ICRC 2001

Clues on the turbulent galactic magnetic field from wave energy dissipation by linear Landau damping in the interstellar medium

I. Lerche[1] **and R. Schlickeiser**[2]

[1]Department of Geological Sciences, University of South Carolina, Columbia, SC 29208, USA
[2]Institut für Theoretische Physik, Lehrstuhl IV: Weltraum- und Astrophysik, Ruhr-Universität Bochum, D-44780 Bochum, Germany

Abstract. The heating rate of the diffuse interstellar medium is calculated from collisionless dissipation of magnetohydrodynamic plasma waves by linear Landau damping. The numerical estimates produced for linear Landau damping indicate that there is little, if any, disparity between radiative cooling rates and wave energy loss rates for the fluctiferous (H II-regions) and the diffuse interstellar medium when the anisotropy in the wave power spectrum is properly accounted for. Our results show that the interstellar turbulence mostly consists of obliquely propagating compressive fast magnetosonic waves. However, the power spectrum of these waves is not isotropic in wavenumber space but has to be very anisotropic, either elongated mainly along or perpendicular to the ordered magnetic field component.

1 Introduction

According to current understanding the interstellar medium consists of at least three distint phases in approximate pressure equilibrium: cold clouds, warm intercloud medium, and hot coronal gas generated by supernova explosions (Cox and Smith, 1974). Spitzer (1956) originally postulated the existence of hot ($3 \cdot 10^5$ K$\leq T \leq 10^6$ K) coronal gas which was later confirmed by observations of ultraviolet absorption lines (Jenkins, 1978; Hartquist and Snijders, 1982) and studies of the diffuse soft X-ray background (McCammon and Sanders, 1990). 21 cm radio studies indicate that the warm medium has a temperature between 6000 and 10^4 K and a mean HI-density of about 0.8 cm^{-3}, but is very stuctured in colder clouds and a warm intercloud medium of density 0.1-0.2 cm^{-3}. Under such conditions atomic and metallic radiative transitions efficiently cool the gas, so that an efficient heating mechanism is required in order to maintain the gas temperature.

Here we investigate the heating of the warm gas by the collisionless damping of interstellar magnetohydrodynamic waves. The collisionless dissipation of various kinds of waves in the interstellar medium has had a long history. Underlying motivations for this continued interest are to obtain a de-

Correspondence to: R. Schlickeiser
(rsch@tp4.ruhr-uni-bochum.de)

tailed understanding of the mechanisms responsible for the heating and/or cooling of the interstellar medium (see the review in Spangler (1991)), and to understand how a balance is achieved or maintained between the competing processes.

In a theoretical description the heating (ϵ) and cooling (λ) rates enter the heat conduction equation of a viscous fluid (Landau and Lifschitz, 1982)

$$\rho T\Big[\frac{\partial s}{\partial t}+\boldsymbol{v}\cdot\nabla s\Big] = \mathrm{div}\,(\kappa\nabla T)+\sigma_{ik}\frac{\partial v_i}{\partial x_k}+\epsilon(\rho,T)-\lambda(\rho,T) \quad (1)$$

where s denotes the fluid entropy, ρ and $\boldsymbol{v}$ the fluid density and velocity, κ is the heat conduction coefficient and σ_{ik} the viscous stress tensor. Formally, Eq. (1) can be regarded as the fifth equation – besides the continuity equation and the three Euler equations – of the hydrodynamical description of a viscous fluid. Here we consider a stationary interstellar medium with large spatial scales and without strong spatial inhomogenities, i.e. far away from phase boundaries and shock waves, so that to a first approximation we can neglect all terms in Eq. (1) with spatial and time derivatives. In this case Eq. (1) reduces to the simple balance of heating and cooling rates

$$\epsilon(\rho,T) = \lambda(\rho,T) \quad (2)$$

Our discussion of the heating by collisionless Landau damping will be based on Eq. (2), and thus is valid only in the stationary and homogenous interstellar medium.

