-3}\mathrm{s}^{-1}(\mathrm{MeV/n})^{-1}$), $\dot{E}_i(E)$ is the energy loss rate, in $(\mathrm{MeV/n})\,\mathrm{s}^{-1}$, of nuclei of species i in the considered propagation medium, and $\tau_i^{\mathrm{tot}}(E)$ is the total 'loss time' taking into account nuclear destruction and particle escape out of the interaction region.

The formal solution of Eq. (2) reads:

$$N_i(E) = \frac{1}{|\dot{E}_i(E)|} \int_E^{+\infty} Q_i(E_{\mathrm{in}}) \mathcal{P}_i(E_{\mathrm{in}}, E) \mathrm{d}E_{\mathrm{in}}, \tag{3}$$

where $\mathcal{P}_i(E_{\mathrm{in}}, E)$ can be interpreted as the survival probability (against destruction and escape), in the propagation medium considered, of a particle injected at energy E_{in} and losing energy down to energy E. It obviously depends on the total loss time at each energy between E_{in} and E and the energy loss function, and can be expressed as follows (see Parizot and Lehoucq, 1999, for more details and the treatment of the general, non stationary and non homogeneous case):

$$\mathcal{P}_i(E_{\mathrm{in}}, E) = \exp\left(- \int_{E_{\mathrm{in}}}^{E} \frac{\mathrm{d}E'}{\dot{E}_i(E') \tau_i^{\mathrm{tot}}(E')} \right). \tag{4}$$

3 Gamma-ray yields of individual EPs

Combining Eqs. (1) and (3), one can rewrite the γ-ray emission rate as follows (specializing to one nuclear reaction for illustration):

$$\frac{\mathrm{d}N_\gamma}{\mathrm{d}t} = \int_0^{+\infty} \mathrm{d}E \int_E^{+\infty} \mathrm{d}E_{\mathrm{in}} \frac{n_0 \sigma(E) v(E)}{|\dot{E}(E)|} Q(E_{\mathrm{in}}) \mathcal{P}(E_{\mathrm{in}}, E), \tag{5}$$

where n_0 is the density of the propagation medium and $Q(E)$ is the EP *source* spectrum.

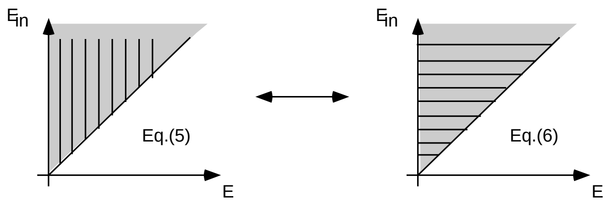

Fig. 1. Graphical demonstration of the equivalence between Eq. (5) and Eq. (6): the shaded area is the integration domain, divided into vertical and horizontal slices, respectively.

Our new approach is based on the rewriting of this expression using a simple mathematical transformation which consists in inverting the order of the two integrations, as shown in Fig. 1:

$$\frac{\mathrm{d}N_\gamma}{\mathrm{d}t} = \int_0^{+\infty} \mathrm{d}E_{\mathrm{in}} \int_0^{E_{\mathrm{in}}} \mathrm{d}E \frac{n_0 \sigma(E) v(E)}{|\dot{E}(E)|} Q(E_{\mathrm{in}}) \mathcal{P}(E_{\mathrm{in}}, E). \tag{6}$$

Getting the source function, $Q(E_{\mathrm{in}})$, out of the integral over E, one then obtains the following expression for the γ-ray emission rate (adding the contribution of all the reactions involved):

$$\frac{\mathrm{d}N_\gamma}{\mathrm{d}t} = \sum_{i,j} \int_0^{+\infty} Q_i(E_{\mathrm{in}}) \alpha_j \mathcal{N}_{i,j;\gamma}(E_{\mathrm{in}}) \mathrm{d}E_{\mathrm{in}}, \tag{7}$$

where $\alpha_j = n_j/n_0$ is the relative abundance of nuclei of species j in the target, and

$$\mathcal{N}_{i,j;\gamma}(E_{\mathrm{in}}) = \int_0^{E_{\mathrm{in}}} \frac{n_0 \sigma_{i,j;\gamma}(E) v(E)}{|\dot{E}(E)|} \mathcal{P}_i(E_{\mathrm{in}}, E) \mathrm{d}E. \tag{8}$$

