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Cosmic ray transport in anisotropic magnetohydrodynamic turbulence

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Abstract. Observations of interstellar scintillations, general theoretical considerations and comparison of interstellar radiative cooling in HII-regions and in the diffuse interstellar medium with linear Landau damping estimates for fast-mode decay, all strongly imply that the power spectrum of fast-mode wave turbulence in the interstellar medium must be highly anisotropic. It is not clear from the observations whether the turbulence spectrum is oriented mainly parallel or mainly perpendicular to the ambient magnetic field, either will satisfy the needs of balancing wave damping energy input against radiative cooling. This anisotropy must be included when transport of high energy cosmic rays in the Galaxy is discussed. Here we calcuate the momentum diffusion coefficient of cosmic ray particles.

1 Introduction

Observations of interstellar scintillations (Rickett, 1990; Spangler, 1991), general theoretical considerations (Goldreich and Sridhar, 1995), and comparison of interstellar radiative cooling in HII-regions and in the diffuse interstellar medium with linear Landau damping estimates for fast-mode decay (Lerche and Schlickeiser, 2001), all strongly imply that the power spectrum of fast-mode wave turbulence in the interstellar medium must be highly anisotropic. It is not clear from the observations whether the turbulence spectrum is oriented mainly parallel or mainly perpendicular to the ambient magnetic field, either will satisfy the needs of balancing wave damping energy input against radiative cooling. Theoretically, Goldreich and Sridhar (1995) prefer turbulence organized in ribbon-like stuctures paralleling the ambient field. But, whichever way the turbulence is organized (and one expects that observations over the next decade or so should resolve the current ambiguity), there is little question that it is highly anisotropic.

This anisotropy must be included when transport of high energy cosmic rays in the Galaxy is discussed. So far, with the noteworthy exception of Jaekel and Schlickeiser (1992), in all the literature concerning the determination of cosmic ray transport parameters, there appears to be consideration given only to turbulence which has a power spectrum either slab-like along the ordered magnetic field or isotropically distributed in wavenumber (e.g. Schlickeiser and Miller (1998)–hereafter referred to as SM). The purpose of the present work is to remedy this defect to some extent by evaluating the relevant cosmic ray transport parameters in the presence of anisotropic wave turbulence.

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2 Turbulence spectrum

A synthesis of current observations would indicate that a plasma wave power spectrum of the form

$$I(\boldsymbol{k}) = I_0\,[k_\parallel^2 + \Lambda k_\perp^2]^{-(2+s)/2} \tag{1}$$

satisfies the needs of the interstellar scintillation observations, the balance of wave energy dissipation and radiative cooling in HII-regions and in the diffuse interstellar medium, and is in accord with the general theoretical arguments advanced by Goldreich and Sridhar (1995). According to Rickett (1990) and Spangler (1991) Eq. (1) is valid for $|\boldsymbol{k}|(\equiv (k_\parallel^2 + k_\perp^2)^{1/2})$ larger than a minimum wavenumber, $k_{\rm min}$, and less than a maximum $k_{\rm max}$. Spangler (1991) identifies these wavenumbers as due to an inner scale length, $l_{\rm min}(\equiv 2\pi/k_{\rm max})$, and an outer scale length $l_{\rm max}(\equiv 2\pi/k_{\rm min})$. Observations indicate that the power spectral index, s, is around $5/3$, while normalization of the power spectrum requires

$$(\delta B)^2 = \int d^3k\, I(\boldsymbol{k}) =$$

$$2\pi I_0 \int_{-1}^{1} d\eta [\eta^2 + \Lambda(1-\eta^2)]^{-(2+s)/2} \int_{k_{\rm min}}^{k_{\rm max}} dk\, k^{-s} \tag{2}$$

where $k_\parallel = k\eta$, $k_\perp = k(1-\eta^2)^{1/2}$, with η being the cosine of the propagation angle of a plasma wave with respect to the ambient magnetic field. Moreover, $(\delta B)^2$ is the fluctuation strenth in the magnetic field, and the constant Λ accounts for the turbulence anisotropy. Note that if the turbulence is isotropic($\Lambda = 1$) then

$$I_0(\Lambda = 1) = \frac{(\delta B)^2}{4\pi} / \int_{k_{\min}}^{k_{\max}} dk\, k^{-s} \tag{3}$$

while for non-isotropic turbulence

$$I_0(\Lambda) = I_0(\Lambda = 1) / {}_2F_1(1 + \frac{s}{2}, 1; \frac{3}{2}; 1 - \Lambda^{-1}) \tag{4}$$

