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Cross helicity of self-generated Alfvén waves downstream a parallel shock

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Abstract. Energetic proton acceleration and transport through self-generated Alfvén waves in vicinity of a low-Mach-number parallel shock wave is considered employing Monte Carlo simulations. The large scattering-center compression ratio predicted by the Alfvén-wave transmission through such a shock leads to a strong increase of the particle pressure as $t \to \infty$ in the test-particle approximation. As the pressure of the energetic particles becomes dynamically important, the energetic particles can modify the cross-helicity of the downstream waves to turn away from the shock at certain distance downstream the shock front. This leads to a finite size of the scattering region with large compression ratio, providing a mechanism for the shock to self-regulate its accleration efficiency.

1 Introduction

The energy spectrum of accelerated particles downstream of a fast shock wave is given by a power law in momentum (e.g., Bell, 1978),

$$\frac{\mathrm{d}J}{\mathrm{d}E} \propto p^{-\Gamma}, \quad \Gamma = \frac{r_c + 2}{r_c - 1} \tag{1}$$

where $r_c = V_1/V_2$ is the scattering-center compression ratio cross the shock and $V_{1[2]}$ is the scattering-center speed (normal to the shock) upstream [downstream] the shock wave. For self-generated Alfvénic upstream turbulence, $V_1 = u_1 - V_{\mathrm{A}1}$ (Bell, 1978), u and V_{A} denoting plasma flow speed and Alfvén speed normal to the shock, respectively. The downstream value for $V_2 = u_2 + H_{\mathrm{C}2} V_{\mathrm{A}2}$ depends on the value of cross helicity in the dowstream region, $H_{\mathrm{C}2}$, which can be calculated from the given upstream plasma and wave properties (Vainio and Schlickeiser, 1999). The calculation gives values close to $H_{\mathrm{C}2} = -1$ and large scattering center compression ratios,

$$r_c \lesssim r \frac{M - 1}{M - r^{1/2}}, \tag{2}$$

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where $r(M) = u_1/u_2$ is the gas compression ratio and $M = u_1/V_{\mathrm{A}1}$ is the Alfvénic Mach number of the shock. This energy spectrum holds in a steady state for a shock with homogeneous properties of the downstream and upstream gases extending to $x = \pm\infty$, respectively.

For shocks with low Mach numbers propagating in low-beta plasmas, $M^2 \sim 4 - 3\beta$, Vainio and Schlickeiser (1999) found extremely hard spectral indices, $1 < \Gamma < 2$, resulting from scattering-center compression ratios far exceeding the limiting value of $r_c = r \leq 4$ for adiabatic shock waves with frozen-in scattering centers and the ratio of specific heats of $\gamma = 5/3$. The spectrum, however, can not be this hard as $p \to \infty$, since it would yield an infinite energetic-particle pressure. Some cut off mechanism must, therefore, be applied around the shock to keep the test-particle approach valid. Both a finite acceleration time and a finite size of the shock can be introduced as physical ways to limit the maximum energy and, thus, the pressure of the accelerated particles, but such an approach obviously limits the general applicability of the theory.

In this paper, we calculate the acceleration and transport of energetic particles close to a parallel shock wave using time-dependent Monte Carlo simulations. We drop the test-particle approximation from the wave–particle interactions, but still consider an unaffected bulk plasma. This paper presents a detailed description of the model and preliminary results from the simulations.

2 Model

We consider a stationary parallel shock wave in a 1-D geometry (Fig. 1). The plasma parameters are taken to have constant values up- and downstream the shock located at $x = 0$. We neglect the effects of the transverse magnetic field caused by the waves to the plasma flow structure around the shock, so the gas compression ratio for $M^2 > 4 - 3\beta$ is given by

$$r = \frac{4}{1 + 3\beta/M^2}. \tag{3}$$

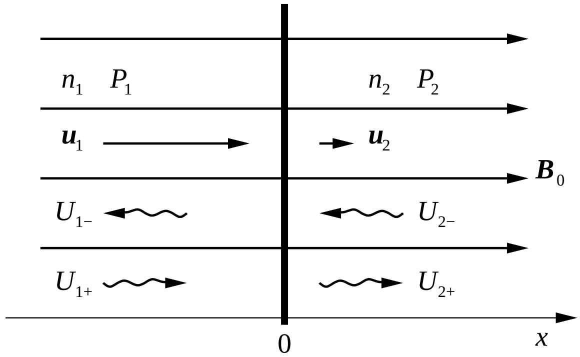

Fig. 1. The model of a shock wave studied in the simulation.

