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A model for geomagnetic deviations of muons in inclined showers

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Abstract. We present a new approach for the description of the muon density patterns at ground level induced by high energy cosmic rays incident at large zenith angles. The approach combines parameterizations of muon lateral distributions in the absence of any magnetic field and a method for calculating the distortions induced by the Earth's magnetic field. As a checking procedure predictions based on this framework are compared to simulation results.

1 Introduction

Cosmic rays induce very inclined air showers at large zenith angles. As the zenith angle increases from zero to 90° the traversed atmospheric depth rises from 1000 to about 36000 g cm^{-2} at sea level. As a result the shower maximum, mainly consisting of photons and electrons, is reached in the upper layers of the atmosphere and most of the electromagnetic component of the shower is absorbed well before the shower reaches the ground. At ground level the shower front contains muons which are mainly produced by charge pion decay. These muons can travel all the slant atmospheric depth and produce density patterns on the ground that are much affected by the Earth's magnetic field. The front also contains electrons and photons which are induced by the muon themselves, through decay and to a lesser extent through muon interactions.

Inclined showers have been known for long to provide an alternative for high energy neutrino detection (Berezinsky and Zatsepin , 1969; Zas et al. , 1993; Capelle et al. , 1998) and understanding backgrounds was the original motivation of studying inclined showers induced by cosmic rays (Ave et al. , 2000b). As the density patterns at ground level become considerably complex, it became quite clear that it is necessary to develop approaches which are completely different from those used for the study of vertical showers (Ave et al. , 1999). We have studied the muon density patterns produced by inclined showers under the influence of the Earth's magnetic field and obtained a model that explains the muon distributions and motivates a satisfactory analytical parameterization. This is reported here and more details can be found in (Ave et al. , 2000a). The potential value of this study goes beyond understanding the background to neutrino induced showers. Part of it is manifest in the recent study of inclined showers detected in Haverah Park (Ave et al. , 2000b) demonstrating that inclined showers induced by cosmic rays can be analysed. This not only nearly doubles the aperture of any air shower array but, when combined with vertical measurements, it has a remarkable potential for the study of primary composition (Ave et al. , 2000c).

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2 Geomagnetic Deviations in a Toy Model

Our approach firstly considers showers in the absence of the magnetic field and then implements magnetic deviations to the muons. The main ideas correspond to a very simple model in which all muons are produced at a single point along the shower axis and with a fixed transverse momentum $p_\perp$. In the absence of magnetic field and neglecting multiple scattering, the tangent of the angle of the muon trajectory with respect to shower axis is $p_\perp/p$. For small deviations this angle is well approximated by $c\, p_\perp/E$. In this model when the muons reach a plane perpendicular to the shower axis at ground level, the (*transverse plane*), they have traveled a distance d measured along the axis, as schematically shown in Fig. 1. In the transverse plane, the distance to shower axis $\bar{r}$, is inversely related to muon energy E and the muon density pattern has full circular symmetry.

When magnetic field effects are considered the muons deviate describing to a first approximation a circle of radius R in a plane perpendicular to $\boldsymbol{B}_\perp$. As a result in the transverse plane the muons deviate a further distance δx which in the limit of R being much larger than d can be easily shown to

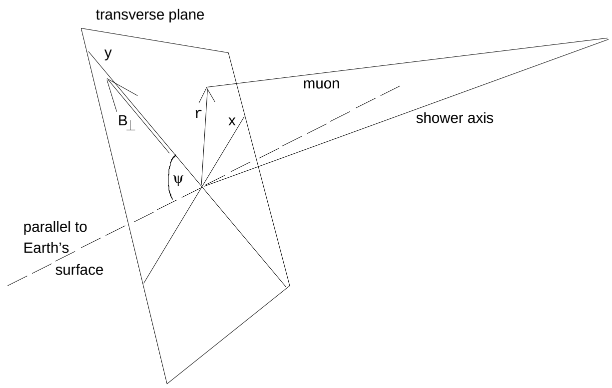

Fig. 1. The plane perpendicular to the shower axis or *transverse plane* and some useful geometrical definitions for Horizontal Air Showers. The y axis is chosen along the direction of $B_\perp$ which is the projection of the magnetic field onto the transverse plane. ψ is the angle subtended by the y axis and the intersection of the transverse and the ground planes.