3 Cosmic ray Fokker-Planck coefficients

On the basis of quasilinear transport theory the general form of the Fokker-Planck coefficients has been given by SM for cosmic ray particles with speeds $v >> V_A$, where $V_A = B_0/\sqrt{4\pi\rho}$ is the Alfven speed in terms of the ambient magnetic field strength, B_0, and the ionized mass density, ρ. Eqs. (17)-(19) of SM are the relevant factors to examine, representing the Fokker-Planck-coefficients $D_{\mu\mu}$, $D_{p\mu}$ and D_{pp}. Here $\mu = p_\parallel/p$ is the cosine of the pitch angle of a cosmic ray particle of total momentum p. These Fokker-Planck coefficients depend on the tensor components of the plasma wave power spectrum $< \delta B_l(\boldsymbol{k})\delta B_m(\boldsymbol{k}) >$. For a magnetic turbulence tensor with no preferred direction, (Batchelor, 1953) notes that $< \delta B_l(\boldsymbol{k})\delta B_m(\boldsymbol{k}) >$ can be written in the general form

$$< \delta B_l(\boldsymbol{k})\delta B_m(\boldsymbol{k}) >= \frac{G(\boldsymbol{k})}{8\pi k^2}[\delta_{km} - \frac{k_l k_m}{k^2}] + \imath \frac{H(\boldsymbol{k})}{8\pi k^2}\epsilon_{lmk}k_k \tag{5}$$

For fast-mode waves propagating either forward (phase velocity $\omega/k = jV_A$, $j = +1$) or backward (phase velocity $\omega/k = jV_A$, $j = -1$) to the ambient magnetic field an index j is used to track the wave direction (SM) and, in principle, the magnetic helicity $H(\boldsymbol{k})$ can also be included in the evaluation of the Fokker-Planck coefficients. However, little is known about any magnetic helicity term in the interstellar turbulence so, in this first investigation of the effects of wave turbulence anisotropy on the cosmic ray transport parameters, we restrict our attention to the anisotropy factor $G(\boldsymbol{k})/(8\pi k^2)$.

With the identification

$$\frac{G(\boldsymbol{k})}{8\pi k^2} = \frac{I_0 k^{-(2+s)}}{[\eta^2 + \Lambda(1-\eta^2)]^{(2+s)/2}} \tag{6}$$

it follows that the anisotropic variants of Eqs. (27)-(29) of SM take the form

$$D_{\mu\mu} = \frac{2\pi^2\Omega^2(1-\mu^2)}{B_0^2} \sum_{j=\pm1} I_0^j \sum_{n=-\infty}^{\infty}$$

$$\int_{-1}^{1} d\eta (1+\eta^2)[\eta^2 + \Lambda(1-\eta^2)]^{-(2+s)/2} \int_{k_{\min}}^{k_{\max}} dk\, k^{-s}$$

$$\delta[kv\mu\eta - jV_A k + n\Omega]\Big(J_n^{'}(\frac{kv(1-\mu^2)^{1/2}(1-\eta^2)^{1/2}}{|\Omega|})\Big)^2 \tag{7}$$

$$D_{pp} = \frac{p^2V_A^2}{v^2} D_{\mu\mu} \tag{8}$$

where I_0^j reflects the two intensity components of turbulence forward and backward to the ambient magnetic field, and we have taken both to have the same spectral shape to be in accord with observations. Then $I_o^+ + I_0^- = I_0$, where I_0 is given by Eq. (4).

These general Fokker-Planck coefficients can be split into two parts: components with $n = 0$ (customarily referred to as transit-time contributions), and components with $n \neq 0$ (customarily referred to as gyroresonant contributions).

4 Comparison of transit-time damping and gyroresonance contributions to particle scattering

Transit-time damping does not contribute to the scattering of particles in the interval $|\mu| < \epsilon$ where the scattering relies solely on the gyroresonant contribution (SM). Outside this interval we can calculate the ratio of the contributions from transit-time damping and gyroresonances as

$$R_{2,3}(\Lambda) \equiv \frac{D^T_{\mu\mu}(\Lambda)}{D^G_{\mu\mu}(\Lambda)} = r_{2,3}(\mu)\frac{A_{TT}(\Lambda)}{A_{G2,3}(\Lambda)} \tag{9}$$

where the indices $2, 3$ refer to the intervals $\epsilon \leq |\mu| \leq 2^{-1/2}$ and $|\mu| > 2^{-1/2}$, respectively. The functions

$$r_{2,3}(\Lambda) \equiv \frac{D^T_{\mu\mu}(\Lambda = 1)}{D^G_{\mu\mu}(\Lambda = 1)} \tag{10}$$

refer to the corresponding ratios for isotropic turbulence (see SM).

