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The longitudinal EAS profile at E$>10^{19}$ eV: A comparison between gil analytical formula and the predictions of detailed Monte Carlo simulations.

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Abstract: The GIL formula is used to describe the longitudinal profile of hadron-initiated EAS in the range E>10^{19} eV, according to the Greisen-Iljina-Linsley parameterisation. The results are compared with the expectation values given by a detailed Montecarlo program for simulating EAS in a wide energy range (CORSIKA-QGSJET/SYBILL), for different primaries and directions. An accuracy of few percent is reached in the description of the longitudinal profile, the number of charged particles at the shower maximum. The depth of the shower maximum X_{max}(E,A) compares within 5%. The use of GIL as a fast generator for EAS longitudinal profile is therefore suggested as a powerful tool to work-out the performances of those experiments which use the longitudinal profile and the fluorescence signal to study the Extreme Energy Cosmic Rays (EECR).

1 Introduction

The knowledge of the longitudinal profile of the Extended Air Showers (EAS) as a function of the atmospheric depth is of critical importance, mostly for the high-energy cosmic ray experiments that detect the nitrogen fluorescence emitted along the shower "track" in the atmosphere. The description of the EAS characteristics through the detailed Montecarlo simulation of the physics processes of each individual particle gets more and more complicate as the shower energy increases. The most popular codes (CORSIKA (Knapp et al., 2000), MOCCA/AIRES (Dova et al., 1997), etc.) need prohibitive computing time to fully simulate showers generated by primaries with energy larger than 10^{17} eV. The introduction of a thinning algorithm partially solves the problem at the price of a loss in the detail of the simulation (when the total energy of secondary particles falls below a given threshold, typically $10^{-5}\div 10^{-6}$ of the primary energy only one of them is followed, selected at random according to its energy E_i with a probability $p_i=E_i/\Sigma_j E_j$, and it is given a weight $w_i=1/p_i$, in such a way to guarantee the energy conservation of the whole algorithm).

A Gaisser-Hillas function (Gaisser et al., 1997) fits well the longitudinal profile of the shower, providing, shower-by-shower, also an estimate "a posteriori" of the fundamental parameters:
x_0≡first interaction point,
x_{max}≡depth of the shower maximum,
N_{max}≡number of charged particles at the maximum.

An analytical description of the longitudinal profile that shortcuts the detailed description of the particle pattern at ground (the space and arrival time distribution, the energy spectrum and the particle nature) is of utmost importance for all those experiments (Fly's Eye (Baltrusaitis et al., 1985), HiRes (Abu-Zayyad et al., 1999), Auger (1997) as ground-based experiments, EUSO (Scarsi et al., 2001), as satellite experiments) that look at the entire atmosphere as an active target and rely on the fluorescence signal induced by the shower's particles to reconstruct position, direction, energy and possibly to have a hint of the elemental composition of the incoming primaries.

In a recent work (Linsley, 2001), J. Linsley proposed a parameterisation for the longitudinal profile of EASs based on the Greisen parameterisation (Rossi and Greisen, 1941) (for the case of EAS initiated by primary gammas) and the Ilijna (Iljina et al., 1992) variant for describing nucleus-initiated showers. The particle content is described as a function of the primary energy E by the expression:

$$N_{EM}(E,t;E_1) = (E/E_1)\, e^{-[t-t_{max}-2t\cdot ln(s)]} \qquad (1)$$

t is, in this formula, directly connected to the atmospheric depth x (in g/cm^2) measured from the first interaction point x_1

$$t = (x - x_1)/ /x_0 \qquad (x_0=37.15\ g/cm^2)$$

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s, the shower age, is analytically described as a function of the depth t:

$$s = [(1+t_{max}/t)/2]^{-1}$$

t_{max}, finally, is the depth at which $s=1$, and holds the same meaning, i.e. the depth at which the shower reaches its maximum development, $N(t=t_{max})=N_{max}$, and depends upon the primary energy E and its mass A. In the same reference a suitable form for t_{max} is proposed:

$$t_{max} = a+b\times[\ln(E/\varepsilon)-\ln A] \quad (2)$$

which gives the main A-dependence of the shower profile. The constant $\varepsilon=81\ MeV$ is, in this formula, the critical energy of the Greisen parameterisation, $b=0.76$ is the value of the elongation rate desumed from a linear fit to the inclusive existing data, $a=1.7$ is an offset constant of the entire formula. The use of the scaled depth t stresses the dependence of the longitudinal shower development from the atmospheric depth measured in units of radiation length, computed from the first interaction point, i.e. from the point where the shower begins rather than from the top of the atmosphere.

