

Fractal analysis of time profile of the gamma ray burst registered by BATSE

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Abstract. GRB time profiles of 1300 GRB from 4B revised BATSE catalog were analyzed by fractal dimension method. Fractal index (dimension) D was determined for each GRB. GRB were divided into three groups in according with the T_{90} burst duration: short, middle and long. The fractal index distributions for these GRB sets are presented. Distribution shapes for short, middle and long GRB are not identical. Distributions for shot and middle bursts have at least two maximums each: at $D=1.47\pm 0.02$ and 1.8 ± 0.02 for shot duration and at $D=1.17\pm 0.02$ and $D=1.42\pm 0.02$ for middle duration of GRB. There is only one wide maximum at $D=1.48\pm 0.02$ in long duration population. While the majority of analyzed GRB probably belong to one class with fractal indexes $D\approx 1.5$, there are two additional statistically reliable subclasses of the bursts: with $D=1.8$ for short bursts and with $D=1.17$ for middle bursts.

1 Introduction

Burst time profiles vary in their characteristics so much that at the first glance it is seen that every burst is individual. Some bursts have one peak in temporal profile, while others exhibit that the count rate rises and falls many times. Nevertheless it is possible to find some common features and pick out the follow time profile types: symmetric, smooth, chaotic, complex, burst with fast rise and exponential decay of count rate, multi-pulsed and quasi-periodical (Kouveliotou, 1964). Probably, the burst duration and type of time profile would be defined by physical processes in which gamma-emission was formed. The existing separation of GRB over the time profiles types is empirical without any quantitative criteria that leads to difficulties in profile analysis.

In according with the energy output duration the GRB can be divided into three groups: short, long and middle with mean duration $\langle T_{90} \rangle \sim 0.7\text{sec}$, $\langle T_{90} \rangle \sim 20\text{sec}$ and $\langle T_{90} \rangle \sim 3\text{sec}$ (Belousova and Rozental, 1998; Tavani, 1998).

In each group there are bursts with any mentioned time profile type. This fact allows us to suppose that there are some additional burst subtypes in each group, that can be revealed by complex analysis on the basis of the burst duration and time profile.

2 Quantity criteria of event time profile definition

Usually for quantity definition of some separation criteria for function $f(x)$ the representation of this function in form of complete and orthogonal basis is used. There are infinitely many families of functions that form complete and orthogonal basis for real functions (burst time profiles are real functions), for example, basis of Dirac delta functions at each point x with amplitude $f(x)$ or Fourier basis.

The Fourier basis is very effective for several reasons. Firstly, the frequencies of basis functions have a clear physical interpretation and, secondary, for periodic functions their Fourier representation is much more compact than the direct presentation of the function. For non-periodic functions, Fourier analysis is often not effective because it gives infinite set of basis functions with its amplitudes and frequencies. Authors do not know any result of GRB profile analysis on the basis of the Fourier transformation.

For analysis of non-periodic functions the wavelet analysis can be used (Abarbane, 1991). Walker and Schaefer (2000) performed the wavelet analysis for 20 GRB using of Haar basis. They revealed fast variations with $0.256\text{msec} < \tau_{\min} < 33\text{msec}$. It seems that there are three GRB groups with $\tau_{\min} < 2.0\text{msec}$, $\tau_{\min} \sim 4.1\text{msec}$ and $\tau_{\min} > 8.2\text{msec}$, but this analysis of set of 20 GRB only is not statistical sufficient to find any significant correlation between burst duration T_{90} and τ_{\min} , although, some correlation probably exists.

In this research we used fractal analysis for event time profiles studying. Fractal index is sensitive to change of shape and indentness of event time profile and gives only one generalized characteristic per analyzable time profile. Fractal indexes would be different for time profiles of

events which caused by different physical processes (Feder, 1988).

Fractal analysis has some features which allow to use it for investigation so widely changes set as GRB. One of them is scaling: two events with the same type of time profiles but the different duration have the same fractal indexes. While wavelet analysis requires the same amount of points (number of points are very different for individual bursts because for GRB $10^{-3} < T_{90} < 10^3$). It is possible to use TTE data (Walker, 2000) which give time target for 65535 photons in burst peak with 10^{-6} sec time resolution but data set duration is ~ 2 sec for each burst. These data can not characterize whole burst if its duration $T_{90} > 2$ sec.

3 GRB fractal index distributions

Time profile fractal index definition methods are usually based on dissection of a profile on bins and analysis of count rate fluctuations in each bin. Received value D is not a real fractal index because burst time profile is just only prefractal (fractal is a limit of some endless process but we have a minimal value - time resolution of the detector). Value of D for prefractal can be shifted relatively real fractal index. That is why, some GRB fractal indexes are out of the band $1 < D < 2$.

