

## Can we predict transport coefficients of heliospheric particles from solar wind observations?

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**Abstract.** Current theories for parallel diffusion of high energy particles in the Heliosphere are fairly well accepted, and supported by both observations and simulation results, if recently found evidence for the geometry of the magnetic fluctuations is taken into account. However, there are important outstanding questions pertaining at medium to low energies where dynamical effects in the solar wind such as propagation and thermal damping of waves, and time dependent decorrelation of magnetic fluctuations have a strong influence on the scattering mean free path. A model is presented which addresses the above effects, and which is able to explain the observations of particle mean free paths ranging from keV electrons to GeV protons. It is found that the dynamical effects, leading to a strongly non-resonant pitch angle scattering through  $90^\circ$  at low rigidities, can be described by a single parameter which is estimated from the observed density, temperature and magnetic field strength in the solar wind. The predictive power of the model is then basically limited by the current lack of knowledge of the exact decomposition of the fluctuations.

### 1 Introduction

The need for a correct quantitative treatment of the interactions between cosmic rays and turbulent magnetic fields still is one of the fundamental problems of modern astrophysics, and the study of energetic solar particle propagation offers a unique possibility to test model predictions with in-situ measurements. Considerable progress has been achieved in recent years towards a better understanding of the nature of the solar wind turbulence, and to overcome some of the deficiencies of the first, pioneering scattering theories of Jokipii (1966) and Hasselmann and Wibberenz (1968)) that could not be reconciled with observations. New approaches to the theory which take into account the dynamical character, and the three-dimensional geometry of the magnetic field fluctuations (e.g., Bieber et al., 1994) and, in another approach the

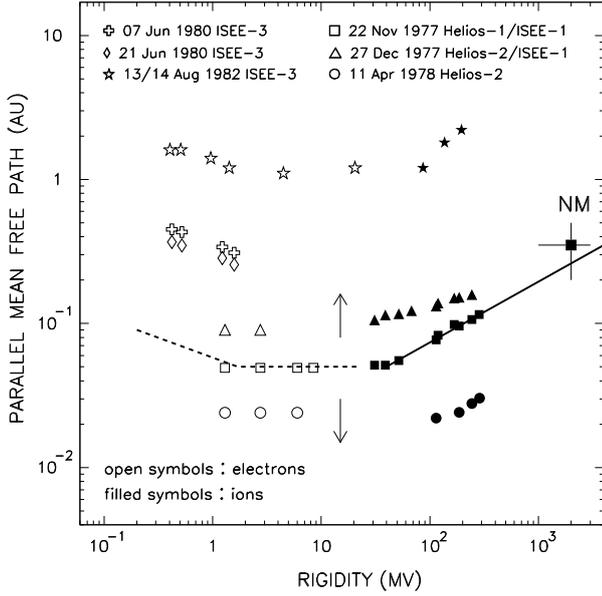
effects of wave propagation and thermal wave damping and resonance broadening (e.g., Schlickeiser and Achatz, 1993) have shown to give better explanations for various aspects of the observations (for a recent review see Dröge, 2000a). These studies predict the correct rigidity dependence of the particle's scattering mean free path (cf., Fig. 1), and have improved the agreement between scattering theory and observations in a statistical sense, averaged over many events. However, a capability to determine the mean free path from the measured properties of the solar wind plasma on an event-by-event basis is still completely lacking. In the present paper we will discuss the possibility to identify a few proxies in the plasma observations which might allow a more accurate estimate for mean free path in a given event.

### 2 Theory

Advanced approaches to QLT which take into account wave propagation effects as well as the dynamical character and the 3-dimensional geometry of the magnetic field fluctuations, and the effects of a dissipation range of the turbulence at high frequencies, predict that the mean free path has an explicit velocity dependence which leads to larger values for electrons than for protons at the same rigidity below a certain threshold. Both "wave" and "turbulence" approaches employ a resonance broadening of the scattering process which can approximately produce the shape of the suggested curve in Figure 1. To reconcile the observations with the still too large absolute levels of the mean free paths, Bieber et al. (1994) suggested a composite model for the fluctuations which consists of  $\approx 20\%$  slab and  $80\%$  2-D fluctuations, the latter contributing little or not at all to particle scattering. In the following, we will adopt the above hypothesis and consider only the slab component as being responsible for the scattering of solar particles. The spectral functions can then be cast into the form (cf., Dröge, 2000a)

$$P_{ij}(\mathbf{k}, \tau) = P_{ij}(k) e^{i\omega_j^r(k)\tau - \Gamma_j(k)\tau} \delta(k_x) \delta(k_y) \quad (1)$$

where we use helical ( $i, j = R, L; k = k_z = k_{\parallel}$ ) coordinates which allow the spectral functions to be related to the spectral