According to (1), the longitudinal profile of an EAS initiated by a primary of energy E and mass A can be described analytically and depends upon the energy scale E_1 which describes the behaviour of the maximum content of the shower as a function of the energy $N_{max}=E/E_1$. $E_1=1.45GeV$ is a value (Linsley, 2001) in good agreement with the conclusions from experimental evidence and, as it will be discussed in section 3, compares well with the maximum shower content predicted by the Montecarlo simulation.

In this paper we compare the longitudinal profile as predicted by the GIL model with the corresponding behaviour given by the Montecarlo simulation program CORSIKA (Knapp et al., 2000) with the QGSJET (Kalmykov et al., 1997) hard interaction model, in the primary energy range ($10^{19}eV\leq E\leq 10^{21}eV$). A cross check of the dependence of the Montecarlo predictions by the hard interaction model has been done by using the SYBILL (Fletcher et al., 1992) model. We simulated showers initiated by proton, oxygen and iron primaries. The simulation program features are summarized in section 2. The simulated events are first used to check the internal consistency of the value chosen for E_1 as the energy scale, as discussed in section 3.

The linear relationship $N_{max}(E) = E/E_1$ is in excellent agreement with the choice $E_1=1.45GeV$ and suggests the measurement of N_{max} as a good "energy estimator" in the energy range $E>10^{19}eV$.

The mass dependence of the position of the shower maximum is discussed in section 4, by comparing the GIL parameterisation and the Montecarlo simulation. Also a possible modification of the parameters in (2) is discussed, which allows to predict a t_{max} behaviour in better agreement between GIL formula and the CORSIKA simulation, in the observed energy range. The longitudinal profile of the shower predicted by GIL is then compared to the profile obtained with CORSIKA varying the nature and the direction of the incoming primary, and the result is discussed in section 5.

2 The simulation.

CORSIKA is a versatile package for simulating air showers over a wide range of primary energies. We have used it in connection with the QGSJET description for simulating high energy hadronic interactions. Electromagnetic sub showers are simulated with the EGS4 code, whereas low energy (<80GeV) hadronic shower development is simulated with the GHEISHA code. The thinning algorithm needed to reduce the CPU time has been selected for those particles whose energy falls below 10^{-6} of the primary energy. Hadrons and muons are then followed down to a lower energy threshold of 300MeV, whereas electrons and photons are followed until their energy falls below 100KeV. The threshold value for the electromagnetic part of the shower turns out to be a critical value for the estimate of the energy leakage of the entire simulation program. Using the value of 100KeV it has been estimated[1] that ~10% of the energy is "lost", carried away by low energy photons and electrons/positrons. This fact leads, in the simulation program, to a corresponding underestimate of the number of particles which produce fluorescence, which has to be taken into due account when predicting the "fluorescence yield" of a shower. As it is explained in the text, the results of GIL formula will be thus scaled down by a factor 0.9 to be compared with the CORSIKA predictions. The energy leakage due to the hadronic part of the shower is less critical as far as the longitudinal profile is concerned. It is in fact mainly due to true "invisible energy" carried by secondary muons and neutrinos down to the shower "dump" on ground, and does not imply an underestimate of the number of particles able to produce fluorescence within the atmosphere.

3 The energy scale E_1

According to the GIL formula (1), the number of particles at the maximum shower development

$$N_{max,GIL}(E)=N_{EM}(t_{max};\ E,A)=E/E_1,$$

grows in a linear way with the primary energy E, at least for large values of E. We compared this expected behaviour with the prediction derived by the CORSIKA shower development code, by varying the primary energy, the primary nature and arrival direction. Fig. 1 summarizes the results as a function of the primary energy in log-log plots for the proton as primary particle. The linear behaviour expected by the GIL model is found to hold also as a prediction based upon CORSIKA. The figure shows the comparison of the maximum number of charged particles in the CORSIKA shower $N_{max,C}(E)$ with the reduced quantity $N'_{max,GIL}(E)=0.9\times N_{max,GIL}(E)$, where the factor 0.9 takes into account the fact that CORSIKA underestimates (Song et al., 2000) the charged particle content by a factor 0.9 because of the lower energy threshold in the electron and photon tracking. The energy scale E_1 is adjusted so as to obtain the best fit of $N'_{max,GIL}$.vs.$N_{max,C}$. The value $E_1=1.45GeV$ compares well with the values $E_1\sim 1.5GeV$ found by Linsley. The same results have been obtained in the case of iron and oxygen nuclei as primary particle, with the same value for $E1=1.45GeV$.

