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On magnetic fluctuation spectra in the solar wind and the influence of mode dispersion

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Abstract. Magnetic fluctuation power spectra in the solar wind are commonly observed to have a power-law form with a spectral index $s = 5/3$ at frequencies lower than about 1 Hz. This characteristic feature of magnetic fluctuation spectra defines what is called the inertial range and may be described, in wavenumber space, by Kolmogorov diffusion. For higher frequencies, it has been suggested that collisionless damping of Alfvén and magnetosonic waves leads to steeper power-laws; this regime is sometimes labeled as the dissipation range. Here we argue, based on numerical calculations, that it is more likely that the observed steeper power-laws result from an increase in the wavenumber diffusion rate caused by whistler-like dispersion than from collisionless damping. The calculations lead to the prediction, that this broken power-law feature of magnetic fluctuation spectra is only observable in low-β_p plasmas.

1 Introduction

Observations on spacecraft show magnetic fluctuations in the solar wind over a broad range of frequencies, from well below the proton cyclotron frequency Ω_p ($\sim 0.1 - 1$ Hz) to several hundred Hz (Coleman, 1968; Gurnett, 1991). In the spacecraft rest frame, the observed power spectrum frequency f shows a power-law $f^{-5/3}$ between $\sim$ 0.001 and 1 Hz. For higher frequencies ($f > 1$ Hz), observed spectra show steeper power-laws with an spectral index of roughly 3 (cf. Goldstein et al., 1994; Leamon et al., 1998). It is usually assumed that the observed frequency spectra correspond to wavevector spectra which are Doppler-shifted by the rapid flow of the solar wind. Under this assumption the discussion of power spectra is usually framed in terms of wavelengths and/or wavenumbers. It is widely accepted that the $k^{-5/3}$ spectrum is the "inertial range" of MHD turbulence in the solar wind, presumably resulting from nonlinear cascade processes from longer to shorter wavelengths. The different power-law dependencies in wavenumber are sometimes interpreted such that the breakpoint between the two ranges represents the onset of collisionless damping (Denskat et al., 1983). Thus, the shorter wavelength regime is sometimes called the "dissipation range" (e.g. Goldstein et al., 1994; Leamon et al., 1998, 1999, 2000). Collisionless damping of Alfvén and/or magnetosonic waves was used by several authors in order to describe this dissipation (see e.g. Marsch, 1991; Leamon et al., 1998; Gary, 1999; Marsch, 1999).

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Assuming that magnetic fluctuations can be treated as an ensemble of linear plasma wave modes, Li et al. (2001) investigated the influence of collisionless damping of left-hand circular polarized Alfvén and right-hand circular polarized magnetosonic modes on magnetic power spectra at $kc/\omega_p > 1$ (where ω_p is the proton plasma frequency) as a function of $\beta_p = 8\pi n_p T_p/B_0^2$ (here n_p and T_p are the proton number density and the proton temperature in energy units, respectively) and different propagation angles with respect to the background magnetic field $\boldsymbol{B}_0$. Using a linear Vlasov theory approach for damping functions, Li et al. (2001) showed that, for most directions of wave propagation, damping rates increase so strongly in wavenumber that they overwhelm the spectral energy input from the cascade mechanism, resulting in sharp cutoffs in fluctuation power spectra. For parallel propagating magnetosonic waves at low β_p, Li et al. (2001) concluded that the small proton cyclotron damping rate leads to continuous Kolmogorov spectra. Thus, it appears that dissipation can not explain the relatively steep power law spectra observed at wavenumbers beyond the inertial range.

Based on the work of Li et al. (2001), Stawicki et al. (2001) considered recently the specific case of the magnetosonic/whistler mode for $\boldsymbol{k} \times \boldsymbol{B}_0 = 0$ and $\beta_p < 1$. Under these conditions proton cyclotron damping is weak, so that dispersion begins at wavenumbers considerably smaller than those at the onset of damping and, therefore, becomes more important for the diffusion of fluctuation energy at short wavelengths. At long wavelengths, i.e. the inertial range, the cascading of wave energy is described by Kolmogorov diffusion until a breakpoint is reached where dispersion sets in. With a faster energy transfer rate resulting from this dispersion, power spectra become steeper at intermediate wavenumbers; this is the dispersion range. At shorter wavelengths, where collisionless electron cyclotron damping dominates, the cascade mechanism is too weak to sustain power law spectra, and magnetic power spectra decline precipitously with increasing wavenumber; this is the dissipation range.