The physical interpretation of $\mathcal{N}_{i,j;\gamma}(E_{\mathrm{in}})$ is straightforward: it is the number of photons of species γ produced in a target made solely of nuclei of species j, by one projectile of species i injected in the ISM at the energy E_{in}, integrated over its entire life (i.e. from its injection until it has lost so much energy that it is below the nuclear excitation threshold). Note that the lower bound of the integral can be replace by the energy threshold of the cross sections. Now the interesting point is that the absolute photon yields, $\mathcal{N}_{i,j;\gamma}(E_{\mathrm{in}})$, can be calculated from physical quantities alone and is independent of astrophysics: as can be seen from Eqs. (8) and (4), it only depends on the nuclear cross-sections and energy loss rates. These can be calculated or measured once and for all, and so is it for $\mathcal{N}_{i,j;\gamma}(E_{\mathrm{in}})$.

The great advantage of this new formulation is that once the quantities $\mathcal{N}_{i,j;\gamma}(E_{\mathrm{in}})$ have been calculated, the actual γ-ray emission rate in a given astrophysical situation can be derived according to Eq. (7) which gathers all the astrophysical information (namely the EP spectrum and composition, and the target composition), but which is now expressed in terms of the *source spectrum*, rather than the *propagated* one. To better understand the signification of this transformation, it suffices to compare Eqs. (1) and (7). We have replaced the

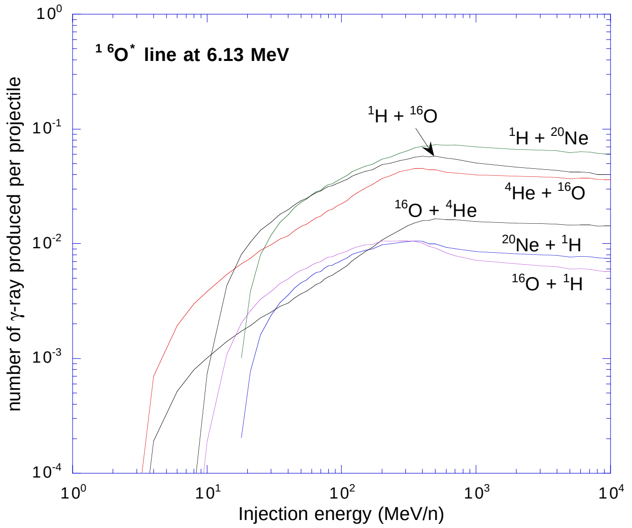

Fig. 2. Absolute photon yields, $\mathcal{N}_{i,j;\gamma}$, in the 6.13 MeV line of ^{16}O through various channels, as a function of the injection energy of the projectile. The latter is the first nucleus appearing in the label, and the target is the second.

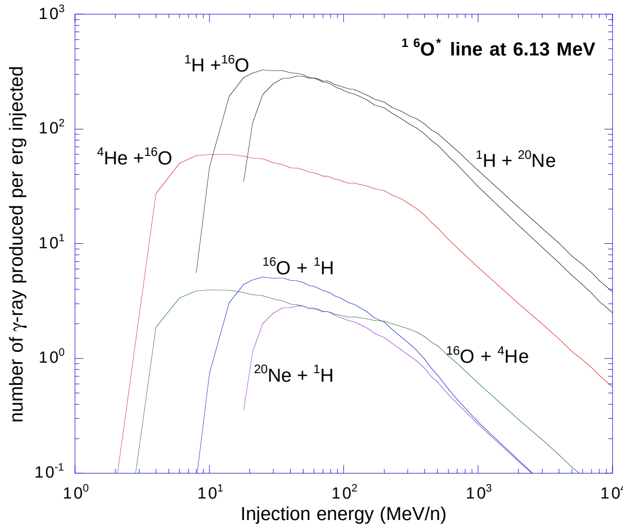

Fig. 3. Gamma-ray production efficiency, in photon/erg, for the 6.13 MeV line of ^{16}O through various channels, as a function of the injection energy of the projectile.

