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Study of the longitudinal development of air showers with CORSIKA

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Abstract. The HiRes air fluorescence experiment directly observes the Nitrogen fluorescence light of air shower cascades in the Earth's atmosphere and is therefore a unique tool to directly study the development of air showers and compare it to various models. In this paper, we fit the average shower profile and compare the quality of fit for a Gaussian function in shower age to the standard Gaisser-Hillas function and discuss how to average shower profiles in terms of age for alignment. To determine the χ^2 for profile fitting in the simulation, effects caused by the use of the thinning algorithm have to be taken into account.

1 Introduction

Fluorescence light detectors like HiRes (Baltrusaitis *et al.*, 1985) observe the longitudinal shower development caused by cosmic ray particles, that is, the number of charged particles as a function of atmospheric depth. However, it is not easy to reconstruct showers without observing X_{max}, the depth at which the shower reaches the maximum size. Often only a part of the shower profile is observed, because the detector may not cover the entire night sky or the air shower stops developing further when it reaches the ground level. Thus, we need to extend the shower profile to both ends in order to reconstruct the air shower and to determine the shower energy which can be obtained by integrating the shower profile (Song *et al.*, 2000). For this purpose, the Gaisser-Hillas (G-H) function has been widely used (Gaisser and Hillas, 1977), but this function has the disadvantage that its parameters are correlated. In this paper, we study characteristics of 4 parameters in the G-H function with simulated air showers, and we try to find a simpler function with fewer parameters giving an equally correct description of the shower development.

The shower profile is very symmetric when plotted as a function of shower age. Consequently, we try a Gaussian function in age as an alternative to the G-H function. The advantage of using a Gaussian function rather than the G-H function is that the number of the fitting parameters is reduced and the function is independent of the starting point of the shower profile (X_0 of the G-H function) which can not be obtained by experiment. In addition, using age rather than depth enables us to align air showers with little fluctuation. Figure 1 shows the average shower profile from data along with the G-H and Gaussian function (Abu-Zayyad *et al.*, 2001). According to this figure, light produced at the early stage of the shower development rarely reaches the detector. In the end, we compare how well both functions fit the shower profile.

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2 Simulations

CORSIKA is a package for simulating air showers over a wide range of primary energies (Heck *et al.*, 1998). Several choices are available for the hadronic interaction model at the highest energies, and we have chosen the QGSJET (Kalmykov *et al.*, 1997) model. Within CORSIKA electromagnetic sub-showers are simulated with the EGS4 code. In order to reduce computing time, a thinning algorithm (Hillas, 1997) is selected within CORSIKA: if the total energy of secondary particles from a given interaction falls below 10^{-5} times primary energy, only one of the secondaries is followed, selected at random according to its energy E_i with a probability of $p_i = E_i/\sum_j E_j$. The sum does not include neutrinos or particles below the threshold energies. In our simulations the threshold energies are 300, 700, 0.1 and 0.1 MeV for hadrons, muons, electrons and photons respectively. We chose an observation level 300 m above sea level and simulate showers with a zenith angle of 45 degrees.

