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Magnetic field line wandering and shock front acceleration: Case of SN1987A

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Abstract. The acceleration time at a shock front of cosmic rays scattered along wandering magnetic field lines is derived as a function of the obliquity and speed of the shock for a general transport exponent of the field lines. For high speed shocks, the magnetic field line wandering can make the acceleration time depart by orders of magnitude from its value estimated in the approximation of simple scattering. This discrepancy is most welcome to explain the acceleration time of GeV-electrons at the reappearance of SN1987A in the radio waveband, as the inferred diffusion coefficient along the shock normal is about five orders of magnitude greater than the limiting Bohm value of a cross-field scattering coefficient. It is also shown that the shock speed and number of scatterings upstream of the shock could be the determining parameters in the reappearance of SN1987A.

1 Introduction

Shock fronts are believed to be involved in a number of high energy astrophysics phenomena, ranging from blazar jets (Kirk et al., 1998) to the radio, X- and γ-ray emission of supernovae remnants (Mastichiadis and de Jager, 1996). Shock fronts of particular importance to the cosmic-ray physicists are the ones encountered at supernovae. The heat input into the interstellar medium is indeed dominated by supernovae and acceleration at the blast waves of supernovae very early became an attractive candidate for the production of high energy cosmic rays in the Galaxy.

The formation of a broad spectrum of energetic cosmic rays, through the bouncing of these particles between the two sides of a shock front and the energy gain at each crossing of the front, is by now well described by the theory under the assumption of a diffusive transport of the particles (Blandford and Eichler, 1987). The predicted spectral index is in reasonable agreement with the spectral index expected for these particles, provided the transport and reacceleration coefficients between production and detection sites take the right form or in the case of a direct determination of the index through the emission in the shock vicinity, provided the appropriate value is chosen for the shock compression ratio.

The way particles are transported through the magnetic turbulence around the shock front is of course of prime importance. It determines the number of crossings of the front and, therefore, strongly influences the efficiency of the acceleration process. A diffusion is commonly assumed for this transport, *i.e.*, a linear increase of the particle spreading with time. The diffusion is supposed to result from the particle scattering in the magnetized medium, but the wandering of the turbulent magnetic field lines is neglected. Under this assumption, the acceleration time of the particles can be expressed as a function of the diffusion coefficient $D_\perp$ along the shock normal (*e.g.*, Duffy et al., 1995):

$$T_{acc} = \frac{3D_\perp}{U_1^2}\frac{1+\rho}{1-1/\rho}\ , \tag{1}$$

where $\rho \equiv U_1/U_2$ stands for the compression ratio of the shock and U_1, U_2 denote the shock speeds (along the normal) relative to the upstream and downstream media, respectively.[1] We will see that this relation leads to a strong inconsistancy between observation and theory in the well-documented case of SN1987A. The acceleration time inferred from observations gives indeed a highly unphysical scattering coefficient for the particles. We will further summarize how, in Ragot (2001), the theory is improved and the problem of SN1987A solved by including the wandering of the magnetic field lines in the description of the transport.

2 Description of the problem

The synchrotron emission in the ambiant magnetic field of the electrons accelerated at the shock along with the heavier

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[1] The case of particle dynamics that would decorrelate from diffusing field lines is also covered by Eq. (1), but it would involve a too long residence time upstream of the shock.

cosmic rays constitutes an important diagnostic of the acceleration process. The observation of this emission in the radio waveband gives direct information on both the spectrum and the acceleration time of the electrons in the GeV range of energies. From the observed delay in the switch-on of SN1987A emission between 843 MHz and 4.8 GHz, Ball and Kirk (1992) could deduce an acceleration time at the shock wave of SN1987A of the order of 15 days. For $\rho = 2.7$ (inferred from the spectral index of the emission in the frame of the diffusive theory), this acceleration time implies a spatial diffusion coefficient $D_\perp$ for the emitting electrons of the order of $2 \times 10^{24}\,\mathrm{cm}^2.\mathrm{s}^{-1}$. This is substantially — five orders of magnitude! — greater than the Bohm diffusion coefficient κ_B. Bohm diffusion occurs when charged particles gyrating around magnetic field lines are scattered across the field by one gyroradius at each half-gyroperiod. This is the fastest possible model of cross-field *scattering*. The large transport coefficient found by Ball and Kirk (1992) could, therefore, certainly *not* be explained by a scattering of the electrons *across* the field lines, the resulting diffusion coefficient $\kappa_\perp$ being precisely limited by the Bohm value.