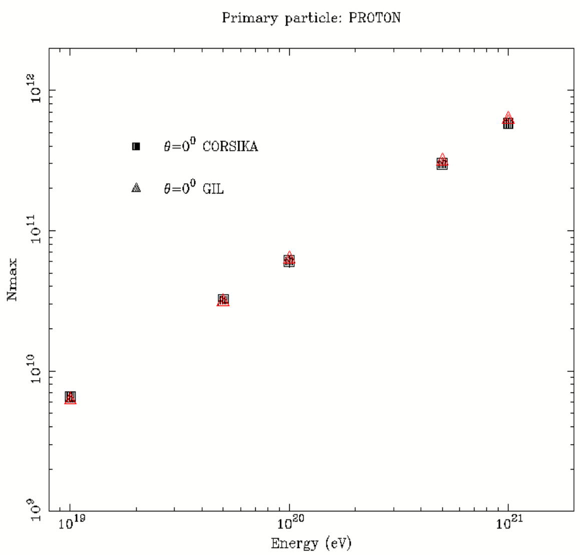

Fig.1– Maximum shower content N_{max}.vs.Energy, from GIL analytical formula and CORSIKA Montecarlo

Fig. 2 shows the dependence from the zenith angle θ of the shower content at its maximum, as computed using CORSIKA (The GIL formula is by construction indipendent from θ). Only proton data are shown.

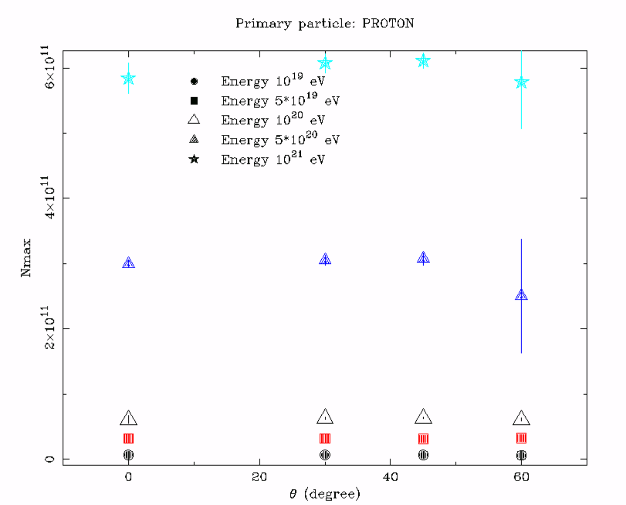

Fig.2- N_{max} behaviour as a function of the zenith angle θ.

4 The A-dependence of $X_{max}(E,A)$

According to the work of J. Linsley (Linsley, 2001), we assume the expression for the depth of the shower as a function of energy and primary mass:

$$X_{max}(E,A) = X_1+X_0\{b[ln(E/\varepsilon)-ln(A)]+a\} \qquad (3)$$

where the A-dependence is derived by the superposition principle and the parameters values choice is described as follows.

According to this expression the inclusive elongation rate is:

$$dX_{max}/d[log_{10}(E)]=X_0b/log_{10}e(1-\partial ln(A)/\partial ln(E)) \qquad (4)$$

and it will coincide with the expression for pure-composition EAS

$$d[X_{max}(E,A=const)]/d[log_{10}(E)]=X_0b/log_{10}e$$

only under the assumption that the elemental composition does not depend from the energy: $\partial ln(A)/\partial ln(E)= 0$.

The existing inclusive UHECR data, where no elemental composition identification is made, exhibit an elongation rate $dX_{max}/d[log_{10}(E)]=65g/cm^2$, as worked out by J. Linsley (Linsley, 2001).

In the same reference, a constant elemental composition with energy $(\partial ln(A)/\partial ln(E)=0)$ and a mean value $<A>=10$ for the atomic number A of the primary have been assumed to calculate the values of the constants $a=1.7$ and $b=0.76$ in (3). The expression of formula (2) for $t_{max}(E,A)$ in GIL, is directly connected to (3).

The CORSIKA Montecarlo simulation, which assumes by definition a pure elemental composition, predicts, according to our results, a value for the elongation rate

$$d[X_{max}(E,A=const)]/d[log_{10}(E)]\sim 60g/cm^2,$$

which is independent, within the statistical errors, from the value of A and from the interaction model used to generate the events. This has been checked by changing the hard interaction model from QGSJET (Kalmykov et al, 1997) to SYBILL (Fletcher et al, 1994, Engel et al, 1992).

