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Extrapolation of hadron production models to ultra-high energy

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Abstract. QCD-inspired models of high-energy hadron production can be used to predict, among others, cross sections, mean production multiplicities and multiplicity fluctuations. These quantities are closely related due to the QCD factorization theorem and Abramovski-Gribov-Kancheli cutting rules. Focusing on the generic structure of models implementing minijet production, we discuss QCD predictions on minijet cross sections and constraints from HERA and Tevatron data.

1 Introduction

Almost thirty years after its proposal, QCD is now the accepted theory of strong interactions. It is clear that a successful high-energy hadron production model has to be based on or compatible with QCD predictions. However, although we have numerous and detailed QCD predictions for large momentum transfer processes, our understanding of the bulk of hadronic interactions is still rather limited. Any detailed calculation of high-energy hadron production, as needed for the simulation of extensive air showers, requires many additional assumptions which cannot be justified on grounds of theoretical predictions. Often different models predict very different particle distributions if extrapolated to high energy. These differences can mainly be understood in terms of different assumptions on QCD-predicted cross sections and their implementation in these models (Engel, 1999b).

In the following we will discuss QCD predictions for jets with transverse momenta in the range of 2 - 5 GeV (minijets) and their relation to the high-energy extrapolation of QCD-inspired models. We will emphasize model-independent quantities at the expense of not always being able to present quantitative predictions.

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2 Inclusive minijet cross section

Thanks to asymptotic freedom, perturbative QCD allows us to calculate jet production in binary parton-parton collisions. The expression for the inclusive cross section reads in leading-order perturbation theory

$$\sigma_{\rm 2jet}(s, p_\perp^{\rm cutoff}) = K \int dx_1 dx_2 d^2p_\perp \times \sum_{i,j,k,l} \frac{1}{1+\delta_{k,l}} f_{A,i}(x_1,Q^2) f_{B,j}(x_2,Q^2) \frac{d\sigma^{\rm QCD}_{i,j\to k,l}}{d^2p_\perp}, \quad (1)$$

where $f_{A,i}(x_1,Q^2)$ and $f_{B,i}(x_2,Q^2)$ are the parton distribution functions of hadron A and B for the parton i. Eq. (1) refers to the integrated minijet cross section for jets with transverse momentum $p_\perp > p_\perp^{\rm cutoff}$. The factor K accounts for neglected higher-order contributions and is expected to be approximately 2.

The QCD factorization theorem states that Eq. (1) will always have a structure which factorizes the parton densities and the hard interaction process, independent of the order in perturbation theory and the particular hard process. QCD factorization holds in the limit $Q^2 \gg \Lambda_{\rm QCD}$ where $Q^2 \sim p_\perp^2$ is the hard scale of the partonic interaction process and $\Lambda_{\rm QCD}$ is the QCD renormalization scale. It is important to notice that the QCD factorization theorem refers to fully inclusive processes, i.e. there are no additional conditions imposed on the interaction of, for example, the hadronic remnants. In particular Eq. (1) does not specify how many hard partonic interactions happen per hadronic collision.

The minijet cross section depends strongly on the transverse momentum cutoff and perturbative QCD does not predict the smallest value of $p_\perp^{\rm cutoff}$ for which Eq. (1) is valid. Naively one would expect an energy-independent cutoff because of the energy-independence of the condition $Q^2 \gg \Lambda_{\rm QCD}$.

Despite these limitations, Eq. (1) is the most fundamental QCD input to hadronic interaction models such as DPMJET (Ranft, 1995; Roesler *et al.*, 2000), neXus (Drescher *et* *al.*, 2000), QGSJET (Kalmykov *et al.*, 1997), and SIBYLL (Fletcher *et al.*, 1994; Engel *et al.*, 1999a). Fig. 1 shows the inclusive cross section for jet pairs in proton-proton collisions as calculated with the GRV98 parton densities (Glück *et al.*, 1998). As reference the plot includes one example of a hard cross section predicted before HERA data were available (EHLQ, set 1) (Eichten *et al.*, 1985). In addition, collider data and the fit of Donnachie & Landshoff (1992) (DL) for the total proton-proton cross section are also shown.