If amount of experimental points k in bin j is not enough for statistical analysis (usually if $k < 20$) then the cell algorithm of fractal index definition is used (Shakura, 1994): the part of plane in which is analyzable curve is covered by cells with side δ . Let $N(\delta)$ -amount of cells, which have at least one common point with this curve:

$$L = N(\delta)\delta^D \quad (1)$$

For usual (nonfractal) curve $L=0$ for $\delta \rightarrow 0$ but for fractal curve gauge L is nonzero for certain $D \neq 1$. For practical application it is more suitable to plot dependence $N_i(\delta_i)$ for set of different δ_i . If it looks as:

$$N(\delta) = a \cdot \delta^{-D} \quad (2)$$

for $a > 0$, then the fractal index of this curve is equal to D .

In the investigation we used data with the time resolution of 64msec from 4B revised BATSE catalog. We have analyzed 1300 GRB. The cell sides were 200, 300, 400, 500 and 600msec.

The cell mechanism is correct only if a straight line is the best approximation of the relation

$$\lg N_i(\delta) = -D (\lg \delta) \quad (3)$$

Approximation (3) by straight line is correct if the next expressions (4) and (5) with $r=1$ are true for equation (3):

$$f_r = \frac{\omega_r}{R_r / (n-1-r)} > F_{1, n-r-1} \quad (4)$$

$$f_r = \frac{\omega_r}{R_r / (n-1-(r+1))} > F_{1, n-(r+1)-1}, \quad (5)$$

where $F_{1,m}$ is Fisher function (Hudson, 1964), ω_j - set of weights for orthonormal polynomials Q_{ij} , where

$$N(\delta) = \sum_{j=0}^r \omega_j \cdot Q_j. \quad (6)$$

The distributions of f_1 and f_2 for 1300 GRB are shown in fig. 1.

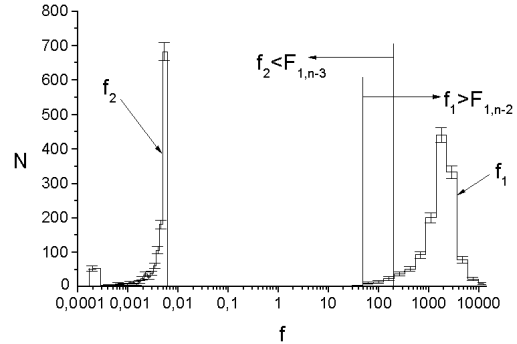


Figure 1. f_1 and f_2 distributions for 1300 GRB.

It is seen that the cell mechanism is correct for each of 1300 GRB from 4B revised BATSE catalog.

The similar attempt of studying GRB time profile fractal indexes was undertaken by Shakura et al. (1994). They have analyzed 4 events from different detectors and have estimated only the range of fractal indexes: $1.9 \pm 0.4 < D + 1 < 2.6 \pm 0.4$.

The distribution of fractal indexes D for all analyzed GRB is shown in fig. 2. It is seen that $0.80 < D < 2.42$ for this population. The maximum at $D=1.50$ shows that many bursts are smooth and fractal index for such bursts is equal to fractal index of background, which is 1.5 (see fig. 2). This fractal index characterizes data not only from BATSE LAD detectors but from other detectors which were analyzed in (Shakura et al., 1994). Moreover, in this work fractal index $D=1.5$ was received for set of 512 random numbers from 0.000 to 1.000. One can see that distributions in fig. 2e-fig. 2h are wider than distributions for uniform subset shown in fig. 2a-fig. 2d.

We suggested that wide distributions in fig. 2e - fig. 2h would be sum of some narrow distributions like fig. 2a - fig. 2d and several maximums may be seen if we will use additional criterion - duration T_{90} for GRB separation on groups.

The distribution of fractal index D for short GRB is shown at fig. 3a. There are two maximums $D=1.47 \pm 0.02$ and $D=1.8 \pm 0.02$. The distribution of fractal index D for middle GRB (see fig. 3b) has two maximums $D=1.43 \pm 0.02$ and $D=1.17 \pm 0.02$. Group of bursts with small fractal index $0.90 < D < 1.05$ is presented only in this population. The distribution of fractal index D for long GRB is shown at fig. 2c. There is only one maximum in this population, but it is more wide than maximum for uniform subset.