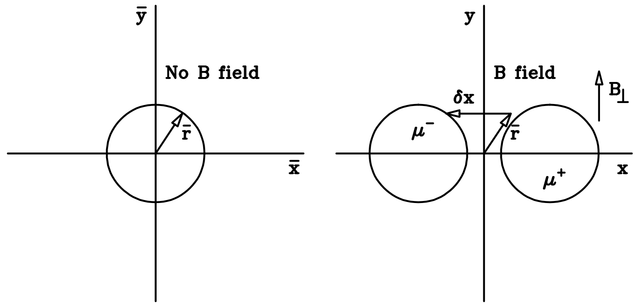

Fig. 2. Composition of deviations due to transverse momentum and to the geomagnetic field. The second graph illustrates how the positive and negative muons that would be a distance r away from shower axis in the absence of magnetic field, are split and deviated an amount δx into two opposite directions along the x axis (perpendicular to $\boldsymbol{B}_\perp$).

be given by:

$$\delta x \simeq \frac{d^2}{2R} = \frac{e|B_\perp|d^2}{2p} = \frac{0.15|B_\perp|d}{p_\perp}\,\bar{r} = \alpha\,\bar{r} \qquad (1)$$

For the numerical expression (the fourth expression) $B_\perp$ is to be expressed in Tesla, d in meters and $p_\perp$ in GeV/c. The relation between $\bar{r}$ and E implies that δx is proportional to $\bar{r}$, α being the proportionality constant.

The muon deviation due to the magnetic field is typically small compared to d and it can be added as a displacement vector in the transverse plane. If one looks at muon deviations in the transverse plane rotated in such a way that $B_\perp$ points upwards (see Fig 2), positive (negative) muons of energy E that lie on the circle of radius $\bar{r}$ are shifted to the right (left) of $B_\perp$ a distance proportional $\bar{r}$.

As $\bar{r}$ determines the muon energy the muon density pattern is modified in a purely geometrical fashion and is a relatively simple transform of the circularly symmetric pattern:

$$\begin{aligned} x &= \bar{x} + \frac{eB_\perp d^2 c}{2E}, \\ y &= \bar{y}. \end{aligned} \qquad (2)$$

where $\bar{x}$ and $\bar{y}$ and $\bar{r}$ refer to coordinates in the absence of a magnetic field and the muon charge e has a sign depending on the muon sign. The dimensionless quantity α parameterizes the relative importance of magnetic deviations. For $\alpha << 1$ magnetic effects are small and the muon isodensity are slightly deformed circles. For α above the critical value of 1 each circle is shifted a distance larger than its radius. The resulting pattern of circles is constrained between the two straight lines $y = \pm\alpha x$ and contains the x axis. In this case *shadow* regions with no muons are expected in the muon density profiles which are in agreement with detailed simulations.

The parameter α depends on d, the distance traveled by the muon and on $B_\perp$. The d dependence implies that magnetic effects become much more important as the zenith angle rises. The projection of B onto the transverse plane depends the arrival directions of the cosmic rays. The magnitude of $B_\perp$ depends on both zenith and azimuthal angles and determines the extent of the magnetic shift. On the other hand the orientation of $B_\perp$ in the transverse plane, which also depends on both angles, affects the projected density pattern on the ground plane. As a result the density patterns at ground level can be very different depending on the Earth's magnetic field at the location being considered.

3 Inclined Shower Features

The two assumptions implied in the previous model have been checked against simulation results. In the absence of magnetic field the correlation between the *mean* muon energy and distance to shower core is patent and can be seen in Fig. 3. If the variable $\epsilon = \log_{10} E$ is used for the distribution of energies at a given $\bar{r}$, its average is well correlated with $\bar{r}$:

$$< \epsilon >= A - \gamma \log_{10} \bar{r}. \qquad (3)$$

There is however a significant spread in energies which has to be accounted for, if a realistic quantitative description of the muon distributions at ground level is wanted. The ratio of this spread to the average decreases slowly as the distance to shower axis increases.

As a second assumption the distance traveled by the muons is assumed to be known for each shower, or equivalent all muons are produced at the same point in shower development. This is an approximation because the real distance traveled by the muons also has a significant spread. As the showers become more horizontal however the approximation improves because the relative importance of the spread with respect to d decreases. Table 1 shows both the average distance and the rms deviation for different zeniths. This distance changes by close to two orders of magnitude as the zenith ranges from vertical to horizontal and results in large differences between inclined showers of 60° and 80° for example.