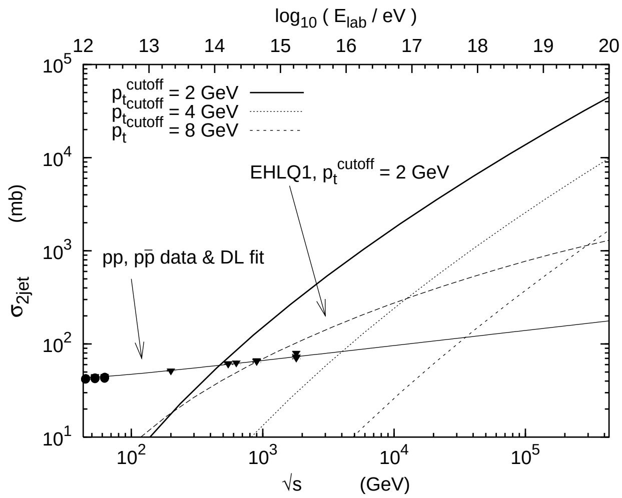

Fig. 1. Inclusive two-jet cross section for proton-proton collisions.

Typically models implement minijet cross sections calculated with a transverse momentum cutoff of about 2 GeV (or an equivalent type of constraint). It is clear from the comparison of the GRV98-based prediction for $p_\perp^{\rm cutoff} = 2$ GeV with the EHLQ cross section that the change from pre-HERA to post-HERA parton densities will change the model predictions considerably. Whereas the predictions for $E_{\rm lab} \sim 10^{20}$eV are rather uncertain due to the extrapolation of the parton densities to small x, the hard cross section at Tevatron energies ($E_{\rm lab} \sim 10^{15} - 10^{16}$eV) is almost completely determined by HERA data. Currently HERA measurements extend down to $x \sim 10^{-4}$ (quarks) and $x \sim 3 \cdot 10^{-4}$ (gluons) for $Q^2 \sim 4$ GeV^2 (Adloff *et al.*, 2000).

Because understanding the minijet cross section is the key to understanding particle production at high energy, it is of great importance to determine reasonable values for the parameter it is most sensitive to, the transverse momentum cutoff $p_\perp^{\rm cutoff}$. Jet cross section measurements at Tevatron are published only for large transverse momenta, $p_\perp \gtrsim 50$ GeV. However, one can use the transverse momentum distribution of charged particles to derive some information on the minijet cross section. In Fig. 2 we show the inclusive cross section for charged particle production at Tevatron. The leading-order perturbative QCD predictions are calculated with PHOJET (Engel & Ranft, 1996) and $K = 2$. Although this comparison might be somehow biased by the limitations of the jet fragmentation model, it strongly disfavors the calculation with $p_\perp^{\rm cutoff} = 1.5$ GeV. Assuming a smooth turn-over in the cross section when approaching the transverse momentum cutoff implies that the disagreement between data and leading-order QCD starts already at $p_\perp \sim 2.5$ GeV. On the other hand, transverse momentum spectra of charged particles are fully compatible with $p_\perp^{\rm cutoff} = 1.5$ GeV at low energy.

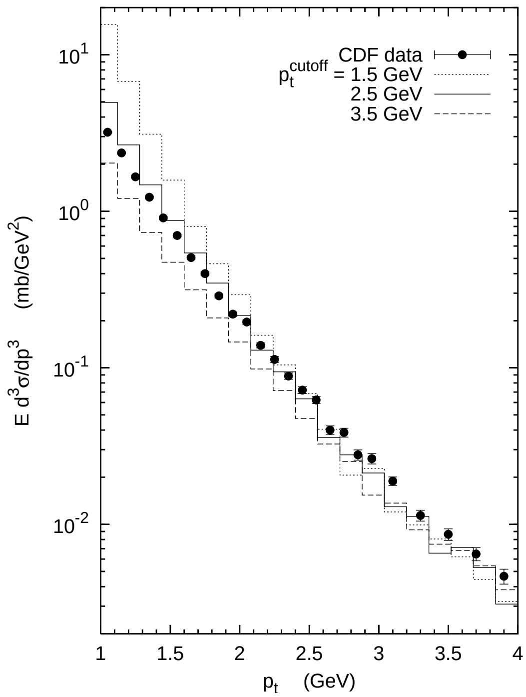

Fig. 2. Inclusive charged particle cross section in $p\bar{p}$ collisions at $\sqrt{s} = 1800$ GeV. Data are from Abe *et al.* (1988).