	$Log(E)$	X_1	X_0	N_{max}	X_{max}	λ	FWHM
p	17.0	52.8 $\pm$ 54.9	-33.3 $\pm$ 69.5	(6.60 $\pm$ 0.35)10^7	672 $\pm$ 77	67.1 $\pm$ 9.2	512 $\pm$ 26
	18.0	49.3 $\pm$ 49.5	-68.0 $\pm$ 64.1	(6.58 $\pm$ 0.26)10^8	731 $\pm$ 65	63.1 $\pm$ 5.7	529 $\pm$ 21
	19.0	46.5 $\pm$ 44.8	-86.1 $\pm$ 55.2	(6.46 $\pm$ 0.22)10^9	782 $\pm$ 68	62.4 $\pm$ 4.8	548 $\pm$ 21
Fe	17.0	12.8 $\pm$ 12.5	-30.1 $\pm$ 20.0	(5.97 $\pm$ 0.15)10^7	574 $\pm$ 19	79.1 $\pm$ 4.5	517 $\pm$ 12
	18.0	10.3 $\pm$ 10.4	-59.7 $\pm$ 17.7	(6.18 $\pm$ 0.13)10^8	632 $\pm$ 17	72.2 $\pm$ 3.4	528 $\pm$ 9
	19.0	10.5 $\pm$ 9.8	-89.1 $\pm$ 12.4	(6.29 $\pm$ 0.11)10^9	692 $\pm$ 15	66.6 $\pm$ 2.0	539 $\pm$ 7
γ	17.0	46.9 $\pm$ 43.3	-74.2 $\pm$ 64.3	(7.87 $\pm$ 0.32)10^7	764 $\pm$ 49	50.1 $\pm$ 4.6	483 $\pm$ 19
	18.0	49.8 $\pm$ 47.9	-50.3 $\pm$ 66.7	(7.46 $\pm$ 0.31)10^8	852 $\pm$ 54	52.0 $\pm$ 5.4	510 $\pm$ 24
	19.0	50.8 $\pm$ 50.2	-32.4 $\pm$ 60.0	(6.97 $\pm$ 0.33)10^9	958 $\pm$ 67	54.7 $\pm$ 6.1	548 $\pm$ 32

Table 1. The first interaction depth, the width of shower profile and 4 parameters of the G-H function for p, Fe and γ induced showers.

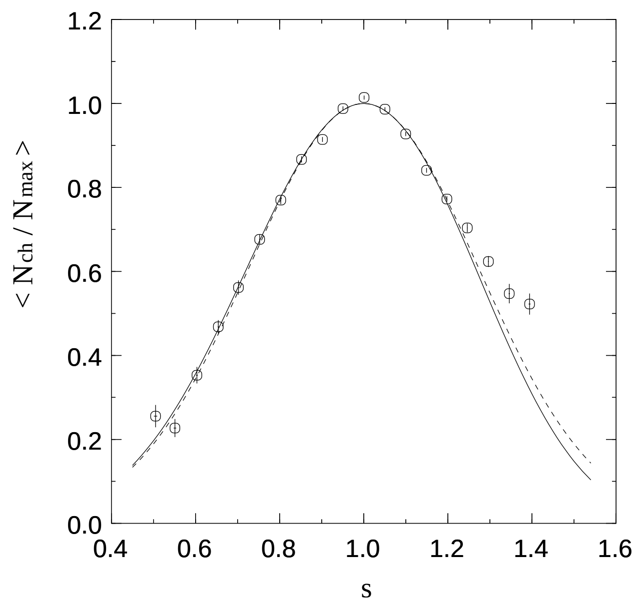

Fig. 1. The average shower profile is shown in terms of age parameter along with the G-H function (solid line) and Gaussian function (dashed line). The open circles refer to the MIA-HiRes prototype data taken from (Abu-Zayyad *et al.*, 2001).

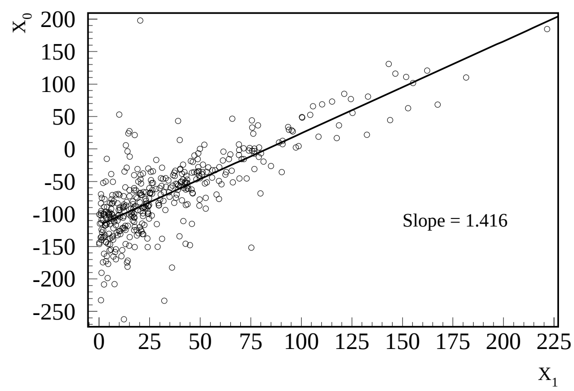

Fig. 2. The correlation between X_0 and X_1 for 400 proton induced showers at 10^{18} eV. The QGSJET model in CORSIKA was used to generate events.

3 Gaisser-Hillas Function

The G-H function is given by

$$N(X) = N_{max}\left(\frac{X_{max}-X}{X_{max}-X_0}\right)^{\frac{X_{max}-X_0}{\lambda}} e^{\frac{X_{max}-X}{\lambda}} \qquad (1)$$

where λ is a parameter depending on primary mass and energy, and X_0 has been interpreted as the first interaction depth. In fact, X_0 is a physically meaningless parameter and has mostly negative values. To show this, we define X_1 as the first interaction depth to distinguish it from X_0. Under this definition, we have $N(X_0) = 0$ and $N(X_1) = 1$. The correlation between X_1 and X_0 is shown in Figure 2. The average X_0 for different primaries is shown in Table 1.