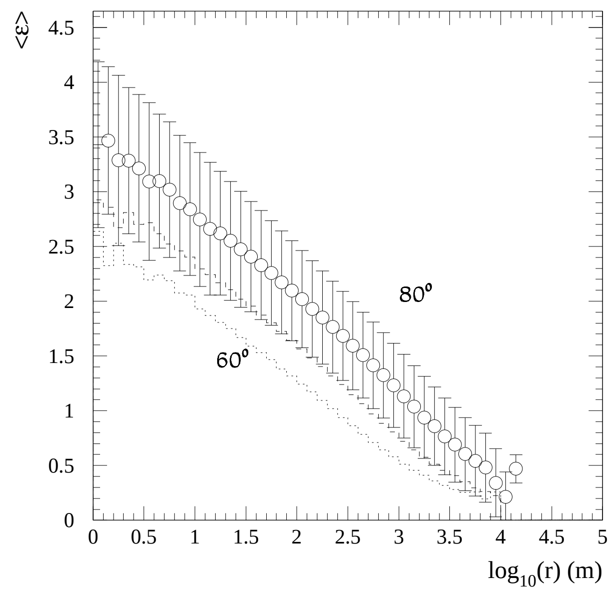

Fig. 3. Correlation between $<\epsilon>=<\log_{10} E_\mu>$ and $\log_{10} r$ for zenith angles 60°, 70°, and 80° from bottom to top, see text. Error bars show the width of the ϵ distribution for 80°.

To obtain a quantitative description of muon densities we first consider a muon "spectral density" in the variable ϵ ignoring the magnetic field:

$$\rho(\bar{r}, \epsilon) = P(\epsilon; <\epsilon>, \sigma)\ \rho(\bar{r}), \tag{4}$$

where P is a distribution of mean $<\epsilon>$ and standard deviation σ. The distribution is normalized to 1 so that the muon density in the absence of magnetic field, $\rho(\bar{r})$, is recovered after integration in ϵ.

The muon number density in the presence of the magnetic field is obtained using the transformation implied by Eq. 2 which depends on ϵ through E. The muon number density becomes:

$$\rho(x, y) = \int d\epsilon\ P(\epsilon, <\epsilon>, \sigma)\ \rho(\bar{r}), \tag{5}$$

with $\bar{r}$ given by:

$$\bar{r} = \sqrt{(x - \frac{eB_\perp d^2 c}{2E})^2 + y^2}. \tag{6}$$

θ(deg)	d (km)	Δd (km)	$<E>$ (GeV)	$N_\mu \times 10^{-6}$
0°	3.9	2.8	8.1	29.
60°	16	6.5	18.9	13.3
70°	32	10	32.9	7.8
80°	88	17	77	3.3
87°	276	31	204	1.2

Table 1. Relevant parameters for muon production as obtained in 100 proton showers of energy 10^{19} eV with a relative thinning of 10^{-6} simulated with AIRES using SIBYLL 1.6 cross sections. Average values and RMS deviations for production altitude (d), muon energy at production ($<E>$), and total number of muons at ground level (N_μ).

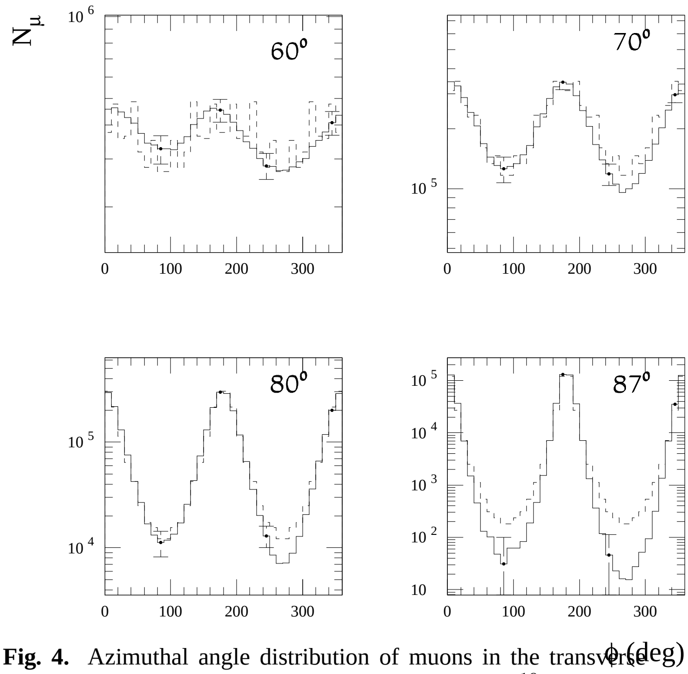

Fig. 4. Azimuthal angle distribution of muons in the transverse plane (integrated up to $r = 6$ km) as obtained in 10^{19} eV proton shower simulations (continuous histogram) compared to the model (dashed histogram) for zenith angles 60°, 70°, 80°, and 87°.