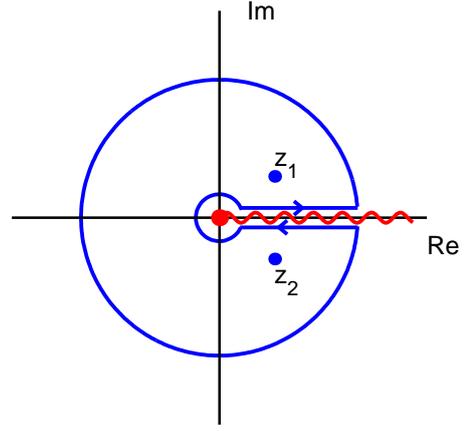
**Fig. 1.** Local parallel mean free path vs. particle rigidity for selected solar particle events. The form of the rigidity dependence as indicated by the curve seems to be consistent with observations from any given event, only the absolute height of the curve varies (from Dröge 2000b).

densities  $I_{R,L}^{\pm}(k)$  of forward (+) and backward (-) propagating waves with right-hand (R) or left-hand (L) polarization, and to the helicity of the fluctuations. Here  $\omega_{R,L}^r(k)$  is the real part of the dispersion relation in the wave picture, and  $\Gamma_{R,L}(k)$  describes wave damping effects due to interactions with the warm ( $T > 0$ ) background plasma, or decorrelation effects in the turbulence picture, in which case  $\omega_{R,L}^r(k) = 0$ .

The pitch angle scattering coefficient now can be expressed as

$$D_{\mu\mu}(\mu) = \frac{\Omega^2(1-\mu^2)}{2B_0^2} \int_{-\infty}^{+\infty} dk \times \left\{ \frac{\Gamma_R(k)}{\Gamma_R^2(k) + (k\mu v - \omega_R^r(k) - \Omega)^2} P_{RR}(k) + \frac{\Gamma_L(k)}{\Gamma_L^2(k) + (k\mu v - \omega_L^r(k) + \Omega)^2} P_{LL}(k) \right\} \quad (2)$$

The occurrence of the function  $\Gamma(k)$  in the spectral density (2) leads to a resonance broadening in the particles' interaction with the fluctuations. As a result, particles with a given  $\mu v$  (in particular,  $\mu = 0$ ) can now be scattered by fluctuations within a finite range of wave numbers. A number of different functional forms for  $\Gamma(k)$  have been suggested. Achatz et al. (1993) found that  $\Gamma(k) \simeq kv_{th,p} \exp(-\Omega_p/(\beta^{1/6}kV_A))$  can describe the damping of ion cyclotron waves, where  $v_{th,p}$  is the thermal background proton velocity,  $V_A$  the Alfvén speed, and  $\beta$  the plasma-beta. In the turbulence picture of the fluctuations, Bieber et al. (1994) consider a dynamical scattering model with  $\Gamma_j(k) = \alpha|k|V_A$ , where the parameter  $\alpha$  allows to adjust the strength of the dynamical effects,



**Fig. 2.** Contour used to evaluate the integral in Eq. (3).

ranging from  $\alpha = 0$  (magnetostatic limit) to  $\alpha = 1$  (strongly dynamical).