The maximum number of shower particles N_{max} is found to be correlated with λ as shown in Figure 3 (a). As mentioned previously, the area under the longitudinal shower profile is proportional to the primary energy. Thus, we can infer that N_{max} is inversely proportional to the width of the profile. Figure 3 (b) shows the correlation between N_{max} and the width of the shower profile. λ had been known as the proton interaction length and was set to 70 g/cm^2. According to our study, λ is not the proton interaction length and depends on primary energy and mass. In summary, Table 1 shows the first interaction depth, the width of the profile and 4 parameters of the G-H function with various primary energies and masses. According to the table, the width of the profile fluctuates much less than X_{max}, and the fluctuation of width decreases with primary energy except for gamma induced showers.

4 Gaussian Function in Age

The age parameter has been used to describe energy spectrum and lateral distribution of shower particles. It is interesting to see the shower profile in age (Figure 4). A Gaussian function in age s is given by

$$f(s) = \exp\left\{\frac{1}{2\sigma^2}(s-1)^2\right\}. \qquad (2)$$

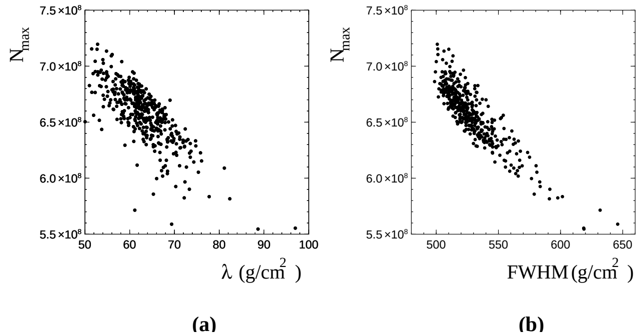

Fig. 3. (a) shows the correlation between N_{max} and λ, and (b) shows the correlation between N_{max} and the width of the profile (FWHM).

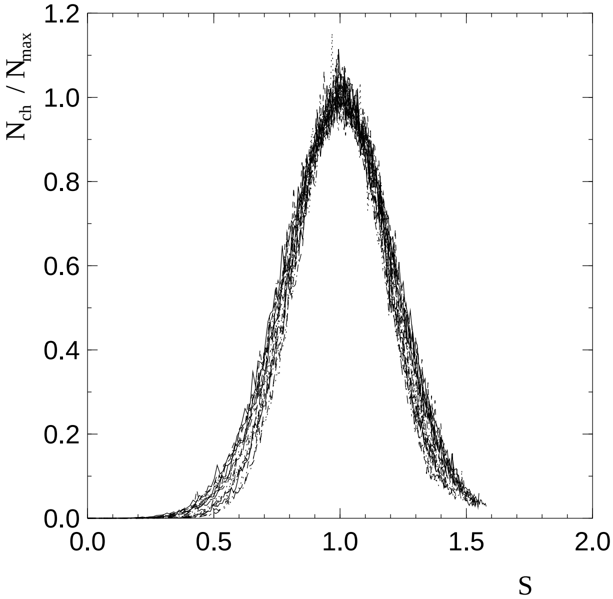

Fig. 4. The shower profiles in age for proton induced showers at 10^{18} eV.