Vainio and Schlickeiser (1999) found this approximation invalid for large wave amplitudes, so our model is limited to cases where the wave pressure at the shock does not develop to a dynamically important parameter of the flow.

Small-scale magnetic-field fluctuations around the shock are modelled as being composed of Alfvén waves. We keep track of the wave intensities on a 2-D grid, $U_{ij}^{\pm} = U_{\pm}(x_i, k_j^{\pm})$, where $U_{\pm}(x,k)$ are the energy densities of the waves, per unit logarithmic bandwidth, propagating parallel (subscript $+$) and anti-parallel ($-$) to the flow. The spatial wave transport is included by moving the contents of one grid cell to the next at appropriate times t that are integer multiples of $\Delta x_i/(u_i \pm V_{\mathrm{A},i})$, where the grid spacing Δx_i has a different fixed value for up- and downstream, respectively. The boundary condition at the shock for the wave intensities is picked up from Vainio and Schlickeiser (1999): the flow is everywhere super-Alfvénic, and the waves incident at the shock from the upstream region excite both parallel and antiparallel waves in the downstream region regardless of their upstream propagation direction, $h_1 = \pm 1$. The transmission (i.e., $A_1^{\pm} \to A_2^{\pm}$) and reflection ($A_1^{\pm} \to A_2^{\mp}$) coefficients are given by (Vainio and Schlickeiser, 1999)

$$T \equiv \frac{\delta B_2^{h_1}}{\delta B_1^{h_1}} = \frac{r^{1/2}(r^{1/2}+1)}{2}\,\frac{M+h_1}{M+h_1 r^{1/2}} \tag{4}$$

$$R \equiv \frac{\delta B_2^{-h_1}}{\delta B_1^{h_1}} = \frac{r^{1/2}(r^{1/2}-1)}{2}\,\frac{M+h_1}{M-h_1 r^{1/2}}. \tag{5}$$

They are used to transport wave energy ($\propto \delta B^2$) accross the shock. The k-spacing is logarithmic. The waves conserve their shock-frame frequencies, so their wavenumbers change at the shock according to

$$(M \pm r^{1/2}) k_{j2}^{\pm} = r(M+h_1) k_{j1}^{h_1}, \tag{6}$$

and $\Delta(\ln k)$ is constant accross the shock. The largest wavenumber in the upstream region is $k_{\max 1} = \Omega_\mathrm{p}/V_{\mathrm{A}1}$. Because of the compression at the shock, the downstream maximum wavenumber is larger and, thus, falls out of the hydromagnetic range. We do not take this into account in our simulations, but the number of particles actually resonant with these high frequency waves is very small. Resonant wave dissipation by the thermal plasma ions is modelled by cutting off the spectrum with a $U_{\pm} \propto k^{-2}$ law at wave numbers above $k = \Omega_\mathrm{p}/v_{\mathrm{th}\,2}$, and non-linear dissipation by forcing the value of of $U_{\pm}$ to be less than $0.1\,U_B$ everywhere. Here, $v_{\mathrm{th}\,2}$ is the downstream thermal speed and $U_B = B_0^2/8\pi$ is the energy density of the background magnetic field.

We keep track of energetic protons in a Monte Carlo simulation (see, e.g., Vainio et al., 2000), where we move the particles under the guiding-center approximation in small time steps along the homogeneous background magnetic field, $\boldsymbol{B}_0$, directed along the shock normal. After each time step, we allow the particle to be scattered by Alfvén waves propagating along the background field both parallel and antiparallel to the plasma flow. We use a transport model, where a proton of certain wave-frame momentum p_w interacts only with waves at a single wavenumber $k = m\Omega_\mathrm{p}/p_\mathrm{w}$. This artificially sharpened version of the full quasi-linear resonance condition, $k = -m\Omega_\mathrm{p}/(p_\mathrm{w}\mu_\mathrm{w})$, where μ_w is the proton's wave-frame pitch-angle cosine, is a commonly used approximation in the theory of diffusive shock acceleration (e.g., Bell, 1978; Lee, 1983). The scattering frequencies off the parallel (ν_+) and antiparallel (ν_-) propagating waves are, thus, (Skilling, 1975)

$$\nu_{\pm}(x) = \frac{\pi}{4}\,\Omega\,\frac{U_{\pm}(m\Omega_\mathrm{p}/p_\mathrm{w})}{U_B}, \tag{7}$$

where $\Omega = \Omega_\mathrm{p}/\gamma$ is the relativistic gyrofrequency of the proton. The scattering frequency is evaluated by using the constant value of $U_{\pm}(x)$ inside each grid cell, but interpolating the value at the resonant k between the grid values.