4.1 Interval $\epsilon < |\mu| \leq 2^{-1/2}$

4.1.1 Strongly perpendicular anisotropy ($\Lambda << 1$)

For strongly perpendicular anisotropy we obtain

$$R_2(\Lambda << 1) \simeq \frac{\Gamma[\frac{2+s}{2}]}{\pi^{1/2}\Gamma[\frac{2+s}{2}]}$$

$$\begin{cases} \mu^{2+s}\epsilon^{-2(2+s)}\Lambda^{1/2} & \text{for } \Lambda << \epsilon^2 << 1 \\ \frac{s+1}{2s(s+2)}\epsilon^{-(2+s)}\Lambda^{-1/2} & \text{for } \epsilon^2 \leq \Lambda << 1 \end{cases} \tag{11}$$

which is much larger than unity unless $\Lambda \leq \epsilon^{2(2+s)}$ is extremely small.

4.1.2 Strongly ribbon-like anisotropy ($\Lambda >> 1$)

For strongly parallel anisotropy we derive

$$R_2(\Lambda >> 1) \simeq \frac{\Gamma[\frac{2+s}{2}]}{2\pi^{1/2}\Gamma[\frac{2+s}{2}]}$$

$$\begin{cases} \epsilon^{-(2+s)}\Lambda^{1/2} & \text{for } \epsilon \leq |\mu| \leq \epsilon(1 + \frac{1}{2\Lambda}) \\ \epsilon^{-(2+s)}\Lambda^{-(s+1)/2} & \text{for } |\mu| > \epsilon(1 + \frac{1}{2\Lambda} \end{cases} \tag{12}$$

which in the small interval $\epsilon \leq |\mu| \leq \epsilon(1 + \frac{1}{2\Lambda})$ is always much larger than unity. Outside this interval, i.e. $|\mu| > \epsilon(1 + \frac{1}{2\Lambda})$ the ratio $R_2(\Lambda >> 1)$ is much larger than unity unless $\Lambda > \epsilon^{-2(2+s)/(s+1)}$ becomes extremely large.

4.2 Interval $|\mu| > 2^{-1/2}$

4.2.1 Strongly perpendicular anisotropy ($\Lambda << 1$)

For strongly perpendicular anisotropy we obtain

$$R_3(\Lambda << 1) \simeq \frac{\Gamma[\frac{2+s}{2}]}{\pi^{1/2}\Gamma[\frac{2+s}{2}]}$$

$$\begin{cases} \mu^{2+s}\epsilon^{2+s}\Lambda^{1/2} & \text{for } \Lambda << \epsilon^2 << 1 \\ \frac{s+1}{2s(s+2)}\Lambda^{-1/2} & \text{for } \epsilon^2 \leq \Lambda << 1 \end{cases} \tag{13}$$

which is much larger than unity unless $\Lambda \leq \epsilon^{2(2+s)}$ is extremely small.

4.2.2 Strongly ribbon-like anisotropy ($\Lambda >> 1$)

For strongly parallel anisotropy we derive

$$R_3(\Lambda >> 1) \simeq \frac{O_3 g(s)\Gamma[\frac{2+s}{2}]}{2\pi^{1/2}\Gamma[\frac{2+s}{2}]}\Lambda^{-(s+1)/2} \tag{14}$$

which is much smaller than unity.

4.3 Interlude

Summarizing our results in short:

(a) For massively parallel ($\Lambda >> 1$) situations, the ratios of the transit-time contribution to the gyroresonance contribution to pitch-angle scattering in the interval $|\mu| > \epsilon$ of cosmic ray particles with gyroradii $R_L < l_{\max}/2\pi$ behave as follows:

– for large $|\mu| > 2^{-1/2}$ the ratio is smaller than unity indicating that the gyroresonance contribution dominates the transit-time damping contribution,

– in the small interval $\epsilon \leq |\mu| \leq \epsilon(1 + \frac{1}{2\Lambda})$ the ratio is larger than unity indicating that the transit-time contribution dominates the gyroresonance contribution,

– in the interval $\epsilon(1 + \frac{1}{2\Lambda}) < |\mu| \leq 2^{-1/2}$ the ratio is larger than unity (i.e. dominance of the transit-time damping contribution) for anisotropy values smaller than $1 << \Lambda \leq \Lambda_l \equiv \epsilon^{-2(2+s)/(s+1)}$ whereas for extremely large values of $\Lambda > \Lambda_l$ the ratio is smaller than unity (i.e. dominance of the gyroresonance contribution).