Eq (5) can be regarded as a parameterization provided four inputs are specified corresponding to muon densities in the absence of magnetic field, namely a distribution for ϵ, an effective distance to the production site d, a relation between the mean of the ϵ-distribution and $\bar{r}$ and finally a lateral distribution function for the muons in the transverse plane . Al these inputs are relatively easy to obtain with a monte carlo program. In Ref. (Ave et al. , 2000a) a relatively simple example of how these can in practice be implemented is given from results obtained using extensively AIRES (Sciutto , 1999) for protons without magnetic field. They include values for d shown in Table 1, simple parameterizations for the $<\epsilon>$-r correlations of Fig. 3 and ad-hoc parameterizations of the lateral distribution functions. A gaussian distribution for ϵ of fixed width 0.4 was also implemented in rough agreement with simulations.

The comparison between simulated results and those obtained using the prescription described above demonstrates that the proposed description of the showers agrees very well with complete simulations (Ave et al. , 2000a). As an example Fig. 4 shows a comparison of the muon density integrated in r up to a distance of 6 km in the transverse plane as a function of azimuth and for different zenith angles. The results are rather encouraging and we note that the largest discrepancies at high zenith angles correspond to regions where the density is low and are dominated by large r. Both these regions are less important for shower experiments.

For a given zenith angle there are very minor differences between showers. For different primary energies the density pattern without magnetic field remains to a very good approximation unchanged except for a normalization that scales

Model	A	β	N_μ (10^{19} eV)
SIBYLL	1	0.880	3.3 10^6
	56	0.873	5.3 10^6
QGSJET	1	0.924	5.2 10^6
	56	0.906	7.1 10^6

Table 2. Relationship between muon number and primary energy for proton and iron in two hadronic models (see equation 7).

with eshower energy E pretty accurately as:

$$N = N_{ref}\ E^{\beta} \tag{7}$$

where β is a constant. With a magnetic field the scaling also holds but the patterns become in addition azimuth-dependent.

The qualitative behaviour is not a feature of a given interaction model or of a particular particle species. Two alternative hadronic interaction models have been studied, the Quark Gluon String Model (QGSM) (Kalmykov and Ostapchenko , 1993) and SIBYLL (Fletcher et al. , 1994). The similarities between the lateral distribution functions of muons of a given zenith angle apply to both models which differ in normalization. A similar situation applies for heavier nuclei. Moreover the lateral distributions of showers induced by heavier primaries are also practically indistinguishable from those induced by protons independently of the interaction model (Ave et al. , 2000b).

It is remarkable that the shape of the muon lateral distribution does not significantly change for showers of energy spanning over four orders of magnitude. It implies that there is a sort of universal lateral distribution function for each zenith angle that can be used to an excellent approximation to describe showers induced by different primaries and/or hadronic models. The numerical differences between each specific combination of model and primary particle nature can simply be implemented changing the absolute normalization and its energy dependence. Table 2 illustrates the changes for this normalization for the four possibilites that have been studied (Ave et al. , 2000b).

4 Summary and Conclusions

We have developed a new framework for the study of muon density patterns arising in inclined atmospheric showers induced by cosmic rays. The framework requires relatively simple inputs that can be obtained from simulations in the absence of magnetic fields what simplifies the task. These include the average distance traveled by the muons, the correlation between average muon energy and distance to the shower axis, the energy distribution at a given distance and the lateral distribution function for the muons. With very simple choices for these inputs the framework has been shown to provide a smooth numerical parameterization of the muon densities at ground level that reproduces the results obtained in detailed simulations that include the magnetic field of the Earth.

The energy behaviour of these inclined showers has been shown to be quite independent of both the hadronic model or the primary particle. The muon densities of these showers have been shown to scale with shower energy with a simple power law. The discussed behaviour of inclined showers greatly simplifies their analysis. This was demonstrated in the first analysis of inclined showers from Haverah Park which was achieved through fits of observed particle densities to those expected (Ave et al. , 2000c). The model described here not only serves as a simplifying tool for the analysis but it also provides a smooth parameterization of the density profiles which helps in the fitting procedure.

The proposed model also provides a good starting point for understanding the complex muon patterns induced by inclined cosmic ray showers. Many improvements can be foreseen to achieve better agreement between this framework and full shower simulations, some of them are being currently explored. We expect that the extra uncertainty that results from the simplified choices made are well below many other less controled uncertainties. For instance those arising from the extrapolations involved in the hadronic models that are required for the description of these showers. It is our hope that this work will be of help in the analysis of inclined shower data enhancing the effective aperture of existing and future experiments.

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