Given the huge discrepancy between $\kappa_\perp$ and $D_\perp$, the cross-field scattering can safely be discarded. In the frame of a scattering description of transport, this approximation gives $D_\perp = \kappa_\parallel \cos^2\psi$ instead of $D_\perp = \kappa_\parallel \cos^2\psi + \kappa_\perp \sin^2\psi$, where $\kappa_\parallel$ denotes the scattering coefficient along the magnetic field lines and ψ the angle between the average magnetic field and the shock normal.

The scattering along the field lines does contribute to the transport across the shock, but this contribution tends to become relatively small for quasi-perpendicular shocks with $\cos^2\psi \ll 1$. In the case of SN1987A, a quasi-perpendicular shock wave is expected at the time of the radio wake-up, since at that time the blast wave is travelling in the wind of the progenitor star and the magnetic field $\boldsymbol{B}$ in this wind is thought to have the form of a Parker spiral (Ball and Kirk, 1992). If the scattering *along* regular field lines could explain the large value of $D_\perp$, the value of $\cos^2\psi$ would have to be of the order of $10^5\kappa_B/\kappa_\parallel = 10^5 r_g/\lambda_\parallel$, where $\lambda_\parallel$ and r_g denote the parallel mean free path and gyroradius of the particles, respectively. In the interstellar medium, the ratio $\varepsilon \equiv r_g/\lambda_\parallel$ is approximately $\varepsilon_{ISM} \approx 10^{-6}$. Ahead of the shock, the turbulence is enhanced compared to the interstellar one (Achterberg et al., 1994). A reasonable value for ε is, therefore, at least of the order of 10^{-5} and a transport dominated by scattering along regular field lines would imply $\cos^2\psi \gtrsim 1$. This is clearly not compatible with a quasi-perpendicular shock for which $\cos^2\psi \ll 1$. As a consequence, the scattering *along* regular field lines is also insufficient to explain the large value of $D_\perp$.

We thus come to the conclusion that the transport of the GeV electrons accelerated by the shock wave of SN1987A at its reappearance in the radio waveband in 1990 (Staveley et al., 1992; Gaensler et al., 1997) cannot be described as a pure scattering process. The wandering of the magnetic field lines must be taken into account. A general estimate of the acceleration time including the effect of the magnetic field line wandering (hereafter MFLW) has been proposed recently (Ragot, 2001). Summarizing below part of this paper, we will show that MFLW does explain the extremely long acceleration time measured for SN1987A. For each set of shock parameters (speed U_1, obliquity ψ), the quantitative effect of the MFLW on the acceleration time will also be predicted as a function of one single parameter, the number of scatterings N upstream of the shock. Further in section 4 we will show that this number of scatterings N, controlled by the shock speed at a given turbulence level, is the determining parameter in the reappearance of SN1987A.

3 Magnetic field line wandering and acceleration time

We will *not* assume the usual diffusion of the magnetic field lines. The superposition of random fluctuations $\delta\boldsymbol{B}$ on a main, regular magnetic field $\boldsymbol{B}_0$ is usually believed to produce a diffusive random walk of the magnetic field lines, i.e., a spreading ΔX^2 of the field lines, across the direction of $\boldsymbol{B}_0$, linearly increasing with the distance Z along $\boldsymbol{B}_0$ (Jokipii, 1966; Jokipii and Parker, 1968). A recent study of ΔX^2 (Ragot, 1999; 2001b) has proven however that, even in the quasilinear regime of turbulence, the diffusion is not guaranteed. In fact, the transport exponent β defined by $\langle \Delta X^2 \rangle \propto Z^\beta$ is extremely sensible to the spectral index of the turbulence at the corresponding scale, and whenever the spectrum does not completely flatten on the reciprocal of the length scale ($\times$ a factor of the order of 10), β differs from 1. A decreasing spectrum results in a supradiffusion of the field lines, with $2 > \beta > 1$, whereas an inverted spectrum implies a subdiffusion ($0 < \beta < 1$) (Ragot, 1999; see also the analytical proof by Ragot elsewhere in these proceedings).

