

## Atmospheric proton and neutron spectra at energies above 1 GeV

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**Abstract.** We discuss an effective numerical method for solving the transport equations for cosmic ray nucleons in the atmosphere. It is demonstrated that the nucleon attenuation lengths are strongly energy and depth dependent due to the non-power-law behavior of the primary spectrum, growth of the total inelastic cross sections with energy, and scaling violation in the nucleon-nucleus interactions. The numerical results are compared with the available experimental data.

### 1 Introduction

Measurements of the fluxes of secondary cosmic-ray protons and neutrons can furnish valuable information about primary cosmic rays and about the nuclear interactions at high energies. In order to extract this information from experimental data, it is necessary, among other things, to be able to calculate the nucleon attenuation lengths, which are functionals of the primary spectrum and which also depend, in general, on energy and atmospheric depth.

In this paper we discuss some results obtained by using a simple but rather efficient and physically evident numerical method for solving transport equations describing the propagation of cosmic-ray protons and neutrons through the atmosphere (Naumov and Sinegovskaya, 2000). The method is applicable at sufficiently high energies and demands several approximations which are quite traditional for high-energy atmospheric cascade calculations. Namely, we use the one-dimensional (1D) approach, in which all secondary particles in the cascade are supposed collinear with the projectiles. The processes of generation of  $N\bar{N}$  pairs in meson-nucleus collisions are disregarded, but the corresponding contribution is typically small and can be taken into account as a correction (Vall et al., 1986). Besides, we apply the standard superposition model for the collisions of cosmic-ray nuclei and neglect the geomagnetic effects and proton ionization energy

losses. These approximations confine the range of applicability of the method. However, it remains rather broad in order to describe all currently available data on the high-energy nucleons in the atmosphere and at sea level. In particular, it covers completely the depth-energy area relevant to calculations of atmospheric muon and neutrino fluxes at energies above several GeV, – one of the most important fields of application of the method.

We compare our calculations with the experimental data and with the results of a more sophisticated approach by Fiorentini et al. (2001) based on an updated version of CORT code which was designed for applications to low and intermediate energy atmospheric cascade calculations and which takes into account essentially all significant effects (Naumov, 1984; Bugaev and Naumov, 1985a,b; Bugaev et al., 1998).

### 2 The Z factor method

Within the above assumptions, the problem of calculating the differential energy spectra of protons  $D_p(E, h)$  and neutrons  $D_n(E, h)$  at the depth  $h$  consists in solving the following set of 1D transport equations:

$$\left[ \frac{\partial}{\partial h} + \frac{1}{\lambda_N(E)} \right] D_N(E, h) = \sum_{N'} \frac{1}{\lambda_{N'}(E)} \int_E^\infty dE' \times \frac{1}{\sigma_{N'A}^{\text{inel}}(E)} \frac{d\sigma_{N'N}(E', E)}{dE} D_{N'}(E', h) \quad (1)$$

( $N, N' = p, n$ ), with the boundary conditions

$$D_p(E, 0) = D_p^0(E), \quad D_n(E, 0) = D_n^0(E), \quad (2)$$

where  $D_p^0(E)$  and  $D_n^0(E)$  are the differential energy spectra of protons and neutrons at the top of the atmosphere;  $\lambda_N(E) = 1/[N_0\sigma_{NA}^{\text{inel}}(E)]$  is the nucleon interaction length;  $d\sigma_{N'N}(E', E)/dE$  is the differential cross section for inclusive reaction  $N'A \rightarrow NX$ ;  $E'$  and  $E$  are the total energies of the projectile and final nucleons, respectively.

The approximate isotopic symmetry of  $NA$  interactions makes it possible to reduce the set of equations (1) to two independent equations for the linear combinations

$$N_{\pm}(E, h) = D_p(E, h) \pm D_n(E, h). \quad (3)$$

Let us write these combinations in the form

$$N_{\pm}(E, h) = N_{\pm}(E, 0) \exp \left[ -\frac{h}{\Lambda_{\pm}(E, h)} \right]. \quad (4)$$