If we impose a few further simplifications, an analytical solution of equation (2) can be obtained. We neglect wave propagation, and consider the component  $I_R^+(k)$  (in this case equivalent to  $I_L^-(k)$  or static fluctuations of positive helicity), to be of the form  $I(k) = I_5 \cdot k_5^{-q}$  (i.e., neglecting the dissipation range of the fluctuations), where  $k_5$  is the wavenumber in units of  $10^{-5} \text{ km}^{-1}$ , and  $I_5 = I(k_5)$ . Resonance broadening effects are described by  $\Gamma = \delta k V_A$ , with  $\delta = \alpha$  for the turbulence picture, and  $\delta = \sqrt{\beta}$  for the wave picture in the limit of large wave numbers. Equation (2) then transforms into

$$D_{\mu\mu}(\mu) = \frac{1-\mu^2}{2} \frac{\Omega^2}{B_0^2} \delta V_A I_5 \times \int_0^\infty \frac{dx x^{1-q}}{(\delta^2 V_A^2 + \mu^2 v^2)x^2 - 2\mu v \Omega/k_5 x + \Omega^2/k_5^2} \quad (3)$$

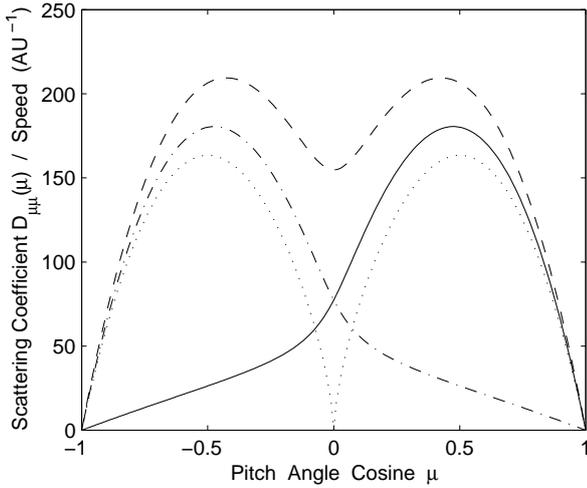
with  $x = k/k_5$ . The integral in equation (3) can be performed by utilizing complex integrals. Poles on the complex plane are located at

$$z_{1,2} = \frac{\Omega}{k_5} \frac{\mu v \pm i\delta V_A}{\delta^2 V_A^2 + \mu^2 v^2} \quad (4)$$

Evaluating the integral along the contour indicated in Figure 3, we obtain

$$D_{\mu\mu}(\mu) = \frac{\pi}{\sin((2-q)\pi)} \frac{1-\mu^2}{2} \frac{\Omega k_5 I_5}{B_0^2} \times \left( \frac{\Omega/k_5}{\sqrt{\delta^2 V_A^2 + \mu^2 v^2}} \right)^{1-q} \sin \left[ (1-q) \operatorname{atan} \left( \frac{\delta V_A}{\mu v} \right) \right] \quad (5)$$

The form of the scattering coefficient (5) for protons of 1 MV rigidity and  $\delta = 1$  is shown in Figure 3, together with the corresponding contribution to  $D_{\mu\mu}(\mu)$  from fluctuations with negative helicity, the sum of both components, and the result of standard QLT. It is evident from the figure that due



**Fig. 3.** Pitch angle diffusion coefficient in the case of resonance broadening for protons of 1 MV rigidity and  $\delta = 1$  for positive (solid line) and negative (dash-dotted line) helicity. Also shown are the sum of both components (dashed line), and the results of standard QLT (dotted line).

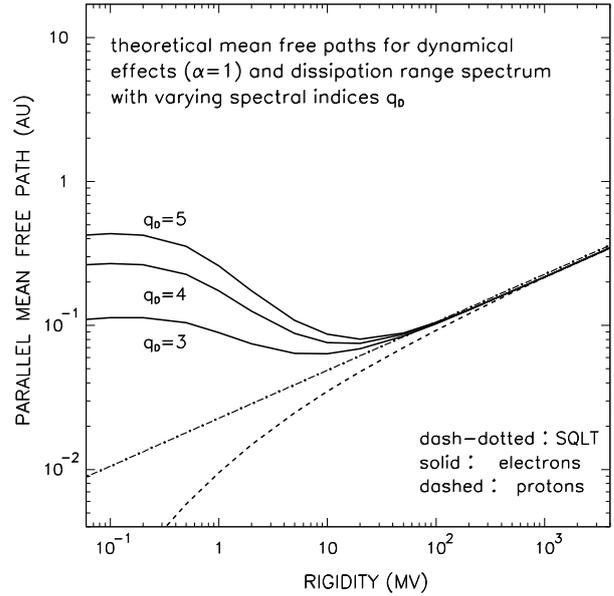
to the resonance broadening effects particles are scattered at all pitch angles, and in particular through  $\mu = 0$ , even if only one wave mode (or helicity) is present.