The scatterings off the waves are elastic in the respective wave frames and, thus, lead to the growth/damping of the waves through the conservation of total (wave + particle) plasma-frame energy in each scattering. As the particle scatters by an amount of $\Delta\mu_\mathrm{w}$ in the wave frame, its plasma-frame energy is changed by $\Delta E = \pm V_\mathrm{A} p \Delta\mu_\mathrm{w}$. This energy is given to (or taken from) the waves that do the scattering, which in our model all have the same wavenumber. The resulting growth rate is, analogous to that of Skilling (1975),

$$-\sigma(k) = \pm\frac{\pi}{2}\,\Omega_\mathrm{p}\,\frac{S_\mathrm{w}(m\Omega_\mathrm{p}/k)}{V_\mathrm{A} n}, \tag{8}$$

where $S_\mathrm{w}(p_\mathrm{w}) = v_\mathrm{w} p_\mathrm{w}^3\, 2\pi \int_{-1}^{+1} \mathrm{d}\mu_\mathrm{w}\, \mu_\mathrm{w} f_\mathrm{w}$ is the wave-frame particle streaming per unit logarithmic momentum interval and n is the number density of thermal ions taken all to be protons, for simplicity. This growth rate per simulated particle is easy to evaluate during each time step. The resulting energy gain of the waves is shared between the k-grid points that lie around the calculated resonant wavenumber.

The particles are accelerated as they cross the shock front many times, because the scattering-centers are compressed with the plasma at the shock. The source of this energy is the bulk plasma flow, so one has to check that energy density of the accelerated particles does not become high compared to this energy reservoir during the simulation.

3 Results and discussion

We performed simulations for a case where the background plasma in the upstream region is taken to be cold ($\beta = 0$) and the shock's Alfvénic Mach number is $M = 3.33$ and the gas compression ratio is $r = 4$. Other plasma parameters are taken to represent values measured in the solar corona, $n_1 = 3 \times 10^8$ cm^{-3}, $V_{\rm A} = 600$ km s^{-1}.

We inject particles at the shock continuously by reflecting a small fraction $\epsilon = 3 \times 10^{-5}$ of the incoming cold plasma ions back to the upstream region. The upstream simulation box size is chosen large enough that no particles escape from the system through the far upstream boundary within the simulation time. We follow the particles until they escape through a downstream free-escape boundary, which is placed at $x = x_0 + (u_2 + V_{\rm A2})\,t$, where $x_0 = 2000$ km $\approx 1.8 \times 10^5\, u_2/\Omega_{\rm p}$ is the initial size of the downstream region. In the initial state, we choose $U_-(x < 0, k) = U_0\,\Omega_{\rm p}/(V_{\rm A} k)$ with $U_0 = 3.5 \times 10^{-6}\, U_B$, $U_+(x < 0) = 0$, and use Eqs. (4–5) to compute the downstream wave-energy densities. The cross helicity of the upstream region is chosen as $H_{\rm C1} = -1$ because the waves quickly attain it anyway as a result of the interactions with the accelerated particles. During the course of the simulation, the box grows to almost 7 times its initial size. As the box grows, the region with $(u_2 - V_{\rm A2})\,t < x - x_0 < (u_2 + V_{\rm A2})\,t$ contains only forward waves, because the backward waves propagate slower with respect to the shock front. We use a spatial grid spacing of $\Delta x_{1[2]} = 280\,[40]$ km in the upstream [downstream] region and a value of $\Delta \ln k = 0.139$.