propagated spectral density of the EPs, $N_i(E)$, by their injection function, $Q_i(E)$, and the cross-sections $\sigma_{i,j;\gamma}$ by our absolute photon yields, $\mathcal{N}_{i,j;\gamma}$, which play the role of 'effective cross-sections' (although their physical dimension is different) taking into account the propagation of the EPs in the ambient medium.

Two comments are in order here. First, the above expression giving the photon yields $\mathcal{N}_{i,j;\gamma}(E_{\rm in})$ may seem to depend on the density, n_0, of the propagation medium (e.g. the ISM). This is actually not the case, as the energy loss rate appearing in the denominator is also proportional to this density. However, and this is our second comment, the exact expression of the energy loss rate depends in principle on the composition of the propagation medium, which cannot be taken into account in the present approach. However, it can be shown that the error induced by considering only the interactions with H and He nuclei is negligible as long as the ambient metallicity is lower than a hundred times the solar metallicity. This approximation will thus be adequate for most of the situations of astrophysical interest, even in regions very much enriched in metals by supernova ejecta and/or winds of massive stars. A more detailed discussion of this and other aspects of our calculations and their use can be found in Parizot and Lehoucq (2001).

4 Results and emission rates reconstruction

In Fig. 2, we show the absolute γ-ray yield, $\mathcal{N}_{i,j;\gamma}$, corresponding to the main ^{16}O line at 6.13 MeV, for various projectiles and targets. The physical inputs are: nuclear excitation cross-sections from Ramaty et al. (1979) and Kiener et al. (2001), Coulombian energy loss rates from J. Kiener (private communication) and total inelastic cross-sections from Silberberg and Tsao (1990).

The evolution of $\mathcal{N}_{i,j;\gamma}$ as the injection energy increases can be interpreted in the following way. Photon production begins when $E_{\rm in}$ becomes greater than the reaction threshold. Then it increases sharply as $E_{\rm in}$ passes through the peak of the cross-section, and increases more smoothly afterwards. As long as particle destruction or escape can be neglected, Eq. (8) makes it clear that the number of photons produced is an increasing function of $E_{\rm in}$, the upper bound of the integral. Physically, the particle produces γ-rays all the way as its energy goes down to below the threshold. If it is injected at higher energy, it will produce γ-rays for a longer time, integrating the cross section over a larger energy range. But when $E_{\rm in}$ increases further, there comes a time when the projectile has a large probability of being destroyed (through a nuclear reaction) or escaping from the region under study, *before* its energy drops below the reaction threshold. In this case, the effective energy range over which the cross section is integrated is reduced from below, and the overall γ-ray yield starts to decrease. For large $E_{\rm in}$, the particle never reaches the most efficient energy range corresponding to the peak of the cross-section.

While the decrease of $\mathcal{N}_{i,j;\gamma}$ at high energy is not very steep, it should be realized that the γ-ray production efficiency, measured in numbers of photons produced per erg of projectiles injected, is falling down more quickly, as shown in Fig. 3. This Figure gives a visual representation of the most efficient energy range for an EP to produce a given γ-ray line. It shows that this range starts at higher energy, and extends to even higher energies than the cross-section peak. The curves can be thought of as simple phenomenological tools: a simple look at them gives an idea of the kind of source spectrum and composition required to reproduce any γ-ray line observational data.