The mean free path  $\lambda_{\parallel}$  which relates the pitch angle scattering rate to the spatial diffusion parallel to the ambient magnetic field is easily obtained with the well-known formula

$$\lambda_{\parallel} = \frac{3v}{8} \int_{-1}^{+1} d\mu \frac{(1 - \mu^2)^2}{D_{\mu\mu}(\mu)} \quad (6)$$

It turns out that equation (5) is a fairly good approximation for the ion's mean free path in the case that the resonance broadening is sufficiently strong ( $\delta \sim 0.3$  or larger). However, for electrons with energies typical for solar events, the dissipation range must not be neglected. The reason is that electrons simply have less time, because of their higher speed, to 'feel' the decaying of the correlations and therefore interact with wave numbers over a smaller range and experience weaker scattering, compared to ions of the same rigidity.

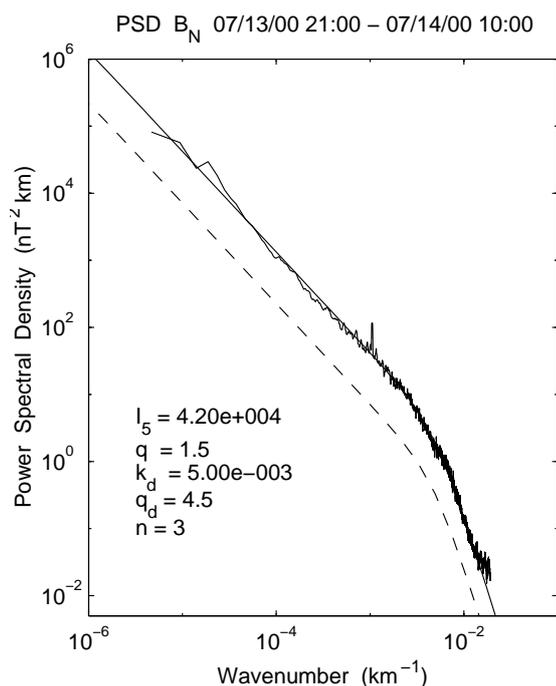
To implement a more realistic spectrum exhibiting a dissipation range, we have applied a numerical technique for the integration of equation (2). In Figure 4 we present mean free paths calculated for a fluctuation spectrum assuming  $\delta = 1$  and an onset of the dissipation range at  $0.02 \text{ km}^{-1}$ , and for three values of the dissipation range spectral index typically within the range of observations. The curves resulting from the combined effects of the dissipation range and resonance broadening are in good agreement with the shape of the rigidity dependence shown in Figure 1. As can be seen from the figure, at rigidities above 100 MV standard QLT is a good approximation for the mean free path, whereas below that value the mean free path is drastically larger for electrons, but even smaller for protons, compared to standard QLT.



**Fig. 4.** Theoretical mean free paths for resonance broadening and a power spectrum with spectral index 5/3 in the inertial range, but with varying spectral indices in the dissipation range. Only electron mean free paths below a rigidity  $\sim 50$  MeV are affected. For comparison, the result of dissipationless standard QLT is also shown.

### 3 Observations

To test the model, and the simplifications made, one would ideally want to compare the scattering properties of electrons and ions over a large range in rigidity and, in particular, electrons and ions at the same rigidity, e.g., electrons at several MeV and ions at  $\sim \text{keV}$  energies. This has turned out to be difficult to observe, because solar particle events with electron fluxes at 10 MV or above which are large enough to derive meaningful values of  $\lambda$  are scarce, whereas the propagation of ions in that range, due to their low speeds is often affected by varying conditions in the solar wind and also, or even totally dominated by coronal mass ejections and interplanetary shocks. A possible candidate is the "Bastille" event of 14 July 2000, in spite the fact that interplanetary disturbances from preceding events were present and spacecraft data were affected by high interplanetary particle fluxes. Mean free paths were determined for electrons observed with the Wind 3DP instrument (Lin et al., 1995) in the range 27 - 179 keV, and for relativistic protons (for details see Bieber et al., these proceedings). A spectrum of the magnetic fluctuations was determined from high resolution (3 vectors/s) ACE MAG observations for the time period 07/13/01 21:00 UT to 07/14/00 10:00 UT (cf., Fig. 5). Theoretical mean free paths were computed from the power spectrum and averages of plasma parameters for the above period, i.e.,  $B = 5.31 \text{ nT}$ ,  $n = 3.49 \text{ cm}^{-3}$ ,  $T = 175 \text{ 000 K}$ , and  $\beta = 0.825$ . The results were compared with the observations for i) the thermal damping model with an assumption of a 17% slab portion (indicated by the dashed line in Fig. 5) being the only free