2 Energy Diffusion in Wavenumber Space

In order to describe the evolution of magnetohydrodynamic turbulence by diffusion of fluctuation energy in wavenumber space, Zhou and Matthaeus (1990) derived, based on phenomenological and dimensional arguments, a transport equation for the wave spectral density including terms for spatial convection and propagation, nonlinear energy transfer across wavenumber space and a source and/or sink of wave energy. For isotropic turbulence the diffusion equation for the omnidirectional spectral density $W(k)$ is

$$\frac{\partial W(k)}{\partial t} = \frac{\partial}{\partial k}\left[k^2 D(k)\frac{\partial}{\partial k}\left(k^{-2}W(k)\right)\right] + \gamma(k)W(k) + S(k) \qquad (1)$$

where the first term of the right-hand side represents diffusion of fluctuation energy in wavenumber space, expressed by a diffusion coefficient

$$D(k) = k^2/\tau_s(k)\ . \qquad (2)$$

Here $\tau_s(k)$ denotes the spectral energy transfer time scale. The last two terms of the right-hand side represent, by the damping rate $\gamma(k)$, collisionless dissipation of the fluctuation and a source function $S(k)$ for wave energy injection, respectively (see also Miller et al., 1996).

Even though equation (1) is derived from phenomenological and scaling arguments, it offers an attractive and tractable way of modeling the energy cascade process in wavenumber space. Assuming that damping is negligible for $k < k_d$, where k_d is the dissipation wavenumber beyond which $\gamma(k)$ becomes important, the spectrum $W(k)$ in this (inertial) range is mostly determined by $D(k)$, which depends upon the cascade phenomenology, i.e. the spectral energy transfer time scale τ_s. For the Kolmogorov phenomenology, Zhou and Matthaeus (1990) proposed

$$D(k) = C^2 v_A k^{7/2}\left[W(k)/2U_B\right]^{1/2}\ , \qquad (3)$$

where C^2, v_A and $U_B = B_0^2/8\pi$ denote a constant, the Alfvén speed and the energy density of the background magnetic field $\boldsymbol{B}_0$, respectively. Upon substituting this into equation (1) and assuming a steady state with no damping, we obtain the usual Kolmogorov spectrum in the inertial range $W(k) \propto k^{-s}$, where $s = 5/3$.

3 Collisionless Damping and Mode Dispersion

To obtain damping rates for the magnetosonic/whistler mode at $\boldsymbol{k} \times \boldsymbol{B}_0 = 0$ and to develop quantitative criteria for two important parameters, i.e. for the dissipation wavenumber k_d and the dispersion wavenumber k_ω, Stawicki et al. (2001) used the linear Vlasov theory for fully electromagnetic fluctuations under the assumption of a collisionless and homogeneous electron-proton plasma in which both species have Maxwellian velocity distributions and $T_p = T_e$.

Using the numerical solutions, Stawicki et al. (2001) derived fitting functions for the damping rates of the magnetosonic/whistler mode in the proton as well as in the electron cyclotron damping range and for the real frequency $\omega_r(k)$ in order to use them in the solution of the transport equation describing the evolution of fluctuation energy in wavenumber space.

For the proton cyclotron damping regime, which corresponds roughly to $0 < kc/\omega_p < 10$, an appropriate fitting function is

$$\frac{\gamma(k_\parallel)}{\Omega_p} = -\mu_1 \exp\left(-\mu_2 k_\parallel^2 c^2/\omega_p^2\right)\exp\left(-\mu_3\omega_p^2/k_\parallel^2 c^2\right) \qquad (4)$$

where the fitting parameters, on the domain $0.50 \le \beta_p \le 10.0$, are

$$\mu_1 = 0.33\beta_p^{0.54}\exp\left(-3.97/\beta_p^2\right)$$
$$\mu_2 = 0.80/\beta_p^{1.07} \ \text{and}\ \mu_3 = 1.73/\beta_p^{0.91} \qquad (5)$$

For the electron cyclotron damping regime, Stawicki et al. (2001) obtained, on the domain $0.10 \le \beta_e \le 10.0$, for the fitting function the expression

$$\frac{\gamma(k_\parallel)}{\Omega_p} = -\nu_1\left(k_\parallel c/\omega_p\right)^2\exp\left(-\nu_2\omega_p^2/k_\parallel^2 c^2\right) \qquad (6)$$

with the corresponding fitting parameters

$$\nu_1 = 0.46\beta_e^{0.26}\ \text{and}\ \nu_2 = 893/\beta_e^{0.57}\ . \qquad (7)$$

Gary (1999) defined the dissipation wavenumber k_d as the smallest wavenumber corresponding to $\gamma/\Omega_p = -0.10$. Generalising this definition in order to accomodate the higher frequencies of the whistler mode, we define the dissipation wavenumber correspondingly to the onset of cyclotron damping by $k_d^2c^2/\omega_p^2 = \mu_3$ and ν_2 in the fitting functions (4) and (6). Using the fitting parameters μ_3 and ν_2, one obtains in the proton and electron cyclotron regimes

$$k_d c/\omega_p = 1.32/\beta_p^{0.46}\ \text{and}\ k_d c/\omega_p = 29.9/\beta_e^{0.29} \qquad (8)$$