Below, the functions  $\Lambda_{\pm}(E, h)$  will be referred to as effective attenuation lengths. It is also convenient to introduce the dimensionless functions

$$Z_{\pm}(E, h) = 1 - \lambda_N(E)/\Lambda_{\pm}(E, h) \quad (5)$$

(“ $Z$  factors”). In just the same way as the effective attenuation lengths, the  $Z$  factors contain full information about the kinetics of nucleons in the atmosphere. Substituting Eqs. (3), (4), and (5) into transport equations (1) and integrating by part, we find that the  $Z$  factors satisfy the integral equation

$$Z_{\pm}(E, h) = \frac{1}{h} \int_0^h dh' \int_0^1 dx \eta_{\pm}(x, E) \Phi_{\pm}(x, E) \times \exp[-h' D_{\pm}(x, E, h')], \quad (6)$$

where

$$D_{\pm}(x, E, h) = \frac{1 - Z_{\pm}(E', h)}{\lambda_N(E')} - \frac{1 - Z_{\pm}(E, h)}{\lambda_N(E)},$$

$$\Phi_{\pm}(x, E) = \frac{E}{\sigma_{NA}^{\text{inel}}(E)} \left[ \frac{d\sigma_{pp}(E', E)}{dE} \pm \frac{d\sigma_{pn}(E', E)}{dE} \right],$$

$$\eta_{\pm}(x, E) = \frac{1}{x^2} \left[ \frac{D_p^0(E') \pm D_n^0(E')}{D_p^0(E) \pm D_n^0(E)} \right], \quad x = \frac{E}{E'}.$$

Although Eq. (6) is nonlinear, it is much more convenient to solve it by an iterative procedure than the original transport equations.<sup>1</sup> The rate of convergence of the procedure depends on the choice of zero-order approximation. The simplest choice is  $Z_{\pm}^{(0)}(E, h) = 0$ , in which case  $D_{\pm}^{(0)}(x, E, h)$  is independent of  $h$  and in first approximation we have

$$Z_{\pm}^{(1)}(E, h) = \int_0^1 dx \left[ \frac{\eta_{\pm}(x, E) \Phi_{\pm}(x, E)}{h/\lambda_N(E/x) - h/\lambda_N(E)} \right] \times \{1 - \exp[h/\lambda_N(E/x) - h/\lambda_N(E)]\}.$$

Obviously, the recurrent relations for  $n$ -th approximation (for  $n = 1, 2, \dots$ ) are given by

$$Z_{\pm}^{(n)}(E, h) = \frac{1}{h} \int_0^h dh' \int_0^1 dx \eta_{\pm}(x, E) \Phi_{\pm}(x, E) \times \exp[-h' D_{\pm}^{(n-1)}(x, E, h')],$$

$$D_{\pm}^{(n)}(x, E, h) = \frac{1 - Z_{\pm}^{(n)}(E/x, h)}{\lambda_N(E/x)} - \frac{1 - Z_{\pm}^{(n)}(E, h)}{\lambda_N(E)}.$$

Numerical analysis has revealed that the rate of convergence of this algorithm is quite sufficient for practical uses.

<sup>1</sup>The main reason is in the comparatively weak energy dependence of the  $Z$  factors (see Fig. 1 below).

### 3 Numerical results and discussion

In the present calculations we employ three models for the primary cosmic ray spectrum and composition: NSU (Nikolsky et al., 1984; Nikolsky, 1987), EKS (Erlykin et al., 1987), and FNV (Fiorentini et al., 2001). The NSU and EKS models describe the high-energy range of the spectrum ( $\sim 10^2$  to  $\sim 10^8$  GeV/nucleon) and take into account the change of the spectral index in the “knee” region. The FNV model is a parametrization of the data from the most recent measurements of the primary spectrum below the “knee” (0.1 to about  $10^4$  GeV/nucleon) for minimum of solar activity. All three spectra are extrapolated up to energy  $E = E_c = 3 \times 10^{10}$  GeV/nucleon above which a soft cutoff is supposed.

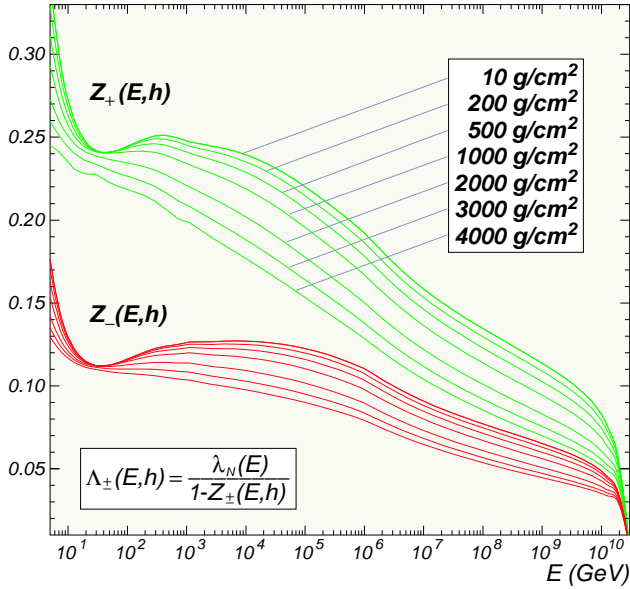
For the differential cross sections  $d\sigma_{N'N}(E', E)/dE$  we use slightly revised semiempirical model by Kimel' and Mokhov (1974, 1975). For the total inelastic cross section  $\sigma_{pA}^{\text{inel}}$  we apply the parametrization by Mielke et al. (1994).