(b) For massively perpendicular ($\Lambda << 1$) situations, the ratio of the transit-time contribution to the gyroresoance contribution to pitch-angle scattering in the interval $|\mu| > \epsilon$ of cosmic ray particles with gyroradii $R_L < l_{\max}/2\pi$ is much larger than unity for anisotropy values larger than $\epsilon^{2(2+s)} \equiv \Lambda_s \leq \Lambda << 1$ indicating that the transit-time damping contribution dominates the gyroresonance contribution.

For extremely small anisotropy values $\Lambda < \Lambda_s << 1$ the ratio is smaller than unity indicating the dominance of the gyroresonance contribution over the transit-time damping contribution.

4.4 Cosmic ray scattering in the interstellar medium

Using the estimates of the Alfven speed in the diffuse interstellar medium of $V_A \simeq 3 \cdot 10^6$ cm s^{-1} (Minter and Spangler (1997)) yields for relativistic cosmic ray particles the value $\epsilon = V_A/v \simeq V_A/c = 10^{-4}$. With a turbulence spectral index of $s = 5/3$ (Rickett, 1990) we obtain for $\Lambda_l = \epsilon^{-2(2+s)/(s+1)} = \epsilon^{-11/4} = 10^{11}$ and $\Lambda_s = \epsilon^{2(2+s)} = \epsilon^{22/3} = 10^{-88/3} = 2 \cdot 10^{-29}$, respectively.

Now, estimates of the anisotropy parameter Λ in the strongly parallel situation ($\Lambda >> 1$) based on linear Landau damping balancing radiative loss in the diffuse interstellar medium, provide the value $\Lambda \simeq 7400$ (Lerche and Schlickeiser, 2001) which is much smaller than Λ_l. Hence, it would seem that in the diffuse interstellar medium the transit-time damping contribution to $D_{\mu\mu}$ is dominant in the pitch-angle angle interval $\epsilon \leq |\mu| \leq 2^{-1/2}$ whereas the gyroresonant contribution dominates in the interval $\mu| > 2^{-1/2}$. The same conclusion holds in HII-regions (the fluctiferous domain of Spangler (1991)), for which Lerche and Schlickeiser (2001) estimated $\Lambda \simeq 17.7$.

Estimates of the anisotropy parameter Λ in the strongly perpendicular situation ($\Lambda << 1$) based on linear Landau damping balancing radiative loss in the diffuse interstellar medium, provide the value $\Lambda \simeq 6 \cdot 10^{-5}$ (Lerche and Schlickeiser, 2001) which is much larger than Λ_s. This indicates that the transit-time damping contribution dominates the gyroresonance contribution throughout the whole pitch-angle interval $|\mu| \geq \epsilon$ in the diffuse interstellar medium. The same conclusion holds in HII-regions, for which Lerche and Schlickeiser (2001) estimated $\Lambda \simeq 10^{-3}$ in this case. Here, we restrict our analysis on the momentum diffusion coefficient which, according to SM, is solely determined by the transit-time damping contribution.

5 Cosmic ray momentum diffusion from fast-mode waves

We obtain for the momentum diffusion coefficient of cosmic rays with gyroradii much less than $R_L << l_{\max}/2\pi$

$$a_2 = \frac{\pi}{2}(s-1)c_1(s)\frac{(\delta B)^2}{B_0^2}(k_{\min}R_L)^{s-1}\frac{v\epsilon^2 p^2}{R_L}h(\Lambda, \epsilon, s) \tag{15}$$

with

$$c_1(s) = \int_0^\infty du\, u^{-(1+s)}J_1^2(u) = \frac{2^{1-s}s}{4-s^2}\frac{\Gamma[s]\Gamma[2-\frac{s}{2}]}{\Gamma^3[1+\frac{s}{2}]}$$

and the anisotropy function

$$h(\Lambda, \epsilon, s) \equiv \int_\epsilon^1 d\mu\; A_{TT}(\mu, \Lambda)\frac{1-\mu^2}{\mu}$$

$$[1 + \frac{\epsilon^2}{\mu^2}][(1-\mu^2)(1-\frac{\epsilon^2}{\mu^2})]^{s/2} \tag{16}$$