Not only will this non-diffusive spreading of the field lines imply different transport regimes of the cosmic rays while scattered along the field lines, but it might as well delay the decorrelation from the field lines themselves. The filamentation and mixing of the field lines are indeed in the diffusive theory responsible for the decorrelation and appearance of the large scale diffusion of the cosmic rays by reducing the distance particles have to be scattered across the field to reach uncorrelated field lines (Rechester and Rosenbluth, 1978). This means that the transport of particles accelerated at a shock front is likely to result from the combination of scattering along magnetic field lines and wandering of magnetic field lines. The particular case where the MFLW is diffusive — resulting in a subdiffusion of the particles with a transport exponent $1/2$ — has been studied by Duffy et al. (1995) and Kirk et al. (1996) for a perpendicular shock. Motivated by the results of Ragot (1999), we consider now the more general case of a MFLW of transport exponent 2α, resulting in a transport exponent α for the particles.

Underlying assumptions are that the particles do not have time to decorrelate from the field lines between two successive crossings of the shock front and that the cosmic rays are diffusively scattered along the magnetic field. The field lines spread following the general rule $\langle \Delta X^2 \rangle = D_{m_{2\alpha}} Z^{2\alpha}$ and the particles are scattered as $\kappa_{\|}t$ in Z. We assume *no* specific relations between $D_{m_{2\alpha}}$, $\kappa_{\|}$, α and the spectrum of turbulence. We calculate below the resulting acceleration time as a function of the transport coefficients and exponent, and the geometric characteristics of the shock, making use of the one-dimensional propagator $P_{\beta,1}$ of anomalous transport derived by Ragot and Kirk (1997).

The acceleration time T_{acc} of a particle of speed v is related to the average residence times t_{res_1}, t_{res_2} upstream and downstream of the shock by:

$$T_{acc} = \frac{3v}{4(U_1 - U_2)} \left(t_{res_1} + t_{res_2}\right) \ . \tag{2}$$

The average residence time ahead of the shock can be expressed as the total time spent upstream, $t_{up} = \int_0^\infty dt \int dZ\, Q(Z,t) \int dX\, P_{2\alpha}(X,|Z|)\, H(x - U_1 t)$, divided by the average number of times, N_{up}, particles leave the upstream region (Duffy et al., 1995), which is given by the flux of particles at the shock integrated over all positive times, $N_{up} = \frac{v}{4} \int_0^\infty dt \int_0^\infty dZ\, Q(Z,t) \int_0^\infty dX\, P_{2\alpha}(X,|Z|) \times\delta(x - U_1 t)$. As for t_{res_2}, it can be deduced from t_{res_1} by substituting U_2 for U_1. In the equations above, H stands for the Heaviside function and $x = Z\cos\psi + X\sin\psi$. x is the abscissa along the normal to the shock and X along the normal to the main magnetic field, increasing with the distance to the shock front. Z is the ordinate associated to X, along the main magnetic field (also increasing with the distance to the shock). Q denotes the one-dimensional propagator of diffusion $P_{1,1}$ with the diffusion coefficient $\kappa_{\|}$, and $P_{2\alpha}$ the one-dimensional propagator of anomalous transport $P_{2\alpha,1}$ with the transport coefficient $D_{m_{2\alpha}}$.

Integrating the expressions for t_{up} and N_{up} and substituting in Eq. (2), we obtain the following relation for the acceleration time as a function of the transport coefficients (the detailed derivation can be found in Ragot [2001a]):

$$T_{acc} = \frac{3H_\alpha(K)\kappa_{\|}\cos^2\psi}{U_1^2} \frac{[1 + \rho H_\alpha(\rho K)/H_\alpha(K)]}{1 - 1/\rho} \tag{3}$$

where $H_\alpha(K) \sim (K/K_l)^{-2(1-\alpha)/(2-\alpha)}$ for $K < K_l$ and $H_\alpha(K) \sim 1$ for $K \geq K_l$. The new parameter K is given by $K = 4A_{2\alpha}\kappa_{\|}/(U_1 D_{m_{2\alpha}}^{1/[2(1-\alpha)]}) \times (\cos\psi)^{(2-\alpha)/(1-\alpha)}/(\sin\psi)^{1/(1-\alpha)}$; $K_l = 4A_{2\alpha}\left[(2-\alpha)/(4A_\alpha)\right]^{(2-\alpha)/[2(1-\alpha)]}$. The coefficient A_α equals $(2-\alpha)\alpha^{-1}(\alpha/2)^{2/(2-\alpha)}$.

