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Photonuclear interaction of muons and generalized vector dominance model

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Abstract. Possibility of a description of the photonuclear interactions of high energy muons using generalized vector dominance model (GVDM) is discussed. It is shown that the consistent GVDM scheme (i.e., the scheme which operates with more or less realistic vector mesons rather than with effective ones) alone is not able to describe the photonuclear interactions at very high energies of muons. Two-component picture of photonuclear interaction (GVDM + perturbative QCD) is proposed.

1 Introduction

According to GVDM the imaginary part of the transverse forward Compton scattering amplitude (or the transverse photon absorption cross section) can be expressed in a form of the mass dispersion relation,

$$\sigma_T(Q^2,s) = \int \frac{\rho_T(m^2,m'^2,s)m^2m'^2}{(m^2+Q^2)(m'^2+Q^2)}dm^2dm'^2. \quad (1.1)$$

The spectral weight function ρ_T is given by the formula of GVDM (in zero-width approximation):

$$\rho_T(m^2,m'^2,s) =$$

$$= \sum_{n,n'} \delta(m^2-m_n^2)\delta(m'^2-m_{n'}^2)\frac{e}{f_n}\frac{e}{f_{n'}}\frac{ImT_{nn'}(s)}{s}. \quad (1.2)$$

Here, m_n is the vector meson mass, f_n is the meson-photon coupling constant, $T_{nn'}(s)$ is an amplitude for the forward meson-nucleon scattering,

$$V_n + N \longrightarrow V_{n'} + N. \quad (1.3)$$

The main problem is the description of $\rho_T(m^2,m'^2,s)$ in a region of large vector meson masses $(m_n >> m_\rho, m_\omega, m_\varphi)$. A correct treatment of the heavy masses would provide, in particular, the convergence of the integral (1.1). The information about vector meson properties in the heavy mass region is rather scarce, therefore in pre-QCD era for the proper choice of the mass dependence of ρ_T the motivation based on parton models was used. It was shown that, in general, ρ_T can be chosen in the form compatible with Bjorken scaling. In particular, in diagonal approximation, when

$$\rho_T(m^2,m'^2,s) = \delta(m^2-m'^2)\rho_T(m^2,s), \quad (1.4)$$

one needs, for this compatibility, the hadronic state continuum term in $\rho_T(m^2,s)$. If the ρ_T-function is nondiagonal, the scaling behavior of $Q^2\sigma_T$ is possible in the more realistic case of isolated hadronic states as well. If, e.g., the ρ_T-function contains large negative off-diagonal contributions, scaling can be achieved through the destructive interference effects, i.e. through the strong cancellations of diagonal and off-diagonal contributions in the integral (1.1) (such a picture was confirmed by the direct calculation of $\rho_T(m^2,m'^2,s)$) in a framework of the covariant parton model). The nondiagonal GVDM based on this picture was really very successful in a description of the nucleon structure functions at small Q^2, besides, the qualitatively correct Q^2-dependence of the nuclear shadowing was obtained. It was shown recently that this model even predicts, similarly to the parton model, the color transparency effects. One should note, however, that the choice of the nondiagonal elements of $T_{nn'}$-matrix in this model has in fact no connection with the predictions of hadronic models. In this sense nondiagonal GVDM uses some fictitious vector mesons. But the original GVDM's idea is that a photon transforms virtually just into the genuine hadron states (those observed in e^+e^--annihilation) which subsequently scatter from the target nucleon. Correspondingly, the f_n-constants in Eq.(1.2) are expressed through the leptonic widths of these states. The logic of GVDM should be such that the hadronic physics is a starting point and the scaling behavior of $Q^2\sigma_T$ is the (nonnecessary, in principle) consequence.

The consistent carrying out of nondiagonal GVDM calculations using the physically well-grounded conceptions of the

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meson-nucleon scattering amplitudes and structures of vector mesons (it is the main topic of this paper) gives the following results: i)the convergence of GVDM sums can be obtained only by introduction of some cut-off factors and ii)an agreement with data on structure functions is unattainable in the region of very low x even at Q^2 ∼few Gev^2. It means that the process of cutting is sufficient: the GVDM describes only the part of the Compton amplitude. Evidently, the remainder should be described by pertubative QCD. It leads to a two-component picture (the idea of two-component description of virtual photon absorption is, actually, very old).