To determine the dispersion wavenumber k_ω, which corresponds to the value at which the real part of the dispersion relation, $\omega_r(k)$, begins to depart significantly from the dispersionless relation ω_r/k =constant, Stawicki et al. (2001) derived, based on numerical solutions to the linear dispersion equation, an approximate empirical fit for the real frequency on the wavenumber domain $0 \le kc/\omega_p \le 4.0$, which is the range of significant proton cyclotron damping for β_p lower than about 4,

$$\omega_r/\Omega_p = kc/\omega_p + 0.75\,(kc/\omega_p)^2 \ , \tag{9}$$

confirming the suggestion of Gary (1993) that $\omega_r(k)/\Omega_p$ versus kc/ω_p is relatively independent of β_p at low frequencies [see Fig. 6.3 of Gary (1993)]. Estimating the onset of a clear departure from $\omega_r = v_A k$ near $kc/\omega_p \simeq 1$, Stawicki et al. (2001) found that the dispersion wavenumber for the magnetosonic/whistler mode scales as

$$k_\omega c/\omega_p = 0.9 \ . \tag{10}$$

To take into account the influence of dispersion properties on the diffusion of fluctuation energy beyond the dispersion wavenumber, Stawicki et al. (2001) assumed that the spectral energy transfer time scale is proportional to the inverse of the mode frequency, yielding for the corresponding diffusion coefficient, after substituting Eq.(9) into Eq.(2) and, since dispersive effects are appreciable for kc/ω_p greater than about unity, neglecting the first term in Eq.(9),

$$D(k) \simeq Ck^4 \ , \tag{11}$$

where C is a constant.

4 Numerical Calculations

To solve Eq.(1) numerically we used the Crank-Nicholson technique. We chose 10^{-4} Gauss for the background magnetic field and 30 km s^{-1} for the Alfvén speed. Through the source function $S(k)$, we injected fluctuation energy at $kc/\omega_p = 0.002$ with a rate of 10^{-15} erg cm^{-3} s^{-1}. In contrast to Li et al. (2001) who assumed that Kolmogorov diffusion is the only energy transfer process, we here used equation (11) for $D(k)$ at intermediate wavenumbers.

Figure 1 illustrates the temporal evolution of a magnetic power spectrum for $\beta_p = 0.5$, showing that steady-state conditions are attained relatively quickly, due to the increased energy flux rate in the dispersion range [cf. Fig. 2 in Li et al. (2001)]. We obtain the usual $k^{-5/3}$ power spectrum in the inertial range and, beyond the dispersion wavenumber $k_\omega c/\omega_p \simeq 1$, a steeper power-law with $s = 3$ in the dispersion range, followed by an even more rapidly diminishing "cutoff" spectrum beyond the electron cyclotron dissipation wavenumber $k_d c/\omega_p \simeq 30$. Since proton cyclotron damping is relatively weak at low values of β_p, an influence of the ion cyclotron dissipation wavenumber is negligible.

Figure 2 illustrates the influence of magnetosonic/whistler dispersion and damping on late-time, steady-state magnetic fluctuation power spectra in our model. Here we consider three values for β_p and obtain in the inertial range the $k^{-5/3}$ power spectra in each case. Near the dispersion wavenumber, $k_\omega c/\omega_p \simeq 1$, the character and properties of the power spectra changes sensitively as functions of β_p. Therefore, we regard the wavenumber domain which is defined by k_ω and the electron cyclotron dissipation wavenumber k_d as the dispersion range and shorter wavelengths as the dissipation range,

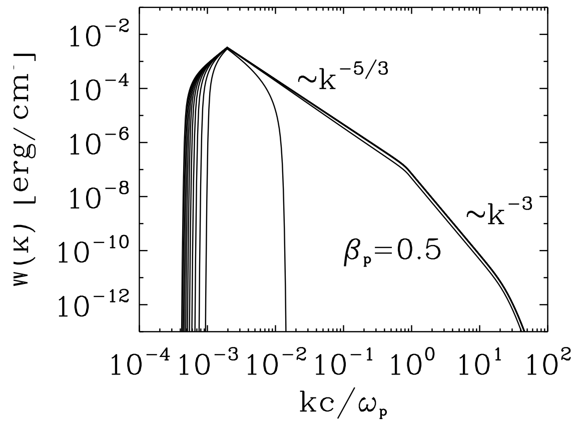

Fig. 1. Numerical result from solving equation (1) showing the temporal evolution of the power spectrum $W(k)$ for $\beta_p = 0.5$ as a function of the dimensionless wavenumber kc/ω_p, where the diffusion coefficient for the inertial range is given by Eq.(3), and for the intermediate wavenumber regime by Eq.(11). The proton and electron cyclotron damping rates are given by equations (4) and (6) with the corresponding fitting parameters.