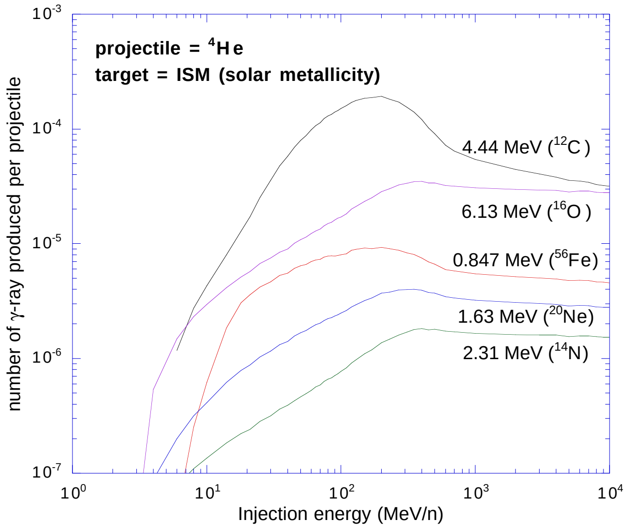

Fig. 4. Gamma-ray yields of a ^{4}He nucleus injected in a medium of solar metallicity, as a function of the injection energy. Contributions to various γ-ray lines are shown.

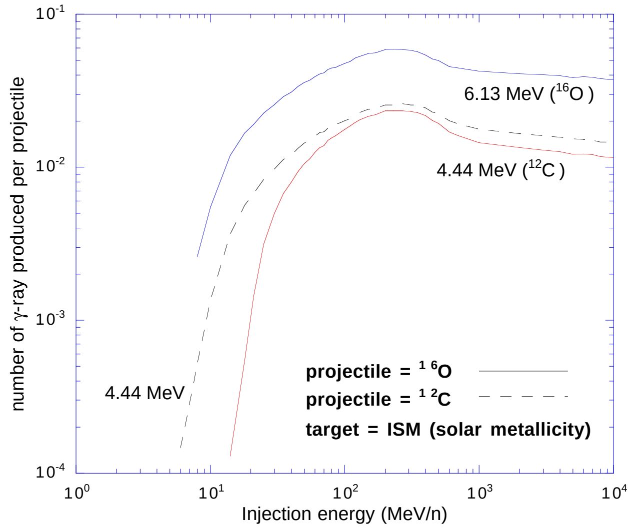

Fig. 5. Same as Fig. 4 for ^{12}C and ^{16}O projectiles.

Conversely, and from a practicle point of view, the quantities $\mathcal{N}_{i,j;\gamma}(E_{\mathrm{in}})$ also allow one to straightforwardly calculate the γ-ray line emission in a given astrophysical situation, once a source spectrum and a target composition is choosen. It suffices to sum the contributions of every contributing reaction, weighted according to the desired chemical abundances of both the source and the target. In other words, one can calculated the γ-ray line emission rate for *any* EP spectrum and composition in *any* medium (except maybe the most extremely metal-rich), without needing to worry about particle propagation and energy losses at all, as intended.

Such a weighting is illustrated in Figs. 4 and 5, where we show the γ-ray yield of He, C and O nuclei in a medium of solar metallicity. In particular, one sees that ^{16}O nuclei are nearly as efficient as ^{12}C nuclei to produce the 4.44 MeV line. Note that although the γ-ray yields of C and O projectiles appear much higher than those of He (or H), these have to be weighted by the relative abundances of the various projectiles among the EPs.

Additional curves for other important lines can be found in Parizot and Lehoucq (2001). Numerical tables and electronic versions of the results are available from the authors upon request.

5 Conclusion

We have shown that standard γ-ray line emission calculations can be simplified by working out once and for all the most difficult step of the calculation, namely the propagation of the energetic particles involving destruction and escape. This is done by calculating the number of photons produced by one projectile of any nuclear species in the ISM, as it slows down from its injection energy to below the nuclear excitation thresholds. These γ-ray yields can then be used to calculate the γ-ray line emission induced by EPs with any spectrum and any composition without having to worry about particle, i.e. by convolving by the EP source spectrum instead of the propagated spectrum, as in the usual approach. This also provides a visual, intuitive tools for γ-ray line phenomenology, making it much easier to interpret the data.

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