2 Magnetic field fluctuation spectrum in the Galaxy

Due to observations of Faraday rotation and pulsar signal dispersion there has been substantial progress in the understanding of wave spectra (see e.g. Rickett (1990)). A synthesis of available data would seem to indicate that a power spectrum of waves in the form

$$P(\boldsymbol{k}) = C_B^2\,[k_\parallel^2 + \Lambda k_\perp^2]^{-(2+s)/2}, \quad \int d^3k\,P(\boldsymbol{k}) = (\delta B)^2 \quad (3)$$

can account for the observations (Spangler, 1991), where δB denotes the total fluctuating magnetic field component. Here $k_\parallel$ ($k_\perp$) is the wavenumber parallel (perpendicular) to the

ambient magnetic field; s is the spectral index which is estimated to lie between about $3/2$ and $5/3$; Λ is the anisotropy parameter. Isotropy occurs if $\Lambda = 1$, whereas if the wave turbulence is more along the lines of thin "platelets" paralleling the ambient field, as suggested by Goldreich and Sridha (1995), then $\Lambda >> 1$. The form of the wave spectrum given by Eq. (3) is taken to operate only between some small wavenumber, $k = k_{min}$, and a large wavenumber, $k = k_{max}$, with $k = |\boldsymbol{k}| = [k_\parallel^2 + k_\perp^2]^{1/2}$. Spangler (1991) suggests that these wavenumbers are related to outer and inner scale lengths, l_{min} and l_{max}, respectively with $l_{min} = 2\pi/k_{max}$, $l_{max} = 2\pi/k_{min}$. The physical bounds of l_{min} and l_{max} are not precisely known, but probably related to the whistler wave resonance limit of the interstellar electrons and the physical size of the warm intercloud medium i.e. the mean cloud distance, respectively. The inner scale is estimated by Spangler (1991) to be of the general order of $l_{min} = 10^7 l_7$ cm, ($l_7 = 1$), with the outer scale of order $l_{max} = 10^{17} L_{17}$ cm with $L_{17} = 1$ in hot, ionized regions, and $L_{17} = 30$ in the diffuse phase of the interstellar medium. In our study here we assume that the power spectrum (3) holds in the stationary and homogenous warm intercloud medium.

The power spectrum itself results from the balance of all wave damping and driving processes, although a detailed theory and explanation currently is not available. Therefore our calculation will be limited in the sense that we assume a given and fixed power spectrum to determine the heating rate of the interstellar medium, but we do not self-consistently investigate the effect of this energy loss rate on the form of the power spectrum.

3 Energy loss rate

For waves damping at a rate $\gamma(\boldsymbol{k})$, the energy loss rate ϵ is conventionally written in the form (Spangler, 1991)

$$\epsilon = \frac{1}{4\pi} \int d^3k \, P(\boldsymbol{k}) 2\gamma(\boldsymbol{k}). \tag{4}$$

3.1 Oblique wave reduction

For an obliquely propagating magnetosonic wave the damping rate is given by (Ginzburg 1961, p.218, Eq. 14.56) as

$$\gamma = (\frac{\pi}{8})^{1/2} \sin^2\theta \sec\theta k v_i \big[(v_i/v_e)+$$

$$5 \exp[-V_A^2/(2v_i^2 \cos^2\theta)]\big], \tag{5}$$

where $\sin\theta = |k_\perp|/k$, V_A is the Alfven speed, v_i (v_e) is the ion (electron) thermal speed. Conditions are also attached to the domain of validity of Eq. (5). These conditions, as spelled out by Ginzburg (1961), are:
(1) Both ions and electrons are taken to be at the same temperature and both are described by isotropic Maxwellian distributions, $v_e = (k_B T/m_e)^{1/2}$, $v_i = v_e(m_e/m_i)^{1/2}$;
(2) One must have $V_A >> v_i$ and $v_e >> v_i$;

(3) Eq. (5) is valid only in the angular range described through

$$v_e|\cos\theta| >> [V_A^2 + 3v_i^2 \sin^2\theta]^{1/2} >> v_i|\cos\theta|. \tag{6}$$

The left part of inequality (6) provides the restriction

$$|\cos\theta| \geq \mu_L \equiv [\frac{V_A^2 + 3v_i^2}{v_e^2 + 3v_i^2}]^{1/2}. \tag{7}$$

In general, $\mu_L \simeq V_A/v_e << 1$ for the cases of interest later in this paper. The right part of inequality (6) is always satisfied when condition 2 is in force.

The exponential factor $\exp[-V_A^2/(2v_i^2 \cos^2\theta)]$ varies between $\exp[-V_A^2/(2v_i^2)]$ and $\exp[-v_e^2/(2v_i^2)]$ as $|\cos\theta|$ traverses $1 \geq |\cos\theta| \geq \mu_L$. Because both of these exponential factors are exceedingly small one can write, to a very accurate approximation, that

$$\gamma \simeq (\frac{\pi}{8})^{1/2} \sin^2\theta \sec\theta k v_i^2 v_e^{-1}. \tag{8}$$

As mentioned, Eqs. (5) and (8) are valid for isotropic Maxwellians with $T_e = T_i$. This assumption is adopted here because we want to evaluate the heating rate with the same input parameters as Spangler (1991) to allow the direct comparison of our calculation with his. In future work we want to consider collisionless heating rates also for an interstellar mediumin NLTE conditions as X-ray absorption studies lead to $T_e > T_i$ and ordered magnetic fields may enforce anisotropic temperatures ($T_\parallel \neq T_\perp$). Both cases require the determination of appropriate, more general damping rates in these changed plasma conditions.