2 The model of the hadronic amplitudes

For a calculation of vector meson-nucleon scattering amplitudes we used an aproach based on constituent quark model of vector mesons and two -gluon exchange approximation for hadron-hadron elastic scattering amplitudes (see, e.g. Nikolaev and Zakharov, 1991). Starting point is the general expression for V_nN amplitude:

$$F_{nn}(s,t) = \int d^2 r_\perp dy F_{\vec{\boldsymbol{r}}_\perp}(s,t)\Phi_n^2(\vec{\boldsymbol{r}}_\perp, y) \equiv$$

$$\equiv < n \mid F_{\vec{\boldsymbol{r}}_\perp}(s,t) \mid n >, \qquad (2.1)$$

Here, $F_{\vec{\boldsymbol{r}}_\perp}$ is "eigenamplitude" , i.e. , an amplitude for the scattering of the $q\bar{q}$-pair with a fixed transverse size $\vec{\boldsymbol{r}}_\perp$ on the nucleon, $\Phi_n(\vec{\boldsymbol{r}}_\perp, y)$ is a quark-antiquark wave function of the vector meson , y is the light-cone variable connected with longitudial momentum partition of quarks inside of the vector meson. The concrete expression for $F_{\vec{\boldsymbol{r}}_\perp}$ is determined by the two-gluon exchange diagrams:

$$F_{\vec{\boldsymbol{r}}_\perp}(s,t) = i\frac{16}{3}\alpha_s^2 s \int \frac{d^2 k_\perp V(\vec{\boldsymbol{k}}_\perp, \vec{\boldsymbol{Q}})}{(\frac{\vec{\boldsymbol{Q}}}{2} - \vec{\boldsymbol{k}}_\perp)^2 (\frac{\vec{\boldsymbol{Q}}}{2} + \vec{\boldsymbol{k}}_\perp)^2} \times$$

$$\times \{ e^{-i\frac{\vec{\boldsymbol{Q}}}{2}\vec{\boldsymbol{r}}_\perp} - e^{-i\vec{\boldsymbol{k}}_\perp \vec{\boldsymbol{r}}_\perp} \}. \qquad (2.2)$$

Here, $t = -\vec{\boldsymbol{Q}}^2$, $\vec{\boldsymbol{k}}_\perp$ is a transverse momentum of the meson‘s quark, and the V-factor describes the ggNN-vertex. Approximately, this factor is given by the expression

$$V(\vec{\boldsymbol{k}}_\perp, \vec{\boldsymbol{Q}}) \simeq e^{-\frac{<r_N^2>}{6}\vec{\boldsymbol{Q}}^2} - e^{-\frac{<r_N^2>}{6}(\frac{\vec{\boldsymbol{Q}}^2}{4} + 3\vec{\boldsymbol{k}}_\perp^2)}. \qquad (2.3)$$

where $< r_N^2 >$ is the mean square radius of the nucleon.

Integrating over asimutal angles in Eq.(2.1) we reduce the problem to calculation of $F_{|\vec{\boldsymbol{r}}_\perp|} \equiv F_{r_\perp}$. This amplitude depends only on two parameters, α_s and μ_g, effective gluon mass (omitted in Eq.(2.2) for brevity's sake). Going into impact parameter space, we introduce the opaque function

$$\Omega_{r_\perp}(s,b) = \frac{1}{2\pi i}\int \frac{1}{4\pi s} F_{r_\perp}(s,t)\, e^{\,i\vec{\boldsymbol{Q}}\vec{\boldsymbol{b}}} d^2 Q. \qquad (2.4)$$

The numerical calculation shows that $\Omega_{r_\perp}$ can be parametrized with a large accuracy by the Regge-type expression:

$$\Omega_{r_\perp}(s,b) = \frac{\sigma(r_\perp)}{4\pi B_{r_\perp}} exp(-\frac{b^2}{2B_{r_\perp}}), \qquad (2.5)$$

where

$$\sigma(r_\perp) = \frac{1}{s} Im F_{r_\perp}(s,0); \;\; B_{r_\perp} = \frac{\sigma(r_\perp)}{4\pi\Omega_{r_\perp}(s,0)}. \qquad (2.6)$$

