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Test of the Diffusion Halo and the Leaky Box Model by means of secondary radioactive cosmic ray nuclei with different lifetimes

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Abstract. ^{10}Be/^{9}Be is the best known and therefore the most commonly used ratio to determine the average density of traversed matter in the Leaky Box Model (LBM) and the Halo size in the Diffusion Halo Model (DHM). A new calculation will be presented including other radioactive Isotops like ^{36}Cl and ^{26}Al , which have different lifetimes. Results will be compared to recent measurements of the corresponding ratios. Unfortunately the production cross sections for the mentioned unstable Isotops are not known well enough to distinguish between the Leaky Box and the Diffusion Halo Model.

1 Introduction

It has been pointed out many years ago (Ginzburg, Khazan & Ptuskin, 1980) that the sources of CR should be distributed within the thin galactic disk and that the escape from the disk into the halo and finally into the intergalactic space is determined by diffusion. This would lead to a gradient of cosmic ray density which has it's highest value in the galactic disc. In the literature this Diffusion Halo Model (DHM) competes with the very popular Leaky Box Model (LBM). The LBM describes an equilibrium model, in which the cosmic ray sources, and the primary and secondary cosmic ray particles are homogeneously distributed in a confinement volume (box, galaxy) and constant in time with no gradient of CR density into any direction. Thus in the LBM the transport of CR is not controlled by diffusion but by a hypothetic leakage process at the imaginary boundaries. After traversing a mean interstellar gas density of $\lambda_{esc}(g/cm^2)$ the particles escape from the confinement volume but the mechanism of their escape is not addressed as well as the physical size of the volume. These two, the DHM and the LBM, are currently the basic competing models in CR propagation calculation. As pointed out by Ginzburg, Khazan & Ptuskin (1980), secondary radioactive nuclei such as ^{10}Be ($\tau_d = 2.3 \cdot 10^6 a$), ^{26}Al ($\tau_d = 1.2 \cdot 10^6 a$) and ^{36}Cl ($\tau_d = 4.4 \cdot 10^5 a$) can be used to check on the physical reality of these two models. We here present a calculation in the framework of these two models including reacceleration and compare the results with recent data of radioactive isotopes from ACE (Yanasak et al. 1999) and ISOMAX (Hams et al. 2001 and de Nolfo et al 2001, for Li-Data see also Göbel et al. 2001).

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2 The transport equations for the Leaky Box Model and the Diffusion Halo Model

In order to obtain the equilibrium spectra of radioactive secondary cosmic ray particles in these two models one has to solve the corresponding equilibrium equations. For the LBM we solved:

$$\frac{N_i(E)}{\tau_{esc}(E)} + \frac{N_i(E)}{{}_i\tau_{int}(E)} + \frac{N_i(E)}{\gamma(E)\cdot {}_i\tau_{dec}} = - \frac{\partial}{\partial E}\left\{\left(\left\langle\frac{\partial E}{\partial t}\right\rangle_{ion} + \left\langle\frac{\partial E}{\partial t}\right\rangle_{reacc}\right)\cdot N_i(E)\right\} + \frac{1}{2}\frac{\partial^2}{\partial E^2}\left\{\left\langle\frac{\Delta E^2}{\Delta t}\right\rangle_{reacc}\cdot N_i(E)\right\} + \sum_{k>i}\frac{N_k(E)}{\tau_{int}^{ki}} \quad (1)$$

and for the one dimensional DHM we solved:

$$0 = \frac{\partial}{\partial z}\left\{D(E,z)\cdot\frac{\partial}{\partial z}N_i(E,z)\right\} - \frac{N_i(E,z)}{{}_i\tau_{int}(E,z)} + \frac{N_i(E,z)}{\gamma(E)\cdot {}_i\tau_{dec}} - \frac{\partial}{\partial E}\left\{\left(\left\langle\frac{\partial E}{\partial t}\right\rangle_{ion} + \left\langle\frac{\partial E}{\partial t}\right\rangle_{reacc}\right)\cdot N_i(E,z)\right\} + \frac{1}{2}\frac{\partial^2}{\partial E^2}\left\{\left\langle\frac{\Delta E^2}{\Delta t}\right\rangle_{reacc}\cdot N_i(E)\right\} + \sum_{k>i}\frac{N_k(E,z)}{\tau_{int}^{ki}} \quad (2)$$