Substituting the definition of age ($s = 3X/(X + 2X_{max})$) into Eq.(2) gives

$$F(X) = \exp\left\{\frac{2}{\sigma^2}\left(\frac{X - X_{max}}{X + 2X_{max}}\right)^2\right\}. \tag{3}$$

Before we do a fit with the above equation, we need to know the error of the weighted shower profile due to the thinning algorithm (Hillas, 1997). For a weighted air shower, the total number of charged particles at certain depth is determined by summing up all weights:

$$N_{ch} = \sum_{i=1}^{N_o} w_i = \bar{w} \cdot N_o, \tag{4}$$

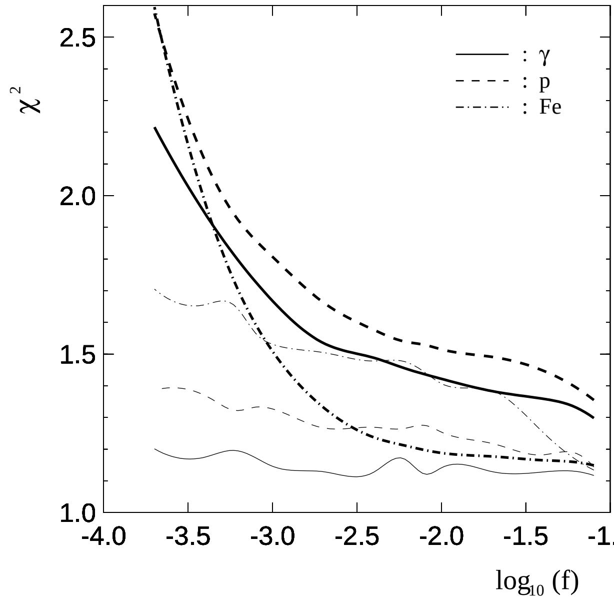

Fig. 5. χ^2 as a function of f. The thick line is for a Gaussian function in age and the thin line is for the G-H function.

where N_o is the unweighted number of particles and w_i is the weight of each particle. From the above equation, we have

$$\frac{\delta N_{ch}}{N_{ch}} = \sqrt{\left(\frac{\delta\bar{w}}{\bar{w}}\right)^2 + \left(\frac{\delta N_o}{N_o}\right)^2}. \tag{5}$$

Using $\delta\bar{w} = \sigma(w)/\sqrt{N_o}$ and $\delta N_o = \sqrt{N_o}$, we obtain

$$\delta N_{ch} = N_{ch}\sqrt{\frac{1}{N_o}\left\{\left(\frac{\sigma(w)}{\bar{w}}\right)^2 + 1\right\}}. \tag{6}$$

Finally, we define χ^2 as following:

$$\chi^2 = \sum_{i=1}^{N} \frac{(F(x_i) - N_{ch}^i)^2}{(\delta N_{ch}^i)^2}, \tag{7}$$

where $\delta N_{ch}^i = \sqrt{N_o^i \cdot \bar{w_i^2}}$.

However, when we do a fit with a Gaussian function in age, χ^2 is large. It turns out that the Gaussian function cannot fit the profile at an early stage of the shower development. We therefore introduce a parameter f, where the fitting is limited to $N_{ch} \geq f \cdot N_{max}$. Figure 5 shows χ^2 as a function of f. The χ^2 for the Gaussian function increases quickly below $f = 10^{-2}$, while that of the G-H function increases very slowly. The residual is shown in Figure 6, and χ^2 with $f = 0.01$ is on the plots. $f = 0.01$ corresponds to $s \approx 0.4$. According to Figure 1, the detector observes the shower profile within $0.5 < s < 1.4$. Therefore, a Gaussian function is expected to work well with the measured shower profiles. We can also use even larger f to determine N_{max} and X_{max}. Then, the primary energy can be determined by integrating