resulting from electron cyclotron damping overwhelming the fluctuation energy which cascades down from longer wavelengths in the dispersion range. With increasing β_p ion cyclotron damping becomes more appreciable over a limited wavenumber range in the dispersion regime resulting in a short-range dip in the power spectrum, followed by a dispersive regime in $W(k)$ with $s = 3$ until the cutoff spectrum of the electron cyclotron dissipation range is reached at $k_d c/\omega_p \simeq 20$. Finally, at $\beta_p = 4.5$, proton cyclotron damping becomes strong enough to completely absorb the fluctuation energy cascading down from the inertial range, no dispersion range is evident, and at $k_d c/\omega_p \simeq 0.4$ the proton cyclotron dissipation range, with its cutoff power spectrum, begins.

In summary, increasing β_p leads to a shrinking of the dispersion range, resulting from the shifted onset of the electron cyclotron dissipation range to lower wavenumbers. Finally, at higher β_p values the dispersion range disappears and is replaced by the ion cyclotron dissipation range with its strong cutoffs.

5 Conclusions

We used a model of weakly turbulent magnetic fluctuations, Kolmogorov diffusion of fluctuation energy at long wavelengths and collisionless proton and electron cyclotron damping at intermediate and short wavelengths, respectively. To consider the influence of wave dispersion on solar wind magnetic power spectra we used, based on dispersion and damping of the magnetosonic/whistler mode derived from linear Vlasov theory, a modified fluctuation energy transfer rate

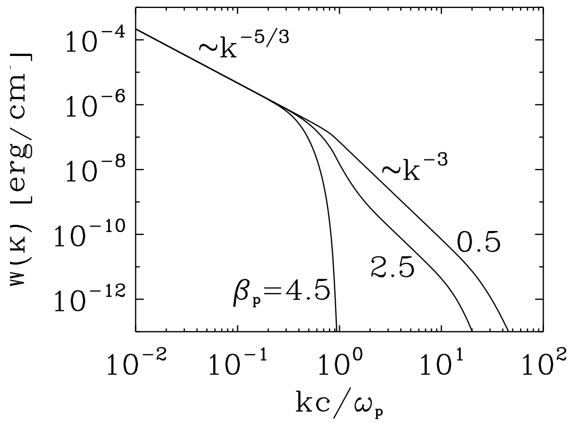

Fig. 2. Computational model results for the power spectrum: late-time values of magnetic fluctuation power spectra for three given values of β_p as functions of the dimensionless wavenumber kc/ω_p, using the same diffusion coefficients and damping functions as for Fig. 1.

at intermediate wavenumbers. Proton cyclotron damping is very weak for $\beta_p < 1$ and $\boldsymbol{k} \times \boldsymbol{B}_0 = 0$, so that in this case the onset of whistler dissipation is due to electron cyclotron damping, at considerably smaller wavelengths than the onset of dispersion.

Using this scenario in a numerical code which describes the transfer of fluctuation energy in wavenumber space, we find that $W(k)$ shows the major features of magnetic power spectra observed in the solar wind, i.e. a long wavelength inertial range with $s = 5/3$ and a steeper power-law spectrum at intermediate wavenumbers with $s = 3$. We call this intermediate regime the "dispersion range". The true dissipation range is at still shorter wavelengths where we find cutoffs in the power spectrum. Finally, increasing β_p leads to a stronger proton cyclotron damping at $kc/\omega_p \approx 1$ where dispersion also begins. In this case, we find sharp cutoffs instead of steep power-laws for the power spectra.

The results support the conclusion of Li et al. (2000) that the relatively steep power-law magnetic spectra observed at intermediate frequencies in the solar wind are not due to collisionless damping. We have extended this result by concluding that such power-law spectra are likely due to an increase in the energy transfer rate of magnetic fluctuation energy at wavenumbers associated with wave dispersion in the absence of damping. We find a dispersion range for magnetosonic/whistler modes at $\beta_p < 2.5$ and $\boldsymbol{k} \times \boldsymbol{B}_0 = 0$. Therefore, we predict that such steep power law magnetic fluctuation power spectra in the solar wind should be observed primarily at $\beta_p < 2.5$, with right-hand polarization, and with wavevectors parallel or antiparallel to the background magnetic field. The power spectra observed by Denskat et al. (1983), which exhibit f^{-3} behaviour to frequencies much above the Doppler-shifted proton cyclotron frequency, would appear to support this whistler-based prediction.

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