There is no singularity in $\alpha = 1$ as K always appears to some power multiple of $1 - \alpha$, which compensates the powers in $1/(1-\alpha)$ in its definition: $H_1(K) = 1$ for all K. K measures the relative importance of the field line wandering and of the inclination of the field lines average direction on the shock plane. When K is large ($\gg K_l$), the scattering of the particles along quasi-regular magnetic field lines making an angle $\pi/2 - \psi$ with the shock plane provides a good description of the transport along the normal to the shock: the usual expression of a simple-scattering transport, $T_{acc_{scatt}} = (3\kappa_{\|}\cos^2\psi/U_1^2)(1+\rho)/(1-1/\rho)$, is recovered. For smaller K, the field lines wandering comes into play. Its effect on the acceleration time increases as K to the power $-2(1-\alpha)/(2-\alpha)$.

Our new expression (3) for the acceleration time can also be written as:

$$T_{acc} = H_\alpha(K) T_{acc_{scatt}} \frac{[1 + \rho H_\alpha(\rho K)/H_\alpha(K)]}{1 + \rho} \ . \tag{4}$$

Since the last factor in Eq. (4) is practically of the order of one (or a few), $H_\alpha(K)$ nearly represents the ratio of the actual acceleration time over its value in the simple-scattering regime (with no field line wandering). As K decreases, $H_\alpha(K)$ can become orders of magnitude larger than one when $\alpha \neq 1$. This accounts for the very long acceleration time observed at SN1987A and solves the problem raised in section 2. Namely, the product $H_\alpha(K)\cos^2\psi$ replaces $\cos^2\psi$ in the relation to ε, which now leads to $H_\alpha(K)\cos^2\psi \gtrsim 1$ instead of $\cos^2\psi \gtrsim 1$, and easily allows for $\cos^2\psi \ll 1$.

The acceleration time depends on two transport coefficients. Both coefficients are related through the spectrum of turbulence, so that they both can be expressed as functions of one *common* parameter, which we choose to be the number of scatterings N upstream of the shock. By choosing the number of scatterings rather than the more commonly used turbulent field amplitude at one frequency, we avoid assuming specific relations between the turbulence and the transport coefficients and keep our results as general as possible. Next, we determine for each shock speed and obliquity the effect of MFLW on the acceleration time as a function of the parameter N. For this we need to find H_α as a function of N.

The squared average distance travelled by the relativistic electrons along the magnetic field upstream of the shock is $\langle \Delta Z^2 \rangle = \kappa_{\|} t_{res_1}$, which can also be expressed as the squared parallel mean free path $\lambda_{\|}^2$ times the number of scatterings N. Since $\kappa_{\|} = \lambda_{\|} v/3$, we have $\kappa_{\|} = v^2 t_{res_1}/(9N)$ and we can write the scattering coefficient $\kappa_{\|}$ as a function of T_{acc} and N. Injecting the obtained relation into the expression (3) for the acceleration time results in:

$$H_\alpha(K) = \frac{U_1}{v} \frac{9N}{4\cos^2\psi} \ , \tag{5}$$

which allows us to view the quantitative effect of MFLW on the acceleration time T_{acc} on a simple diagram, by plotting in Figure 2 the lines of constant H_α in the (U_1, ψ) plane as a function of the parameter N.

Figure 2 shows for $N = 3$ (thick lines) and $N = 10$ (thin lines) the extent of the domains where the transport of the accelerated particles is dominated by the inclination of the average magnetic field on the shock plane, below the continuous lines corresponding to $H_\alpha(K) = 1$, and by the MFLW, above. The effect of the MFLW becomes extremely important above the dashed line $H_\alpha(K) = 10$, but it must already be taken into account above the continuous lines $H_\alpha(K) = 1$. For a shock speed larger than $0.15c$, the MFLW must be included in the description of the transport for any obliquity of the shock. For $N = 10$, it already modifies the acceleration time by a factor 10 at an obliquity of 0.92 and more than 100 at $\psi = 1.4$. For $U_1 \sim 0.1c$ and $N = 3$, the MFLW can