In Figure 2, we present the total energetic-proton pressure

$$P_c(x) = \frac{1}{3} \int \mathrm{d}^3p\ vp\, f(p, x) \tag{9}$$

and the resonant wave-energy densities (integrated over the range of wavenumbers resonant with the particles) around the shock at three different times during the simulation. The upstream region is seen to agree well with the steady-state fluid-theory prediction of $P_c = (M - 1)U_-$ (e.g., Drury, 1983). The downstream region, however, can not be represented by a constant particle pressure, as is usually assumed in fluid-theoretical calculations. As can be seen, the energetic-particle and wave pressures stay well below the ambient magnetic-field pressure and, thus, should have only a minor effect on the thermal plasma flow with momentum flux $\rho u^2 + P \gg U_B$.

We have plotted, in Figure 3, the energy spectrum of the accelerated particles averaged over the whole downstream region at three times in form of partial proton pressure per unit logarithmic momentum interval. Note that the spectrum is not of a pure power-law type; especially the spectrum at the latest time shows a more involved structure. To investigate the reason for this, we plotted the cross-helicity of the waves as a function of position downstream the shock wave at a few different energies (Fig. 4). The spatial coordinate in this figure is

$$\eta(x, p) = \int_0^x \frac{u_2\, dx'}{\kappa(x', p)}, \tag{10}$$

where $\kappa(x, p) = \frac{1}{3} v^2/[\nu_+(x, p) + \nu_-(x, p)]$ is the energetic-proton diffusion coefficient. This dimensionless position variable determines the region in the vicinity of the shock where the particles are trapped by turbulence: the effective compression ratio around a shock is determined by the values of r_c at η less than a few, since at larger distances the particles have a small chance of getting back to the shock. As can be seen, the model produces hard scattering compression ratios near the shock, but self-consistent particle transport and time-dependent effects lead to a decrease of r_c already at $\eta \sim 1$. This is probably triggered by the large compression of the wavelength (by a factor of 7) of the backward waves at the shock meaning that for most part of the presented particle spectrum the resonant backward waves in the downstream region are in fact transmitted from the ambient wave energy densities, not the amplified ones. For forward waves the compression of the wavelengths is much less severe (a factor of 1.75, only). This can probably also account for the observed dip in the spectrum of accelerated particles at around an order of magnitude below the maximum energy.

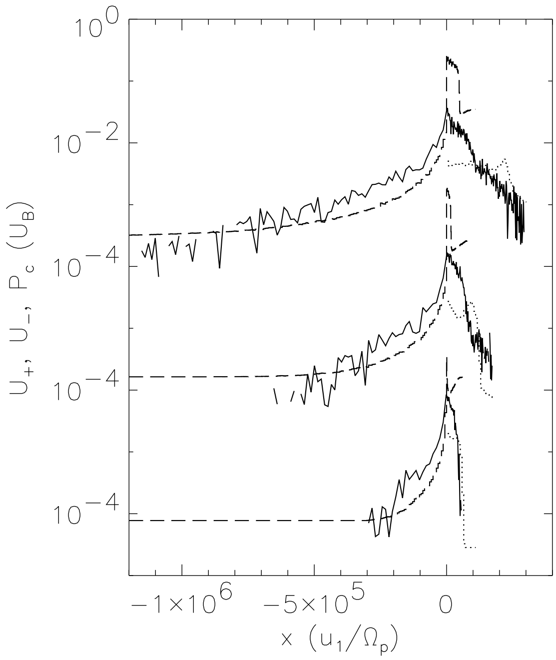

Fig. 2. Energetic-proton pressure (solid curves) and wave-energy densities U_+ (dotted curves) and U_- (dashed curves) around a parallel shock at (from bottom to top) $t = 1.6 \times 10^5\, \Omega_{\rm p}^{-1}$, $3.2 \times 10^5\, \Omega_{\rm p}^{-1}$, and $6.4 \times 10^5\, \Omega_{\rm p}^{-1}$.