With the damping rate (8), using spherical coordinates for $\boldsymbol{k}$, the energy loss rate can then be written

$$\epsilon = (\frac{\pi}{2})^{1/2} C_B^2 \, v_i^2 v_e^{-1} \frac{k_{max}^{2-s} - k_{min}^{2-s}}{2-s} \, I(\mu_L, \Lambda, s) \tag{9}$$

with the integral

$$I(\mu_L, \Lambda, s) \equiv \int_{\mu_L}^{1} d\mu \frac{(1-\mu^2)}{\mu[\mu^2 + \Lambda(1-\mu^2)]^{1+(s/2)}} \tag{10}$$

Note that the lower limit of the μ-integral has been set to $\mu = \mu_L$ whereas the Ginzburg (1961) damping formula is really valid in $\mu >> \mu_L$. Thus Eq. (9) provides an overestimate of the energy loss rate.

3.2 Fluctuating magnetic field

¿From Eqs. (3) we derive

$$(\delta B)^2 = 4\pi C_B^2 \frac{k_{min}^{1-s}}{s-1}[1 - (k_{min}/k_{max})^{s-1}]J(\Lambda, s) \tag{11}$$

where $J(\Lambda = 1, s) = 1$ and for $\Lambda \neq 1$

$$J(\Lambda) = \int_0^1 d\mu[\mu^2 + \Lambda(1-\mu^2)]^{-(2+s)/2}$$

$$= \,_2F_1(1 + \frac{s}{2}, 1; \frac{3}{2}; 1 - \Lambda) \tag{12}$$
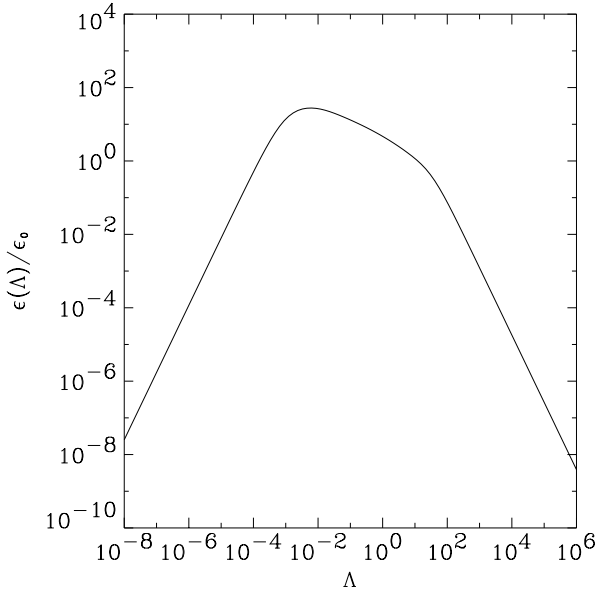
Fig. 1. Variation of the energy loss rate as a function of the anisotropy parameter Λ for a turbulence spectral index $s = 5/3$ and $\mu_L = 0.05$.

3.3 Final energy loss rate

Solving Eq. (11) for C_B^2 and inserting the result in Eq. (9) we obtain the final expression for the energy loss rate

$$\epsilon(\Lambda) = \epsilon_0 \frac{I(\mu_L, \Lambda, s)}{J(\Lambda, s)}, \tag{13}$$

$$\epsilon_{\text{isotropic}} = \epsilon_0 [\ln(\frac{v_e}{V_A}) - 1/2] \tag{14}$$

with

$$\epsilon_0 = (\frac{\pi}{2})^{1/2} (4\pi)^{-1} \frac{s-1}{2-s} \frac{[1 - (k_{\min}/k_{\max})^{2-s}]}{[1 - (k_{\min}/k_{\max})^{s-1}]}$$

$$k_{\max}^{2-s} k_{\min}^{s-1} (\delta B)^2 v_i^2 v_e^{-1} \tag{15}$$

In Fig. 1 we have numerically calculated the variation of the energy loss rate as a function of the anisotropy parameter Λ for the turbulence spectral index $s = 5/3$ and $\mu_L \simeq V_A/v_e = 0.05$. It can be seen that in this case the energy loss rate attains its maximum at values at about $\Lambda \simeq 10^{-2}$ and that it varies with different functional dependences at small or large values of Λ.