3 From inclusive to exclusive cross sections

Exclusive cross sections are needed for the construction of any complete hadron interaction and multiparticle production model. In terms of minijet production this means specifying the probability for having n parton-parton interactions in a single hadron-hadron collision. The exclusive cross sections for the production of n jet pairs, $\sigma_{\rm 2jet}^{(n)}$, have to satisfy

$$\sigma_{\rm tot} = \sum_{n=0}^{\infty} \sigma_{\rm 2jet}^{(n)} \qquad \sigma_{\rm 2jet} = \sum_{n=1}^{\infty} n \cdot \sigma_{\rm 2jet}^{(n)} \tag{2}$$

to reproduce exactly the QCD-predicted, inclusive minijet cross section (1) and the total cross section known from experiment.

In general, Abramovski-Gribov-Kancheli (AGK) cutting rules (Abramovski *et al.*, 1974) can be used to relate exclusive inelastic and elastic cross sections to each other. For instance, the AGK rules specify the sign and size of the contribution of an particular inelastic graph to the total and elastic cross sections. Because of space limitations we won't discuss in the following the cutting rules in general but only one particular realization, the eikonal model.

In a simple two-component eikonal approach the elastic scattering amplitude reads

$$a(s,\boldsymbol{B}) = \frac{i}{2}\left(1-\exp\{-\chi_{\rm soft}(s,\boldsymbol{B})-\chi_{\rm hard}(s,\boldsymbol{B})\}\right), \quad (3)$$

where $\boldsymbol{B}$ denotes the impact parameter of the collision. The eikonal functions for soft and hard interactions are given by $\chi_k(s,\boldsymbol{B}) = \frac{1}{2}\sigma_k(s)A_k(s,\boldsymbol{B})$, with A_k being the normalized density profile function and $k = \mathrm{soft}, \mathrm{hard}$.

Applying AGK cutting rules the inelastic cross section reads

$$\sigma_{\rm ine} = \int d^2\boldsymbol{B}\,(1-\exp\{-2\chi(s,\boldsymbol{B})\}) = \sum_{n_s+n_h>0}^{\infty}\sigma_{n_h,n_s}, \quad (4)$$

with the partial cross section for n_h hard and n_s soft interactions being

$$\sigma_{n_h,n_s} = \int d^2\boldsymbol{B}\,\frac{(2\chi_{\rm hard})^{n_h}}{n_h!}\frac{(2\chi_{\rm soft})^{n_s}}{n_s!} \times\exp\{-2\chi_{\rm soft}-2\chi_{\rm hard}\}. \quad (5)$$

Assuming that each hard interaction produces a minijet-pair, we get

$$\sigma_{\rm 2jet} = \sum_{n_h=1}^{\infty}\sum_{n_s=0}^{\infty} n_h\cdot\sigma_{n_h,n_s} = \sigma_{\rm hard}. \quad (6)$$

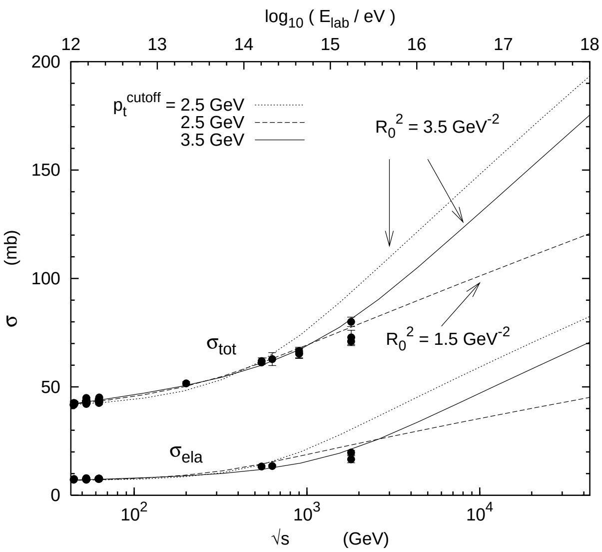

Fig. 3. Eikonal model predictions for total and elastic *pp* cross sections. The data are for *pp* and $p\bar{p}$ collisions (Avila *et al.* (1999) and Refs. therein).