We took in this analysis $\mu_g = \mu_\pi$ and normalized $\sigma(r_\perp)$ on the pion data at medium energies ($\sqrt{s} = 10\ Gev$), in accordance with the additive quark model relation

$$\sigma_{\rho p} = \frac{1}{2}(\sigma_{\pi^+ p} + \sigma_{\pi^- p}). \qquad (2.7)$$

For this normalization we used the unitarized scattering amplitude

$$T_{\rho\rho}(s,0) = \int < \rho \mid 1 - e^{-\Omega_{r_\perp}(s,b)} \mid \rho > d^2 b. \qquad (2.8)$$

Up to now in our model $F_{r_\perp} \sim s$ so that $\Omega_{r_\perp}$ does not depend on the energy. To take into account this dependence we modify Eq.(2.5) adding a new Regge-type term:

$$\Omega_{r_\perp}(s,b) = \frac{\sigma(r_\perp)}{4\pi} \times$$

$$\times \left\{ \frac{1}{B_{r_\perp}} e^{-\frac{b^2}{2B_{r_\perp}}} + \frac{1}{R}\frac{1}{B_{r_\perp} + 2\alpha'_F \xi} e^{\Delta_F \xi} e^{-\frac{b^2}{2(B_{r_\perp} + 2\alpha'_F \xi)}} \right\}, \qquad (2.9)$$

$$\xi = ln\frac{s}{s_0} - \frac{i\pi}{2}; \;\; \Delta_F = \alpha_F - 1 > 0; \;\; \alpha'_F \neq 0.$$

Writing Eq.(2.9) we suppose a two-pole Regge parametrization (Kopeliovich at al ,1989) of the opaque function; by assumption, both trajectories give at small ξ the same diffraction slopes and R does not depend on $r_\perp$. Three new parameters (R, Δ_F, α'_F) are determined from data on s-dependence of hadronic total amplitudes.

Nondiagonal amplitudes are given by a simple generalization of Eq. (2.1):

$$F_{nn'}(s,t) =< n \mid F_{\vec{\boldsymbol{r}}_\perp}(s,t) \mid n' > . \qquad (2.10)$$

For a calculation of the amplitudes $F_{nn'}$ we need also expression for wave functions of vector mesons and meson's mass spectrum. We obtained the wave functions and mass spectrum using the quasipotential formalism in a light-front form (Bugaev, Mangazeev, 1999; Bugaev, Mangazeev and Shlepin, 1999). The resulting wave functions are of oscillator type: e.g., for the first term in ρ-family ($\rho-$meson) one has

$$\Phi_0(\vec{\boldsymbol{r}}_\perp, y) = N_0 exp[-r_\perp^2\beta^2/2] exp[-m_\rho^2 y^2/2\beta^2]. \qquad (2.11)$$

Here, $\beta^2 = 0.094\ Gev^2$. The mass spectrum for the ρ-family is

$$m_n^2 \cong m_\rho^2(1 + 2.55n)\ ; \;\; n = 0, 1, 2, \dots . \qquad (2.12)$$

3 Cut-off factors

The first stage of the photoabsorption is the $\gamma \to qq$ transition. The differential probability of this transition is

$$dP_{q\bar{q}} \cong C\frac{1}{M^2_{q\bar{q}}}[x^2+(1-x)^2]dxdM^2_{q\bar{q}}, \qquad (3.1)$$

where x is the fraction of the photon 3-momentum carried by the quark, $M_{q\bar{q}}$ is an invariant mass of the $\bar{q}q$-pair,

$$M^2_{q\bar{q}} = (p^2_\perp + m^2_q)/x(1-x). \qquad (3.2)$$

Our basic assumption is the following: an interaction of the $\bar{q}q$-pair with the nucleon is meson-dominanted if (and only if) this pair is wide enough (i.e. if $\bar{r}_\perp > r^0_\perp$, where $r^0_\perp$ is some parameter); only in this case confinement forces are effective and pull the pair's particles together. The corresponding restriction of pair's phase volume for a fixed $M_{q\bar{q}}$ leads, evidently, to appearing of some cut-off factors in GVDM's sums over meson states. These cut-off factors are determined from the expression for an average transverse size of the $\bar{q}q$-pair,

$$\bar{r}_\perp = v_{\perp,relative}\cdot\tau_{fe} \cong \frac{p_\perp}{p^2_\perp+m^2_q}(1+\frac{Q^2}{M^2_{q\bar{q}}})^{-1} \qquad (3.3)$$

and from expressions (3.1-3.2). As a result, in our GVDM formulas the followings substitutions must be done:

$$\frac{e}{f}_n \longrightarrow \frac{e}{f}_n\sqrt{\eta}_n \equiv \frac{e}{\tilde{f}_n}. \qquad (3.4)$$

Here $\sqrt{\eta}_n$ is the cut-off factor (determinated using the additional approximation $M_{q\bar{q}} \sim M_n$).