4 Different limits of the anisotropy parameter Λ

In Table 1 we summarize approximations of the energy loss rate for different values of the anisotropy parameter Λ. μ_L is related to the instellar plasma beta $\beta = B_0^2/(8\pi n_e k_B T)$ as

$$\mu_L = (\frac{2m_e \beta}{m_p})^{1/2} = 0.033 \beta^{1/2} \tag{16}$$

According to Table 1 we find the largest energy loss rates in the case of isotropic turbulence ($\Lambda = 1$) and about twice that

loss rate for anisotropic turbulence with $\mu_L << \Lambda << 1$. In the other anistropic cases the energy loss rates are much smaller than in the isotropic case with the relevant reduction factors given in Table 1. In the following section we therefore calculate quantitative energy loss rates in the interstellar medium for the case of isotropic turbulence. With the help of Table 1 it is then straightforward to infer the energy loss rates for any anisotropic case.

5 Results and conclusions

5.1 HII regions

Spangler (1991) has argued that the wave damping is most appropriate for his "fluctiferous" media, taken to be described by HII regions. In our numerical estimate of the isotropic energy loss rate we adopt exactly the same parameter values as given by Spangler (1991), thereby revealing most clearly the differences between our calculations and his values. According to Spangler the parameters of relevance are $l_7 = 1$, $L_{17} = 1$, $s = 5/3$, $v_e = 2 \cdot 10^7$ cm s^{-1} (corresponding to a temperature of about 10^4 K), $V_A = 10^6$ cm s^{-1} (so $v_e >> V_A$ as required for the Ginzburg damping formula to be valid), $v_i = v_e/43$, and $(\delta B) = 0.3\mu G$. With Eq. (14) we obtain

$$\epsilon_{\text{isotropic}} = 6.61 \cdot 10^{-23} L_{17}^{-2/3} l_7^{-1/3} \quad erg \ s^{-1} cm^{-3} \tag{17}$$

If the outer scale length $l_{\max}$ is increased to 1 pc ($L_{17} = 30$) then $\epsilon_{\text{isotropic}}$ is reduced by a factor $30^{2/3} = 9.65$, as are all other cases. If the magnetic field fluctuations are increased to 1 μG from 0.3 μG then also the energy loss rate (17) is increased by almost an order of magnitude.

The corresponding radiative cooling rate in this fluctiferous medium is given by Spangler (1991) as about $10^{-23} n_e^2$ erg s^{-1} cm^{-3} so that, all other factors being equal, there is little disparity between the wave energy loss rate and the radiative cooling rate. Indeed, given the uncertainties on the values of (δB), $l_{\max}$ and $l_{\min}$, it is remarkable that the rates are as close as they are. The factor 6.6 enhancement of the isotropic energy loss rate (17) can be accounted for easily either by
(i) a large anisotropy factor $\frac{3}{5}\Lambda^{5/6} = 6.6$ corrresponding to $\Lambda = 17.7$,
or
(ii) a small anisotropy factor $(400\Lambda)^{11/6} = 1/6.6$ corresponding to $\Lambda = 10^{-3}$.

5.2 Diffuse interstellar medium

For the diffuse interstellar medium Minter and Spangler (1997) give a larger value for $\delta B = 0.9\mu G$, and also give the outer and inner scales, $L_{17} = 30$, $l_7 = 8$, while all other turbulence parameters remain unchanged. The isotropic energy loss rate (17) then reduces slightly to

$$\epsilon_{\text{isotropic}} = 3.08 \cdot 10^{-23} \quad erg \ s^{-1} cm^{-3} \tag{18}$$

Table 1. Energy loss rates for different anisotropy parameters

Anisotropy parameter Λ	$\epsilon/\epsilon_{\text{isotropic}}$
$\Lambda = 1$	1
$\Lambda = 1 - \eta, \|\eta\| << 1$	$1 + \frac{2}{3}(1 + \frac{s}{2})(1 - \Lambda)$
$\Lambda \leq (V_A/v_e)^2 << 1$	$[\frac{\Lambda v_e^2}{V_A^2}]^{(2+s)/2}[\ln(v_e/V_A) - \frac{1}{2}]^{-1} << 1$
$(V_A/v_e)^2 \leq \Lambda << 1$	$\ln(\Lambda v_e^2/V_A^2)[\ln(v_e/V_A) - \frac{1}{2}]^{-1} \leq 2$
$\Lambda >> 1$	$s\Lambda^{-s/2} << 1$