Our calculations based on the  $Z$  factor method are performed for the energy range between 1 and  $3 \times 10^{10}$  GeV at  $h \leq 4 \times 10^3$  g/cm<sup>2</sup>. It should be noted that, at  $h \sim 10^3$  g/cm<sup>2</sup>, the energy losses by protons are important up to  $E \sim 30$  GeV.<sup>2</sup> The  $Z$  factor method (which neglects the energy loss effect) is extrapolated to the low-energy region in order to match the results of a more accurate analysis performed with CORT code. In the absence of experimental data on the fluxes of secondary nucleons arriving from oblique directions, our calculations for  $h > 10^3$  g/cm<sup>2</sup> are used at present only to test convergence of the iterative algorithm.

At all values of  $E$  and  $h$  iterative process converges fast: five to six iterations are sufficient for calculating the  $Z$  factors to precision not poorer than  $10^{-3} - 10^{-4}$ . At moderate depths,  $h \lesssim 300$  g/cm<sup>2</sup>, even the first approximation ensures a precision of a few percent, which is sufficient for many applications of the theory – in particular, for calculating the fluxes of atmospheric muons and neutrinos.

Figure 1 illustrates energy dependence of the  $Z$  factors calculated using the EKS model of the primary spectrum for several oblique depths. The ground of the observed dependence of  $Z_{\pm}$  on  $E$  and  $h$  is in the following three effects: (i) a non-power-law primary spectrum, (ii) energy dependence of the total inelastic cross section, (iii) violation of Feynman scaling in  $NA$  interactions. As a result, the attenuation lengths are also energy and depth dependent. This fact must be taken into account explicitly for determining the nucleon interaction length  $\lambda_N(E)$  from the measured nucleon intensities in the atmosphere. Local minima in the  $Z$  factors that appear in the region around 45 GeV are due to the beginning of the growth of  $\sigma_{NA}^{\text{inel}}(E)$ . At not overly large depths, the shape of the energy dependence visibly changes at  $E \gtrsim 10^6$  GeV, which is caused by artificially introduced freezing of the growth of the quasielastic peak in the reaction  $pA \rightarrow pX$ . The vanishing of the  $Z$  factors at  $E = E_c$  is due to the sup-

<sup>2</sup>By way of example, we indicate that, at  $E = 10$  GeV, the relevant correction is about 20% for protons and about 6% for neutrons (Bugaev and Naumov, 1985a,b).



**Fig. 1.**  $Z_+$  and  $Z_-$  vs energy, computed with the EKS primary spectrum for seven oblique atmospheric depths. Both  $Z_+$  and  $Z_-$  decrease with increasing depth.

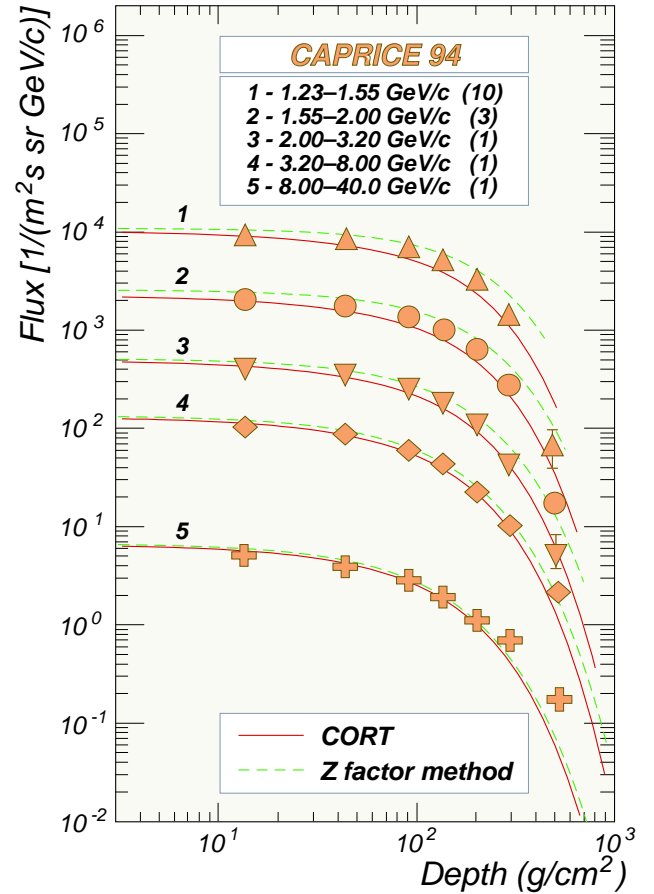
posed cutoff of the primary spectrum at  $E > E_c$ .