These two equations contain similar and different terms. In the LBM the quantities $N_i(E)$ and $N_k(E)[cm^{-3}GeV^{-3}]$ stand for the number densities of different types of nuclei of

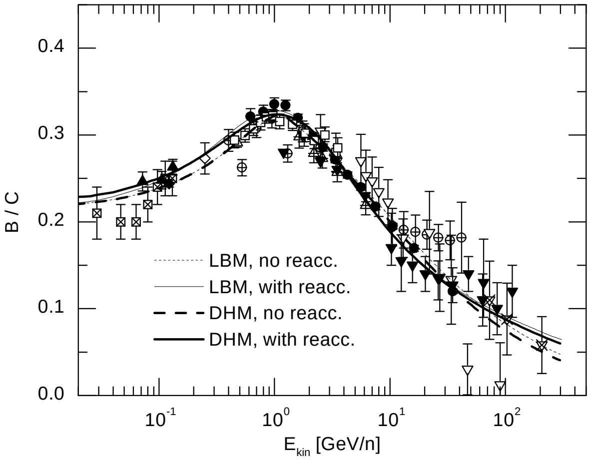

Fig. 1. The curves show the calculated B/C-ratio in the framework of the LBM and the DHM, with and without reacceleration. The curves are solar modulated with modulation parameter $\Phi = 600\,MV$.

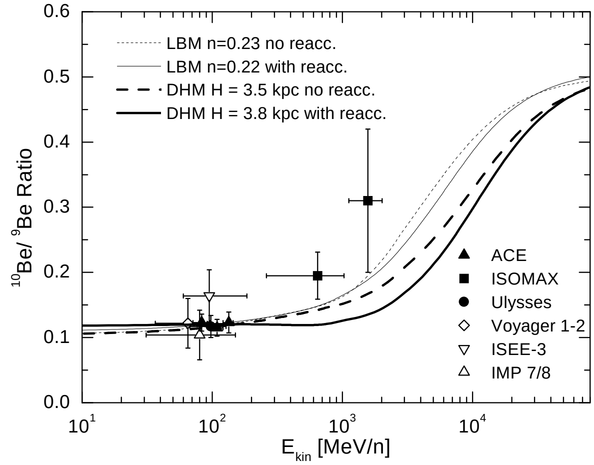

Fig. 2. The curves show the calculated ^{10}Be/^{9}Be-ratio in the framework of the LBM and the DHM, with and without reacceleration. The free parameters in the two models were adjusted so that the low energy data could be fitted. The curves are solar modulated with modulation parameter $\Phi = 600\,MV$. Data are as indicated: ACE: Yanasak et al. (1999); ISOMAX: Hams et al. (2001), de Nolfo et al. (2001); Ulysses: Conell (1998); Voyager: Lukasiak et al. (1994); ISEE-3: Wiedenbeck and Greiner (1980).

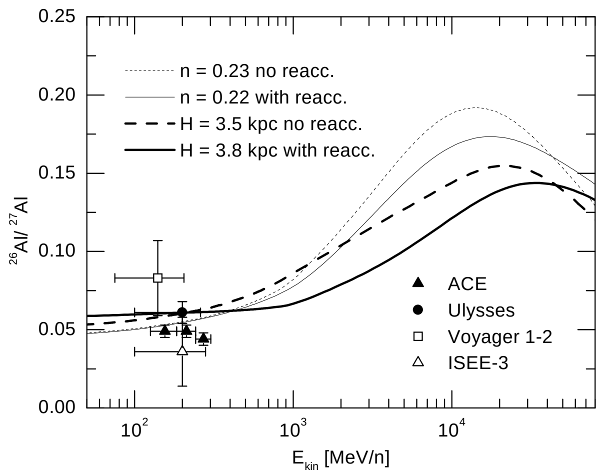

Fig. 3. The curves show the calculated ^{26}Al/^{27}Al-ratio in the framework of the LBM and the DHM, with and without reacceleration. The curves are solar modulated with modulation parameter $\Phi = 600\,MV$. Data: ACE as in Figure 2, Ulysses: Simpson and Connel (1998), Voyager: Lukasiak et al. (1997), ISEE-3: Wiedenbeck (1983).