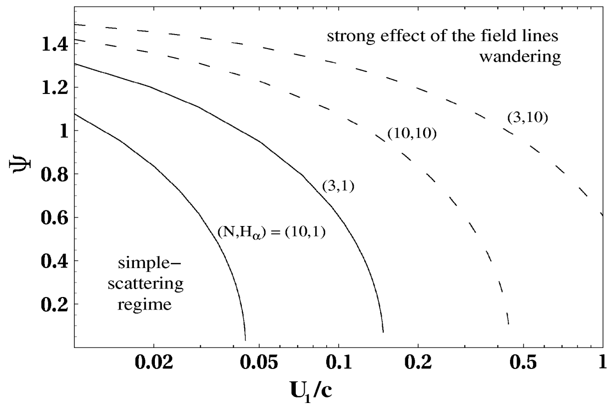

Fig. 1. Niveau lines of the function H_α in the plane $(U_1/c, \psi)$ at $H_\alpha(N, \psi) = 1$ (continuous lines) and $H_\alpha(N, \psi) = 10$ (dashed lines) for $N = 3$ (thick lines) and $N = 10$ (thin lines). Below the continuous lines, the inclination of the average magnetic field on the shock plane dominates the field lines wandering and the transport is well described by a simple scattering of the particles along quasi-regular magnetic field lines. Above these continuous lines, the field lines wandering must be taken into account. Above the dashed lines, its effect is very strong.

only be neglected if the shock is quasi-parallel with $\psi < 0.6$. As the speed of the shock decreases, the condition of small obliquity to enter the regime of simple-scattering transport becomes looser for a given N.

4 Is the switch-on of SN1987A radio emission related to the decrease of the shock speed?

In order to be accelerated at the shock front, the GeV electrons need to be scattered at least a few times, say N_0, between two crossings of the shock front. As the number of scatterings N increases with decreasing shock speed for a given turbulence, the shock speed has to be lower than a threshold value U_{1_0} for the acceleration process to be possible. From the expression (5) for the factor H_α, we can relate the number of scatterings N to the speed of the shock U_1 and the parameter ε. In the case of SN1987A, $N \approx 10^5\varepsilon(4v/9U_1)$ and the threshold speed is given by $U_{1_0} \approx (4/9N_0)10^5\varepsilon c$. We have argued that ε should be at least of the order of 10^{-5}. We do not think however that ε could be much larger than 10^{-5} since it is of the order of 10^{-4} in the inner heliosphere during intense solar activity. — From the extent of unresolved rims of supernova remnants, Achterberg et al. (1994) found levels of turbulence larger than 60 times the interstellar value (which in the quasilinear approximation corresponds to $\varepsilon \geq 60\varepsilon_{ISM} = 6 \times 10^{-5}$), but this was for a magnetic field of about $3\,\mu$G at the shock. For a magnetic field of 1 mG as expected for SN1987A, the lower limit set by Achterberg et al. (1994), Eq. (28), would have barely exceeded $1/10$ of the interstellar level, which just tells us how much improvement might still be needed to resolve the rims of some supernova remnants in the radio waveband. — If we take $\varepsilon = 10^{-5}$, which we think is the most reasonable value for ε, we find for $N_0 \approx 3 - 4$, $U_{1_0} \approx 3 - 4 \times 10^9$ cm.s^{-1}. This is very close to the speed of the shock at the radio onset of SN1987A (Gaensler et al., 1997). We thus come to the conclusion that the number of scatterings N upstream of the shock, controlled by the shock speed, has probably been the determining parameter in the radio reappearance of SN1987A. This explanation could account for a sharp turn-on of the radio emission of SN1987A as expected when corrected for travel-time delays (Ball et al., 1995).

5 Conclusion

Describing the transport of the cosmic rays accelerated at a shock front as scattering along wandering magnetic field lines, we have derived a new expression for the acceleration time that takes into account both the non-diffusive nature of the magnetic field lines spreading and the precise obliquity of the shock front. For a given number of scatterings N upstream of the shock, we can predict for each shock obliquity and speed the quantitative effect of the field line wandering on the acceleration time of the particles. For relatively slow, subluminal shocks (how slow actually depends on N), we recover the usual expression of $T_{acc_{scatt}}$ valid for particles scattered along regular magnetic field lines whereas for faster shocks, T_{acc} can depart by orders of magnitude from its simple-scattering value. This explains the very long acceleration time observed for SN1987A at its reappearance in the radio waveband. Our calculation also argues in favor of a radio silence of SN1987A prior to 1990 due to a lack of scattering of the GeV electrons when the shock speed exceeds about one tenth of the speed of light.

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