5.1 Isotropic turbulence $\Lambda = 1$

This case $A_{TT} = 1$ has been considered before by SM who derived

$$h(\Lambda = 1) = (1 - \epsilon^2) D(\epsilon, \frac{s}{2}) \qquad (17)$$

after substituting $y = [(1 - \mu^2)(1 - \frac{\epsilon^2}{\mu^2})]^{1/2}$ in Eq. (16) with the integral

$$D(\epsilon, k) \equiv \int_0^{(1-\epsilon)^2} dy y^k \Big[[y - (1-\epsilon)^2][[y - (1+\epsilon)^2]\Big]^{-1/2}$$

$$= (1 - \epsilon)(1 - \epsilon^2)^k Q_k(\frac{1 + \epsilon^2}{1 - \epsilon^2}) \qquad (18)$$

which can be solved as an associated Legendre function of the second kind of zeroeth order and degree k. SM noted that the latter approaches $\rightarrow \ln \epsilon^{-1}$ for small $\epsilon << 1$, so that

$$h(\Lambda = 1) \simeq (1 - \epsilon)(1 - \epsilon^2)^{(2+s)/2} \ln \epsilon^{-1} \simeq \ln \epsilon^{-1} \qquad (19)$$

5.2 Strongly perpendicular turbulence $\epsilon^2 \le \Lambda < 1$

Here we obtain

$$h(\epsilon^2 \le \Lambda << 1, \epsilon, s) \simeq \int_\epsilon^1 d\mu \, \frac{1}{\mu}$$

$$[1 + \frac{\epsilon^2}{\mu^2}][(1 - \mu^2)(1 + \frac{\epsilon^2}{\mu^2})]^g \qquad (20)$$

with $g = (s^2 + 4s + 2)/2(s + 2)$. With Eq. (18) we obtain

$$h(\epsilon^2 \le \Lambda << 1, \epsilon, s) \simeq \frac{3 + \epsilon^2}{2} D(\epsilon, q) - \frac{1}{2} D(\epsilon, q + 1)$$

$$\simeq (1 - \epsilon)(1 - \epsilon^2)^q \ln \epsilon^{-1} \simeq \ln \epsilon^{-1} \qquad (21)$$

the same result as in the isotropic case.

5.3 Strongly parallel turbulence $1 << \Lambda << \Lambda_l$

Here we obtain

$$h(1 << \Lambda << \Lambda_l, \epsilon, s) \simeq s\Lambda\epsilon^{-(2+s)} K_1 + \frac{s}{2}\Lambda^{-s/2} K_2 \qquad (22)$$

with

$$K_1 \simeq \epsilon^{2+s} \Lambda^{-(2+s)/2} \qquad (23)$$

and

$$K_2 \simeq \ln \epsilon^{-1} + \frac{1}{2\Lambda} \qquad (24)$$

so that

$$h(1 << \Lambda << \Lambda_l, \epsilon, s) \simeq \frac{s}{2} \Lambda^{-s/2} \ln \epsilon^{-1} \qquad (25)$$

which is strongly reduced compared to the isotropic value.

6 Summary and conclusions

Observations of interstellar scintillations, general theoretical considerations and comparison of interstellar radiative cooling in HII-regions and in the diffuse interstellar medium with linear Landau damping estimates for fast-mode decay, all strongly imply that the power spectrum of fast-mode wave turbulence in the interstellar medium must be highly anisotropic. It is not clear from the observations whether the turbulence spectrum is oriented mainly parallel or mainly perpendicular to the ambient magnetic field, either will satisfy the needs of balancing wave damping energy input against radiative cooling. This anisotropy must be included when transport of high energy cosmic rays in the Galaxy is discussed. We show that in nearly all situations the pitch-angle scattering of relativistic cosmic rays by fast magnetosonic waves at pitch-angle cosines $|\mu| \ge V_A/c$ is dominated by the transit-time damping interaction.

Without considering the influence of the anisotropy parameter on the Fokker-Planck coefficients in the case of shear Alfven waves, we are able to calculate the momentum diffusion coefficient a_2 of cosmic ray particles by averaging the respective Fokker-Planck coefficient over the particle pitch-angle for the relevant anisotropy parameters within values of $10^{-8} \le \Lambda \le 10^{11}$. For strongly perpendicular turbulence ($\Lambda < 1$) we obtain the same cosmic ray momentum diffusion coefficient as in the case of isotropic ($\Lambda = 1$), whereas for strongly parallel turbulence ($\Lambda >> 1$) the momentum diffusion coefficient is reduced with respect to isotropic turbulence by the large factor $2\Lambda^{s/2}/s$. This implies that the acceleration time scale of cosmic ray particles by momentum diffusion for strongly parallel turbulence is smaller by the same reduction factor with respect to the case of isotropic turbulence, which for large enough anisotropy factors Λ would justify to neglect effects of reacceleration in the transport of galactic cosmic rays.

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