The ambient wave-energy density in our simulation is rather small when compared to the pressure of the injected particles, $\epsilon\rho(2M - 1)^2 V_{\rm A1}^2 = 2\epsilon(2M - 1)^2 U_B$. This leads to the dynamical importance of particle pressure already at

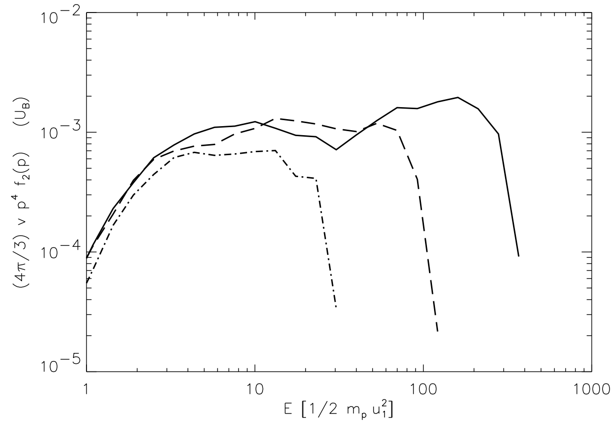

Fig. 3. Partial energetic proton pressure per unit logarithmic momentum interval averaged over the downstream region of a parallel shock wave at times $t = 1.6 \times 10^5\,\Omega_\mathrm{p}^{-1}$ (dot-dashed curve), $3.2 \times 10^5\,\Omega_\mathrm{p}^{-1}$ (dashed curve), and $6.4 \times 10^5\,\Omega_\mathrm{p}^{-1}$ (solid curve).

rather small energies. If the ambient wave-energy density is larger, we can expect a more test-particle-like acceleration to occur at low energies. But since the test-particle partial pressure shows a slope increasing with energy, at some point the particles will become dynamically important in any case.

The physical values of the energies attained in our simulation ($\lesssim 20$ MeV in $t \approx 14$ s) with parameters resembling the solar corona indicate that coronal shocks driven by CMEs are able to accelerate particles to tens of MeV's in a time scale short enough to be considered impulsive with respect to interplanetary propagation time scales.

4 Summary

We have presented a numerical model to calculate acceleration of energetic particles in parallel shocks by self-generated waves in a time dependent manner. We use the Monte Carlo method to calculate energetic-particle transport through the resonant wave spectra, which are transported along the magnetic field through a spatial coordinate grid. The wave–particle interactions are modeled by a simple resonance condition, $k = m\Omega_\mathrm{p}/p$, which allows us to use an isotropic scattering law (ν independent of pitch alngle) and makes the simulation code very efficient. In addition, this a commonly used approximation to the quasi-linear resonance condition in steady-state diffusive shock acceleration theories (e.g., Bell, 1978; Lee, 1983). Thus, the model provides a tool for studying time-dependent effects in these theories.

We performed simulations of particle acceleration and wave generation in a low Mach number parallel shock. In the simulated case of a weak ambient turbulence level, the energetic particles were found to be able to modify the turbulence behind the shock in such a manner, that the effective compression ratio of the shock was significantly less than in the test-particle limit. Looking at energies, where the compression ratio is modified most, we conclude that it is triggered by the strong compression of the backward-mode wavenumbers that in a time-dependent calculation lead to a deficit of resonant downstream backward waves around one order of magnitude below the maximum energy.

The simuations were performed for plasma parameters resembling solar coronal conditions. The shock was shown to be able to accelerate particles to 20 MeV in the simulation time of 14 s that can still be regarded as impulsive. More simulations in realistic magnetic field and density structures are needed to conclude if the coronal/interplanetary shocks can accelerate the highest-energy solar cosmic rays, as the current paradigm assumes.

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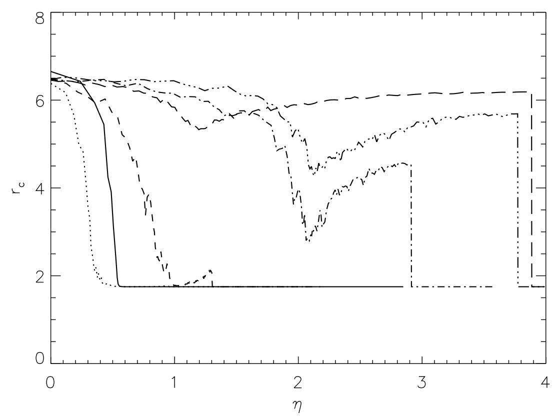

Fig. 4. Scattering-center compression ratio $r_c = V_1/V_2(x,p)$ at time $t = 6.4 \times 10^5\,\Omega_\mathrm{p}^{-1}$ as a function of normalized coordinate η in the downstream region for proton energies $E/\frac{1}{2}m_\mathrm{p}u_1^2 = 17$ (—), 30 (···), 52 (- - -), 91 (— · —), 160 (— ··· —), and 280 (— — —).