4 Structure functions

Electromagnetic structure functions are calculated using the usual GVDM formulas, following from (1.1 - 1.2) and modified by introducing the cut-off factors. In particular, the transverse cross section is given by the expression

$$\sigma_T(Q^2,x) = \sum_{n,n'}\frac{e^2}{f_n f'_n}\sqrt{\eta_n\eta_{n'}}\frac{ImT_{nn'}}{s}. \qquad (4.1)$$

The resulting formulas for Im$T_{nn'}$ are very complicate. If we keep only the linear term in an expansion of the exponent in Eq.(2.8), we have

$$ImT_{nn'} \sim \sigma_{nn'}(1+\frac{1}{R}(\frac{s}{s_0})^{\Delta_F}cos\frac{\pi}{2}\Delta_F). \qquad (4.2)$$

Here, each $\sigma_{nn'}$ is expressed through integrals of the type

$$\int\int r_\perp dr_\perp dy\sigma(r_\perp)f(r_\perp,y),$$

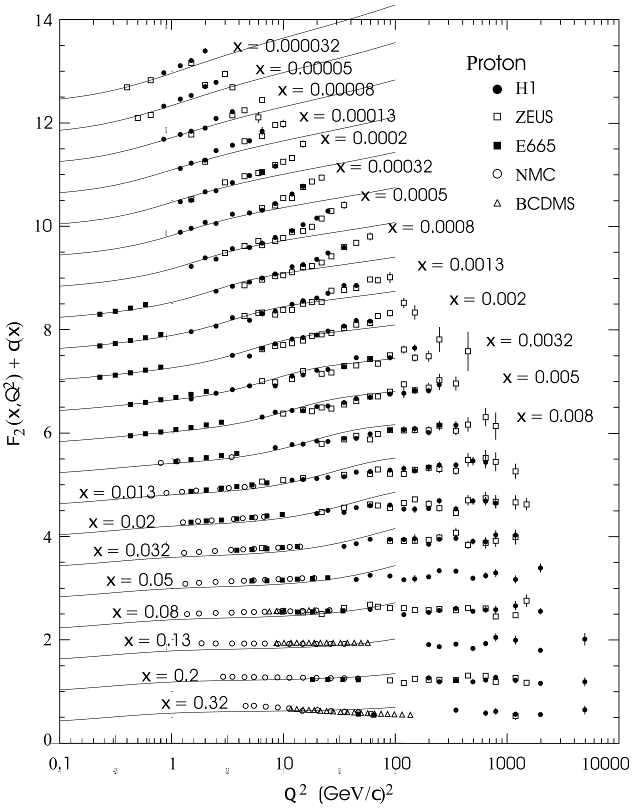

Fig. 1. The proton structure function $F_2(Q^2,x)$.The solid curves are our results. The data are due to H1, ZEUS, BCDMC, E665, NMC. A constant $c(x) = 0.6(i_x - 0.4)$ is added to F_2, where i_x is the number of the x bin ranging from 1 ($x = 0.32$) to 21 ($x = 0.000032$).

where $f(r_\perp,y)$'s are cumbersome functions of $r_\perp, y$ which depend on the wave functions of vector mesons. The connection of s with Q^2 and x is

$$s = \frac{Q^2}{x} + m^2_p - Q^2. \qquad (4.3)$$

The formula for longitudinal cross section σ_L differs from (4.1) only by the amplitude factors:

$$ImT^L_{nn'} = \frac{Q^2}{m_n m_{n'}}\xi(K)ImT_{nn'}, \qquad (4.4)$$

$$K = \frac{s}{2M_p}.$$

For the ratio of the longitudinal VN-amplitude to the transverse one we used the parametrization

$$\xi = 1 - e^{-\frac{K}{K_0}},\ K_0 = 10^3. \qquad (4.5)$$

In Figs. 1-3 the results of our calculations are shown, together with available data on structure functions. For the calculation the following values of parameters have been used:

$$R = 22,\ \Delta_F = 0.25,\ (r^0_\perp)^{-1} = 0.4\text{Gev},\ m_q = 0.$$

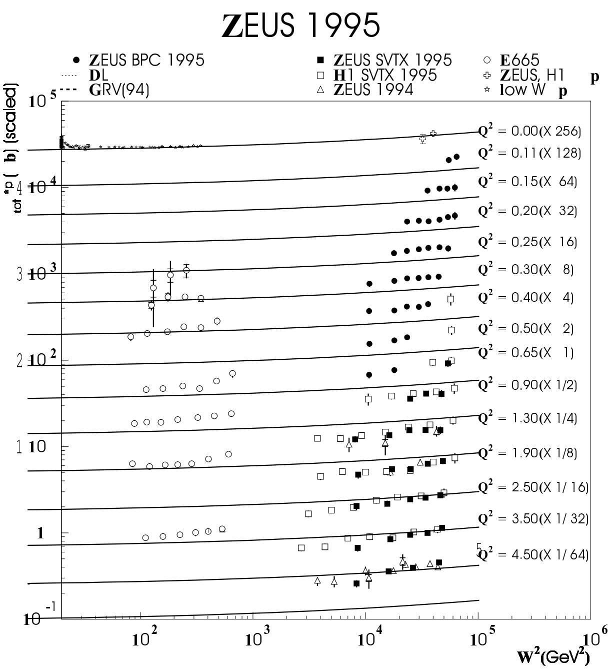

Fig. 2. Transverse cross section $\sigma_T(W^2, Q^2)$. The solid curves are predictions of the present paper.

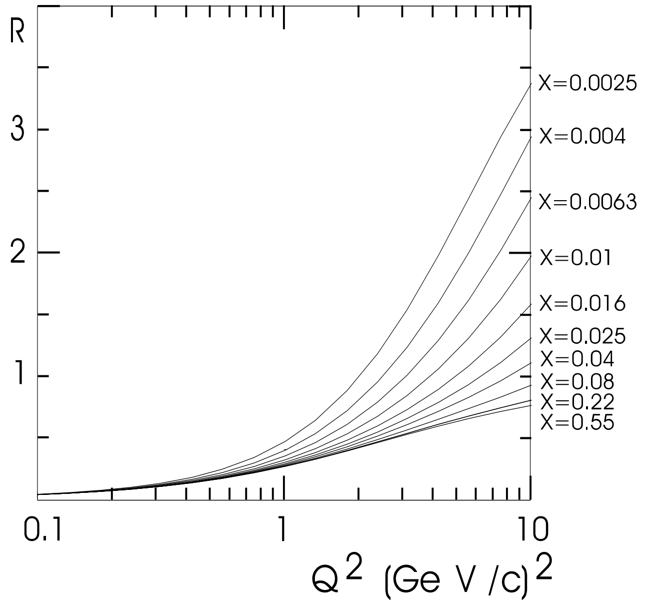

Fig. 3. Longitudinal-to-transverse ratio,$R = \sigma_L/\sigma_T$.

5 Conclusion

The numerical results obtained in the present model lead to the following conclusions.

1. If no cut-offs are introduced, GVDM is not able to describe photoabsorption data. Even the simplest variant of the GVDM containing only ρ and ρ' give too large value of $\sigma_{\gamma N}$. Nondiagonal contributions are not negligibly small. Destructive interference effects proposed in earlier works on nondiagonal GVDM are not large (in particular, the largest nondiagonal term $(\rho\rho')$ is positive).

2. The introduction of the cut-off factors motivated by QCD can give the correct predictions. This is reached without nonnatural break of the meson mass spectrum (the value of the heaviest mass in GVDM expressions is determined solely by the condition that the longitudinal size of the fluctuation must exceed the target size). In the scheme with the cut-offs the nondiagonal contributions have the same order of magnitude as neighbouring diagonal ones, so GVDM developed in the present paper is an essentially nondiagonal model.

3. The GVDM alone is not able to describe correctly the electromagnetic structure functions in the region of very small x(even at $Q^2 \sim 1$ Gev^2). For a full description of the structure functions one needs a two-component model. Evidently, the second component must be based on perturbative QCD.

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