kinetic energy E and in Equation 2 $N_i(E,z)$ and $N_i(E,z)$ describe the number density of particles at a given position z. The first term on the right side of Equation 2 describes the diffusion and $D(E,z)$ means the diffusion coefficient at position z. For simplicity we allow $D(E)$ to be independent of position. The second term on the right side of Equation 2 accounts for the losses of i-type particles similar to those in Equation 1, where ${}_i\tau_{int}(E)$ stands for the mean lifetime of the i-type particles against interaction in the interstellar gas and $\gamma(E) \cdot {}_i\tau_{dec}$ accounts for the loss due to radioactive decay (γ is the Lorentz-factor). The quantity $\tau_{int}^{ki}(E)$ means the mean time which a k-type nuclei needs to produce an i-type secondary in the interstellar gas. This quantity depends on the production cross section and the interstellar gas in terms of density and composition. $\frac{\partial}{\partial E}\{\ldots\}$ and $\frac{\partial^2}{\partial E^2}\{\ldots\}$ account in both equations for the energy changing processes. The losses are due to ionization and the two energy gain terms, which are due to stochastic reacceleration, are given by:

$$\left\langle \frac{\partial E}{\partial t} \right\rangle_{reacc} = \eta \beta E_{tot} \left(\frac{R}{MV} \right)^{-\alpha}$$
$$\left\langle \frac{\Delta E^2}{\Delta t} \right\rangle_{reacc} = \frac{\eta}{2} \beta^3 E_{tot}^2 \left(\frac{R}{MV} \right)^{-\alpha} \qquad (3)$$

$\eta[s^{-1}]$ and α are free parameters. For the LBM η can be written as $\eta = \eta_x n\, m\, c$ where $n[cm^{-3}]$ denotes the mean interstellar gas density and $m[g]$ the mean mass of an interstellar gas particle. We assumed for the interstellar gas a mixture of 90% hydrogen and 10% helium.

For stable secondary particles in the LBM it is more convenient to swich from time parameters as in Equation 1 to path lengths. The main parameter ist then the mean escape length $\lambda_{esc} = n\, m\, \beta\, c\, \tau_{esc}$.

The above equations can be solved by different mathematical techniques and we refer to the literature. We like to note that care has to be taken when energy changing processes are involved (Heinbach & Simon 1995, Stephens & Streitmatter 1998, Garcia-Munoz et al 1987, Gaisser & Schaefer 1992).

3 Results of the Calculation

We solved the above equations under four different situations: the LBM and the DHM with and without reacceleration. In all four cases one deals with free parameters. We used for the four models a collection of data for the B/C ratio

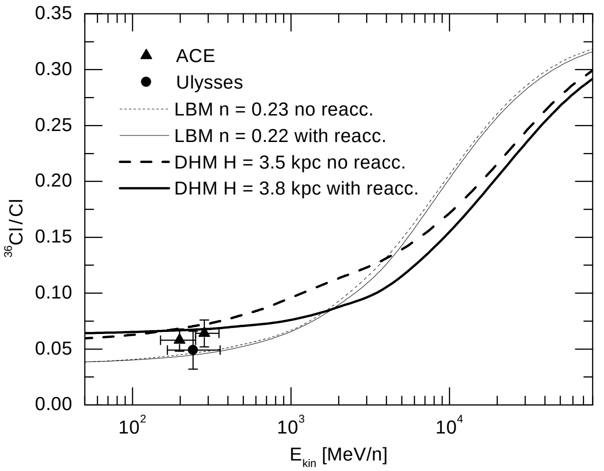

Fig. 4. The curves show the calculated ^{36}Cl/Cl-ratio in the framework of the LBM and the DHM, with and without reacceleration. The curves are solar modulated with modulation parameter $\Phi = 600\,MV$. Data as in Figure 2, except for Ulysses: Connell et al. (1998).