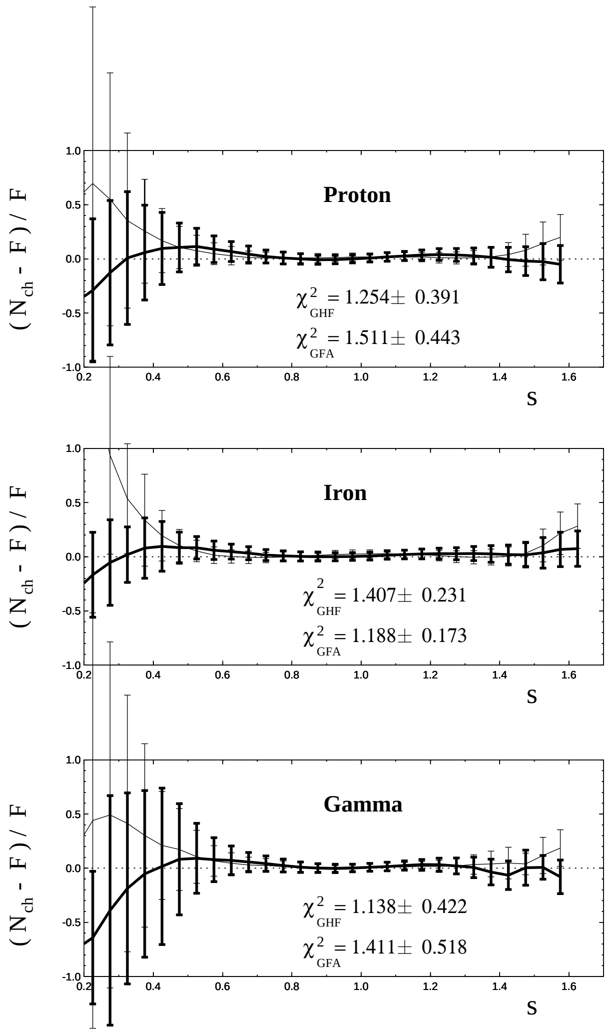

Fig. 6. The residuals for gamma, proton and iron initiated showers are shown. The thick line is for a Gaussian function in age (GFA) and the thin line is for the G-H function (GHF).

either functional form. For example, integrating the profiles initiated by protons at 10^{18} eV gives $(3.66 \pm 0.06) \cdot 10^{11}$ g/cm^2 for the G-H function and $(3.67 \pm 0.07) \cdot 10^{11}$ g/cm^2 for a Gaussian function (for 200 events). The difference is less than 1 %. Figure 7 shows the comparison of N_{max} and X_{max} determined by fitting with two different functions. According to Table 2, σ is changing slowly with energy. Notice that σ for electromagnetic showers is quite different from that of hadronic showers.

$Log(E)$	p	Fe	γ
17.0	0.210 ± 0.019	0.241 ± 0.008	0.176 ± 0.011
18.0	0.200 ± 0.015	0.227 ± 0.005	0.167 ± 0.011
19.0	0.196 ± 0.013	0.214 ± 0.004	0.159 ± 0.011

Table 2. σ for a Gaussian function in age for p, Fe and γ induced showers at 3 different energies.

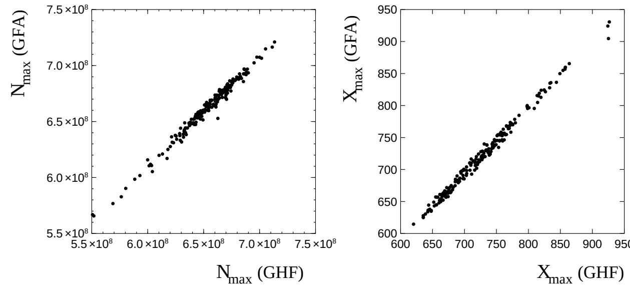

Fig. 7. Comparison of X_{max} and N_{max} from fitting with the G-H function and a Gaussian function in age.

5 Conclusion

The FWHM of the G-H function and σ of the Gaussian function are more stable than other parameters which means that the profile shape does not change much. Therefore, averaging shower profiles is possible if showers are aligned properly. One way to do so is using the age rather than depth as well as normalizing the shower profile at N_{max}. Unfortunately, even though the width of the profile provides important information on the primaries, it is not powerful enough to distinguish the mass of the primary particles.

Using a Gaussian function in age rather than a G-H function reduces the number of correlated parameters. λ is a parameter depending on the primary energy and mass, but the σ of the Gaussian is more stable with changing energy. Both functions give comparable χ^2 at $f > 10^{-2}$ though the Gaussian function is not be able to fit the profile at an early stage of the shower. For fitting without any points which have a small number of charged particles ($f = 10^{-2}$), both functions give consistent X_{max}, N_{max} and energy (the total track length). Therefore, the Gaussian function in age is more useful for the reconstruction of data because of the finite resolution of the detector.

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