A direct comparison of the CORSIKA prediction and the GIL computed value is shown in Fig.3 for proton, oxygen and iron primaries. It can be notice that the slope of the X_{max}.vs.energy is described in the same way by the two methods, whereas the light to heavy primary separation is predicted to be larger in GIL than in CORSIKA. The difference among the prediction of the two methods, as far as the position of X_{max} is concerned is anyway below 6%.

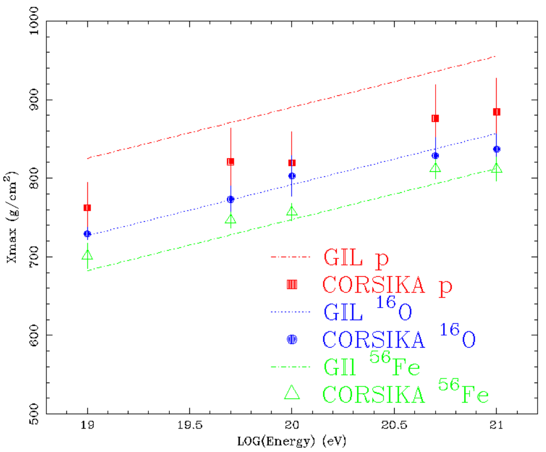

Fig.3 – X_{max}.vs.Energy behaviour for CORSIKA Montecarlo and GIL formula

5 The EAS longitudinal profile

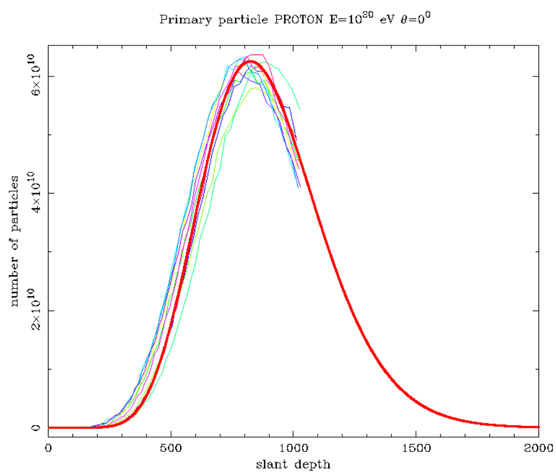

Fig.4- CORSIKA (light full lines) and GIL (thick line) shower profile for proton–induced EAS

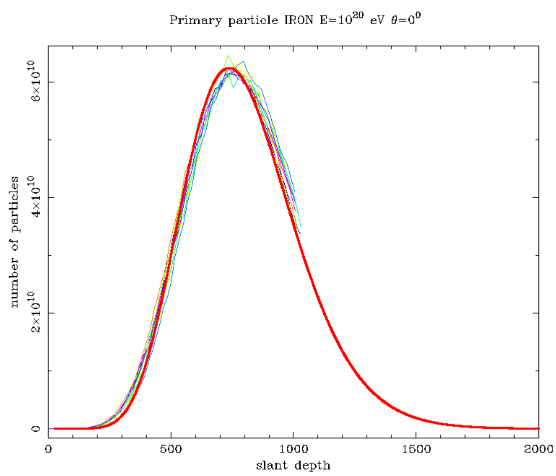

Fig.5 – CORSIKA (light full lines) and GIL (thick line) shower profile for Iron–induced EAS

We compared finally the shape of the longitudinal profile, as predicted by GIL formula and CORSIKA/QGSJET Montecarlo. Figs. 4 and 5 show the profile of 10 CORSIKA showers of energy $E=10^{20}$eV for light (proton) and heavy (^{56}Fe) nuclei and, superimposed, the analytical behaviour of GIL formula. The shower-to-shower fluctuations contained in CORSIKA do not spoil the main conclusion: GIL formula and CORSIKA simulation predict longitudinal profiles with the same behaviour, as far as EECRs are concerned.

6 Conclusion

We have compared the GIL parameterisation of EAS profile to the CORSIKA/QGSJET Montecarlo simulation method for deriving the shower expected features, in terms of particle content at the maximum, position of the shower maximum as a function of the atmospheric depth, overall shape of the longitudinal profile. Under this respect the two methods give comparable result, within the accuracy allowed by the Montecarlo fluctuations.

Since GIL is an analytical expression it shows up to be very fast and can therefore be used as a very powerful tool to derive the expected performances of the experimental approaches to the EAS study, which rely on the shower longitudinal profile to investigate EECRs. A typical example is given by those experiments, which detect the fluorescence produced by the shower along the atmosphere.

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