(see Figure 1) to determine which parameters best fit this ratio. We obtained the following results for the LBM without reacceleration:

$$\lambda_{esc} = \begin{cases} \lambda_0 \left(\frac{R}{4.7\,GV}\right)^{0.8} & for\ R < 4.7\,GV \\ \lambda_0 \left(\frac{R}{4.7\,GV}\right)^{-0.57} & for\ R > 4.7\,GV \end{cases} \quad (4)$$

with $\lambda_0 = 12.8 g\,cm^{-2}$ and for the LBM with reacceleration:

$$\lambda_{esc} = (64\,g\,cm^{-2})\,(R/MV)^{-0.3} \quad (5)$$

with the following parameters for the reacceleration terms: $\eta_x = 0.64(g\,cm^{-2})^{-1}$ and $\alpha = 0.3$. In the framework of the LBM the mean gas density n of the confinement volume can be derived by matching the calculated ratio of the radioactive ^{10}Be to the stable ^{9}Be isotope with the data. Such a fit is given in Figure 2 as indicated. We obtain a mean gas density of $n = 0.23\,cm^{-3}$ in the case of no reacceleration and $n = 0.22\,cm^{-3}$ with reacceleration.

In the DHM without reacceleration the fit to the B/C ratio only allows to determine the ratio of the diffusion coefficient $D(E)$ to the halo size H. The best fit was obtained with the following parameters:

$$\frac{D(R)}{H} = \begin{cases} \nu_0 \beta \left(\frac{R}{4.7\,GV}\right)^{-0.8} & for\ R < 4.7\,GV \\ \nu_0 \beta \left(\frac{R}{4.7\,GV}\right)^{0.57} & for\ R > 4.7\,GV \end{cases} \quad (6)$$

with $\nu_0 = 1.7 \cdot 10^6\,cm\,s^{-1}$. This D/H ratio can be disentangled by matching the calculated ^{10}Be/^{9}Be ratio with data as shown in Figure 2. For the halo size one obtains $H = 3.5\,kpc$. In the presence of reacceleration we also fit the B/C ratio and the ^{10}Be/^{9}Be ratio simultaneously and for the following parameters we obtain the best fit:

$$D = D_0 \beta\,(R/MV)^{1/3} \quad (7)$$

with $D_0 = 2.52 \cdot 10^{27}\,cm^2 s^{-1}$ and $\eta = 3.06 \cdot 10^{-15}\,s^{-1}$, $\alpha = 1/3$ and halo size $H = 3.8\,kpc$.

With these various conditions, which fit the B/C ratio and the $^{10}\mathrm{Be}/^{9}\mathrm{Be}$ ratio simultaneously, we calculated the $^{26}\mathrm{Al}/^{27}\mathrm{Al}$ and the $^{36}\mathrm{Cl}/\mathrm{Cl}$ ratio which are shown in Figure 3 and 4 respectively.

4 Discussion

These results show that under the same propagation conditions which fit the B/C and the $^{10}\mathrm{Be}/^{9}\mathrm{Be}$ ratio simultaneously the calculated ratios of $^{26}\mathrm{Al}/^{27}\mathrm{Al}$ and $^{36}\mathrm{Cl}/\mathrm{Cl}$ also show good agreement with data, as illustrated in Figure 3 and 4. All curves depict the increase of these ratios with energy, which is due to the relativistic time dilatation, but they show an individual energy dependence which allow to distinguish between the models and this works best at energies above 1 GeV/n, where data are unfortunately very scarce. In addition, reacceleration further complicates the conclusions. A distinction between the LBM and the DHM is probably even easier at low energies by taking radioactive isotopes with different decay times into consideration (see also Simon and Molnar 1999). The $^{36}\mathrm{Cl}/\mathrm{Cl}$ ratio is a good example. The data of this ratio lie just between the calculated expectations from the two models, which leaves it open what model might be correct. It should be said that production cross sections of these isotopes are not so well known. We here used the partial cross section calculations of Silberberg and Tsao (1998) which we modified in some cases where measurements of these cross sections were in clear disagreement with the calculated results. The uncertainties of these cross sections are still very large and in individual cases they can exceed even one hundred percent (Tsao, Silberberg and Barghouty 1999), thus comparisons of curves between different authors should be taken with care.

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