Figure 2 shows the atmospheric growth of the proton fluxes for five momentum bins. The data of the balloon-borne experiment CAPRICE 94 (Francke et al., 1999) are compared with our calculations performed for the same bins by using the  $Z$  factor method and CORT code. In both calculations we use the FNV model of the primary spectrum. The nominal geomagnetic cutoff rigidity in the experiment was about 0.5 GV; hence the geomagnetic effect can be neglected. As one can expect, the proton fluxes calculated by the  $Z$  factor method systematically exceed those are obtained with CORT code, which takes into account the energy loss effect. The discrepancies become larger at increasing depth but vanishes at increasing the proton momentum. Minor differences between two calculations at small depths are in part due to different treatments of nucleus-nucleus interactions.

Comparison of the calculated differential energy spectra of nucleons at three atmospheric depths with the data from Fawler et al. (1966); Aguirre et al. (1968); Apanasenko and Scherbakova (1968) are depicted in Fig. 3. The data collected in Fig. 3 were obtained indirectly, from measurements of photon spectra in extensive air showers (Grigorov et al., 1973) and are therefore model-dependent to considerable extent.<sup>3</sup> Nonetheless, our calculation relying on the EKS and NSU models of primary spectrum are by and large consistent with this data sample.

In Fig. 4 we compare the calculated energy spectra of protons and neutrons at sea level with the data from Brooke and Wolfendale (1964); Ashton and Coats (1968); Ashton

<sup>3</sup>In particular, approximate formulas used for recalculating photon energy to nucleon energy lead to a distortion of the nucleon energy spectra.

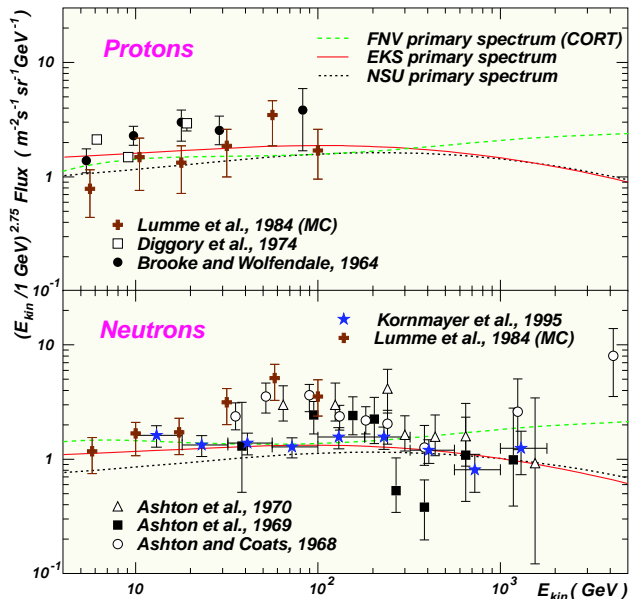


**Fig. 2.** Atmospheric growth curves for protons. The data are from the CAPRICE 94 balloon-borne experiment (Francke et al., 1999). The curves are calculated by CORT and by using the  $Z$  factor method with the FNV model of primary spectrum. The legend shows the proton momentum bins and scale factors (in parentheses).

et al. (1969, 1970); Diggory et al. (1974); Kornmayer et al. (1995). The results of Monte Carlo calculation by Lumme et al. (1984) is also shown. Direct measurements of the proton energy spectra at sea level are very fragmentary and one can speak only about qualitative agreement with the results of our calculations. Estimates reveal (see also Vall et al. (1986); Bugaev et al. (1998)) that the inclusion of processes of nucleon production in meson-nucleus interactions can increase the vertical flux of nucleons at sea level by no more than 10% at  $E = 1$  TeV and by about 15% at  $E = 10$  TeV, but this increase is much smaller, in either case, than the uncertainties in the  $NA$  cross sections and in the primary spectrum. Experimental data on the neutron component at sea level are more detailed, but rather contradictory. The results of our calculations are in quite a good agreement with the data from

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**Fig. 3.** Energy spectra of nucleons at three atmospheric depths.