According to Minter and Spangler (1997) the radiative cooling rate in the diffuse interstellar medium at temperatures of 10^4 K is $L_R = 5 \cdot 10^{-24}n_e^2$ erg s^{-1}cm^{-3}. With an electron density of $n_e = 0.08$ cm^{-3} this implies typically $L_R = 3 \cdot 10^{-26}$ erg s^{-1}cm^{-3}. The factor of 10^3 enhancement in the energy loss rate (18) can again be accounted for by either a large anisotropy factor $\Lambda = 7400$ or a small anisotropy factor $\Lambda = 6 \cdot 10^{-5}$.

5.3 Nature of interstellar turbulence

The rather close agreement of our linear Landau damping heating rates with the radiative cooling rate in interstellar HII regions and in the diffuse interstellar medium modifies the result of Minter and Spangler (1997) who, on the basis of the calculation of Spangler (1991), conclude that the heating rate by linear Landau damping exceeds, in all cases, the radiative cooling rate of the diffuse interstellar medium by 3-4 orders of magnitude. Because we evaluated our heating rates with the exact damping formula and exact input turbulence wave spectra this modification results from the manner of computation of the energy loss rate (4) by Spangler (1991). The integrals must be correctly integrated over all propagation angles of the plasma waves; the final result cannot then depend anymore on the wave propagation angle, unlike Eq. (15) of Spangler (1991) and Eq. (8) of Minter and Spangler (1997).

In particular, the conclusion of Minter and Spangler (1997), that interstellar turbulence does not behave like an ensemble of obliquely propagating fast magnetosonic waves, is not valid. Quite the opposite is true: the close match of Landau damping heating rate and the interstellar radiative cooling rate strongly supports the argument that interstellar turbulence mostly consists of obliquely propagating compressive fast magnetosonic waves. However, the power spectrum of these waves is not isotropic in wavenumber space but has to be very anisotropic with $\Lambda \simeq 10^{-3}$ or $\Lambda \simeq 18$ in the fluctiferous (H II) medium and with $\Lambda \simeq 6 \cdot 10^{-5}$ or $\Lambda \simeq 7000$ in the diffuse interstellar medium. This determination of highly anisotropic wave spectra is in accord with the theoretical arguments advanced by Goldreich and Sridha (1995) who convincingly argued for ribbon-like turbulence roughly paralleling the ambient magnetic field.

5.4 Summary

Because of the ongoing efforts to obtain an understanding of energy production and loss rates in the interstellar medium,

with plasma waves being considered an integral component, it is necessary to continually update wave energy loss rates in the light of improvements in observations and their interpretations.

When considering individual plasma wave types, two factors are of importance. First, the approximations and conditions under which the wave damping rates are produced must be honored to ensure that one does not step outside the domain of validity of the approximation. Second, when integrating over a wave spectrum due care and diligence must be given in performing the integrals as accurately as possible and not approximating too early in the development. Failure to take such care can lead to inappropriate wave energy loss rates which do not reflect correctly the exact expressions.

The numerical estimates produced for linear Landau damping, using values from Spangler (1991), would seem to indicate that there is little, if any, disparity between radiative cooling rates and wave energy loss rates for the fluctiferous (H II) and the diffuse interstellar medium when the anisotropy in the wave power spectrum is properly accounted for. Our results indicate that the interstellar turbulence mostly consists of obliquely propagating compressive fast magnetosonic waves. However, the power spectrum of these waves is not isotropic in wavenumber space but has to be very anisotropic, either elongated highly along or perpendicular to the ordered magnetic field component.

References

Cox, D. P., Smith, B. W., 1974, ApJ 189, L105

Ginzburg, V. I., 1961, Propagation of Electromagnetic Waves in Plasma, Pergamon Press, New York

Goldreich, P., Sridhar, S., 1995, ApJ 438, 763

Hartquist, T. W., Snijders, M. A. J., 1982, Nat 299, 783

Jenkins, E. B., 1978, Comm. Astrophys. 7, 121

Landau, L. D., Lifschitz, E. M., 1981, Hydrodynamik, Akademie-Verlag, Berlin

McCammon, D., Sanders, W. T., 1990, AR A& A 28, 657

Minter, A. H., Spangler, S. R., 1997, ApJ 485, 182

Rickett, B. J., 1990, AR A&A 28, 561

Spangler, S. R., ApJ 376, 540

Spitzer, L., ApJ 124, 20