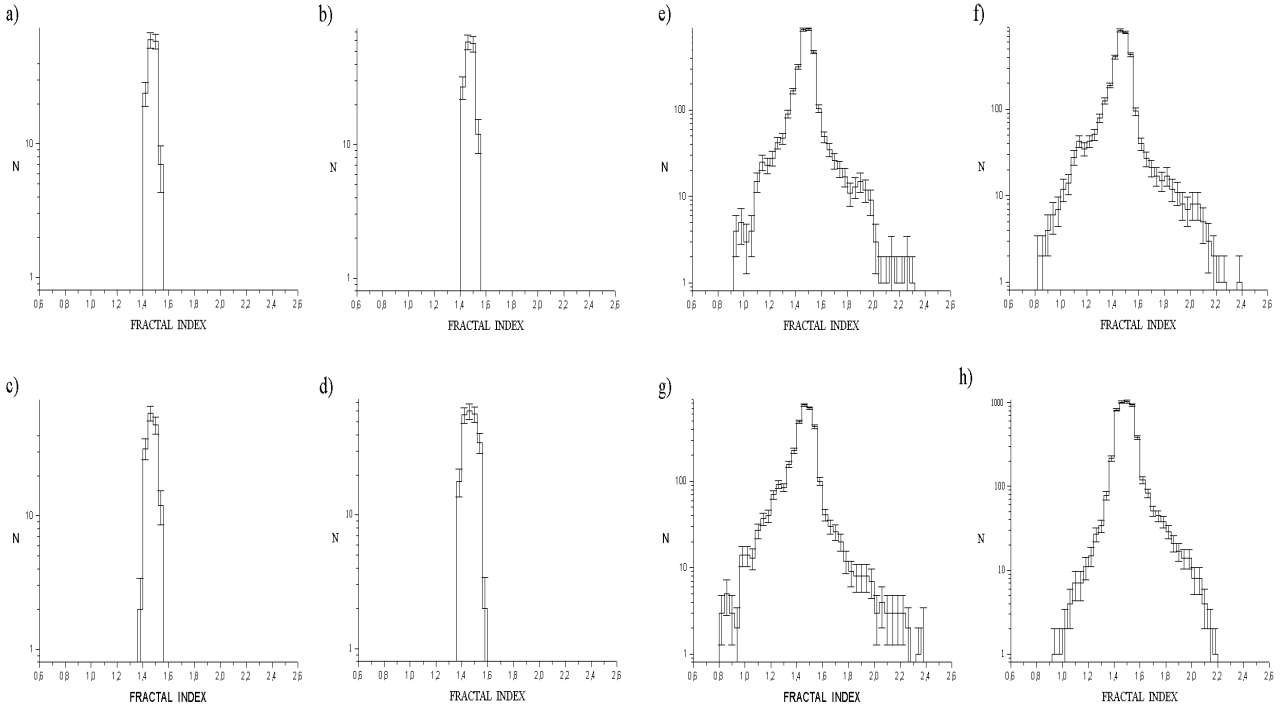


Figure 2. Fractal index distributions for BATSE background (a: $25\text{keV} < E_\gamma < 50\text{keV}$; b: $50\text{keV} < E_\gamma < 100\text{keV}$; c: $100\text{keV} < E_\gamma < 300\text{keV}$; d: $E_\gamma > 300\text{keV}$) and fractal index distributions for 1300 GRB (e: $25\text{keV} < E_\gamma < 50\text{keV}$; f: $50\text{keV} < E_\gamma < 100\text{keV}$; g: $100\text{keV} < E_\gamma < 300\text{keV}$; h: $E_\gamma > 300\text{keV}$)

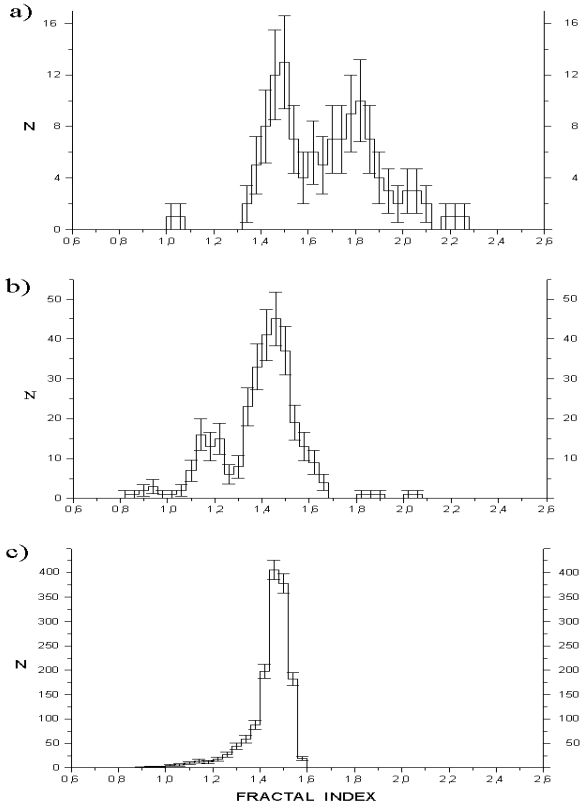


Figure 3. Fractal index distributions for short (a), middle (b) and long (c) GRB in the second BATSE channel.

Thus, there are two maximums in distributions of GRB fractal indexes for short and middle GRB. While the majority of analyzed GRB probably belongs to one class with index $D \approx 1.5$, there are two additional statistically reliable subclasses of the bursts: with $D = 1.8$ for short bursts and with $D = 1.7$ for middle duration bursts.

We can also suggest the existence of one/some GRB subclasses in the population of long bursts. Difference in the GRB time profile fractal indexes may be point out that GRB are forming by various physical processes.

References

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