**Fig. 4.** Energy spectra of protons and neutrons at sea level.

recent measurements at prototype of the KASCADE facility in Karlsruhe (Kornmayer et al., 1995). As can be seen from Fig. 4, the neutron data by Kornmayer et al. (1995) below 200–300 GeV are described by the calculation with the EKS and FNV models of primary spectrum somewhat better than by the calculation with the NSU model.

It can be hoped that further experiments to study the nucleon component of secondary cosmic rays will allow a more detailed test of the method and of the models for the primary spectrum and for nucleon-nucleus interactions.

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## References

- Aguirre, C., et al., *Can. J. Phys.*, 46, 660, 1968.  
 Apanasenko, A. V. and Scherbakova, M. N., *Izv. Akad. Nauk Ser. Fiz.*, 32, 372, 1968.  
 Ashton, F. and Coats, R. B., *J. Phys. A: Gen. Phys.*, 1, 169, 1968.  
 Ashton, F., et al., *Izv. Akad. Nauk Ser. Fiz.*, 33, 1557, 1969.  
 Ashton, F., et al., *Acta Phys. Acad. Sci. Hung.*, 29 (Suppl. 3), 25, 1970.  
 Brooke, G. and Wolfendale, A. W., *Proc. Phys. Soc.*, 84, 843, 1964.  
 Bugaev, E. V. and Naumov, V. A., *Investigations on Geomagnetism, Aeronomy and Solar Physics*, 73, 198, 1985.  
 Bugaev, E. V. and Naumov, V. A., *Institute for Nuclear Research, Reports II-0385 and II-0401*, Moscow, 1985 (unpublished).  
 Bugaev, E. V., et al., *Phys. Rev. D* 58, 054001, 1998.  
 Diggory, I. S., et al., *J. Phys. A: Math., Nucl., Gen.*, 7, 741, 1974.  
 Erlykin, A. D., Krutikova, N. P., and Shabelskiy, Y. M., *Yad. Fiz.*, 45, 1075, 1987.  
 Fowler, P., et al., *Nature*, 209, 567, 1966.

- Fiorentini, G., Naumov, V. A., and Villante, F. L., preprint hep-ph/0103322 (accepted for publication in *Phys. Lett. B*).  
 Francke, T., et al., in *Proc. of the 26th Int. Cosmic Ray Conf.*, Salt Lake City, Utah, Aug. 17–25, Vol. 2, 80, 1999.  
 Grigorov, N. L., Rapoport, I. D., and Shestoporov, V. Ya., *High-energy particles in cosmic rays*, “Nauka”, Moscow, 1973.  
 Kimel’, L. R. and Mokhov, N. V., *Izv. Vuz. Fiz.* 10, 17, 1974.  
 Kimel’, L. R. and Mokhov, N. V., *Problems of Dosimetry and Radiation Protection*, 14, 41, 1975.  
 Kornmayer, H., et al., *J. Phys. G: Nucl. Part. Phys.*, 21, 439, 1985.  
 Lumme, M., et al., *J. Phys. G: Nucl. Phys.*, 10, 683, 1984.  
 Mielke, H. H., et al., *Phys. G: Nucl. Part. Phys.*, 20, 637, 1994.  
 Naumov, V. A., *Investigations on Geomagnetism, Aeronomy and Solar Physics*, 69, 82, 1984.  
 Naumov, V. A. and Sinegovskaya, T. S., *Yad. Fiz.*, 63, 2020, 2000 [*Phys. Atom. Nucl.*, 63, 1927, 2000].  
 Nikolsky, S. I., Stamenov, J. N., and Ushev, S. Z., *Zh. Eksp. Teor. Fiz.* 87, 18, 1984.  
 Nikolsky, S. I., in: *Problems of Cosmic Ray Physics*, “Nauka”, Moscow, p. 169, 1987.  
 Vall, A. N., Naumov, V. A., and Sinegovsky, S. I., *Yad. Fiz.* 44, 1240, 1986 [*Sov. J. Nucl. Phys.* 44, 806, 1986].