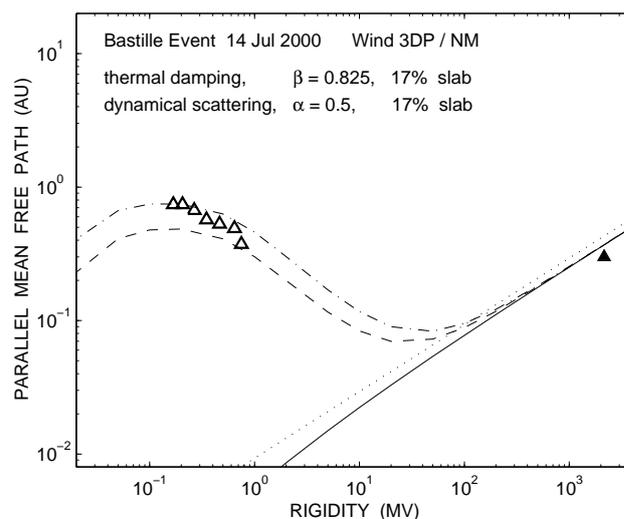


**Fig. 5.** Power spectrum of the magnetic field normal component (in the RTN system, ACE MAG data kindly provided by C. W. Smith / N. F. Ness) for the period indicated in the figure. A double power law fit was performed to the spectrum which is used as a proxy to estimate the mean free path of solar particles in the 14 July 2000 event.

parameter, and ii) for the turbulence model with an additional free parameter of  $\alpha = 0.5$ . As can be seen from Figure 6, the agreement between observations and modeling is very good.

#### 4 Conclusions

The results presented in this work indicate that we finally have arrived at a correct and complete description of solar energetic particle transport in the inner heliosphere. It appears that QLT is a good approximation to particle scattering at all pitch angles if resonance broadening due to dynamic or wave damping effects is taken into account. These effects dominate scattering through  $\mu = 0$  and will probably render the need for investigating other additions to QLT such as non-linear corrections, mirroring, wave propagation effects and details of the dispersion relation unnecessary. Resonance broadening is related to observable plasma parameters  $B$ ,  $n$ , and  $T$ , so a prediction of local scattering conditions should be possible. Particle observations have revealed a strong correlation between near-relativistic protons and electrons, and the detection of one species can be used to forecast the propagation conditions of the other, or of ions in the intermediate rigidity range. Because the scattering strengths of  $< 1$  MV electrons and ions of the same rigidity differ by order of magnitudes (cf., Figs. 4 and 6), these electrons may be used as probes to study the level of interplanetary turbu-



**Fig. 6.** Mean free paths obtained from neutron monitor and Wind 3DP electron data (triangles), and predictions from the dynamical scattering (dash-dotted line, electrons) and thermal damping (dashed line, electrons) models. Solid line shows prediction for ions (similar for both models), dotted line standard QLT result.

lence, and thus the acceleration efficiency of a CME-driven shock wave in the inner heliosphere, in order to predict properties of the particle event associated with the shock when it passes the Earth after two days or so. The major remaining obstacle for a true prediction of scattering conditions in the solar wind remains our lack of knowledge of the exact decomposition of the fluctuations, which is difficult to obtain from single spacecraft measurements, and the question whether those measurements are representative for scattering conditions in the inner heliosphere. If current analysis methods could be further refined, and if routine plasma observations from spacecraft located closer to the sun should become available in the future, major improvements in the forecast of solar particle events should be possible.

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