The total cross section is given by the optical theorem as

$$\sigma_{\rm tot}(s) = 4\int d^2\boldsymbol{B}\,\Im m(a(s,\boldsymbol{B})). \quad (7)$$

Hence the structure of the amplitude allows us to have an arbitrarily large inclusive minijet cross section in the eikonal function if the profile function is of such a shape that the scattering is restricted to impact parameters $B < B_{\rm max}$, with $\sigma_{\rm tot} \sim \pi B_{\rm max}^2$. This means that one can always satisfy the constraint (2). However, the amplitude (3) also predicts the elastic cross section

$$\sigma_{\rm ela}(s) = 4\int d^2\boldsymbol{B}\,|a(s,\boldsymbol{B})|^2. \quad (8)$$

At high energy a "black disk" like amplitude is expected because from $\sigma_{\rm hard} \gg \sigma_{\rm tot}$ follows

$$a(s,\boldsymbol{B}) \stackrel{\chi_{\rm hard}\gg 1}{\longrightarrow} \frac{i}{2} \qquad |\boldsymbol{B}| < \sqrt{\sigma_{\rm tot}}/\pi. \quad (9)$$

As can be seen from (7) the black disk limit leads to $\sigma_{\rm ela} = \sigma_{\rm tot}/2$. Measurements indicate that pp scattering at $\sqrt{s} = 1800$ GeV does not correspond to black disk scattering: $\sigma_{\rm ela}/\sigma_{\rm tot} \sim 0.23-0.25$ (see (Engel, 2000) and Refs. therein), though the black disk limit might have been reached for a very small region about $B \approx 0$.

Using data on cross sections and the t-slope of the elastic cross section $d\sigma_{\rm ela}/dt$, one can derive a limit on the smallest transverse momentum cutoff for which Eq. (2) can be fulfilled in a consistent way. Of course, such a limit will depend to some extent on the particular details of the considered model. In Fig. 3 fits of the amplitude (3) to total and elastic cross section data are shown for different assumptions on the profile function $A_{\rm hard}(\boldsymbol{B})$. For simplicity we use a gaussian profile and vary the parameter R_0

$$A(\boldsymbol{B}) = \frac{1}{4\pi R_0^2}\exp\left\{-\frac{\boldsymbol{B}^2}{4R_0^2}\right\}. \quad (10)$$

The soft cross section is parametrized as $\sigma_{\rm soft} = \sigma_0 s^{\Delta}$. Although the cross section fits are shown only for the eikonal model, the results qualitatively do not change if one considers a two-channel eikonal model as implemented in DPMJET and SIBYLL 2.1 or the quasi-eikonal model which is the basis of QGSJET.

Assuming that the partons involved in jet production are uniformly distributed in transverse space all over the proton, we find $p_\perp^{\rm cutoff} = 3.5$ GeV as a lower limit for the transverse momentum cutoff. The situation changes drastically if we take the possibility of parton clustering into account which might lead to a smaller R_0 parameter. For example, for $R_0^2 = 1.5$ GeV^{-2} we get $p_\perp^{\rm cutoff} = 2.5$ GeV as limit.

Indeed there are experimental indications that partons are distributed in clusters inside the proton. The CDF Collab. measured the ratio of 4-jet events to 2-jet events for a jet transverse energy cutoff of 5 GeV (Abe *et al.*, 1997). To interpret the CDF data it is convenient to express this ratio in terms of the effective cross section (Calucci & Treleani, 1999)

$$\sigma_{\rm eff} = \frac{1}{2}\frac{[\sigma_{\rm 2jet}]^2}{\sigma_{\rm 4jet}} = \frac{\langle n_h\rangle^2}{\langle n_h(n_h-1)\rangle}. \quad (11)$$

Within the eikonal model this can be simplified to

$$\sigma_{\rm eff} = \frac{1}{\int d^2\boldsymbol{B}\,\left[A_{\rm hard}(\boldsymbol{B})\right]^2} \stackrel{\rm gaussian}{=} 8\pi R_0^2, \quad (12)$$

where the RHS is valid only for a gaussian distribution. The CDF result of $\sigma_{\rm eff} = 14.5\pm 1.7^{+1.7}_{-2.3}$ mb corresponds to $R_0^2 = 1.5$ GeV^{-2}.

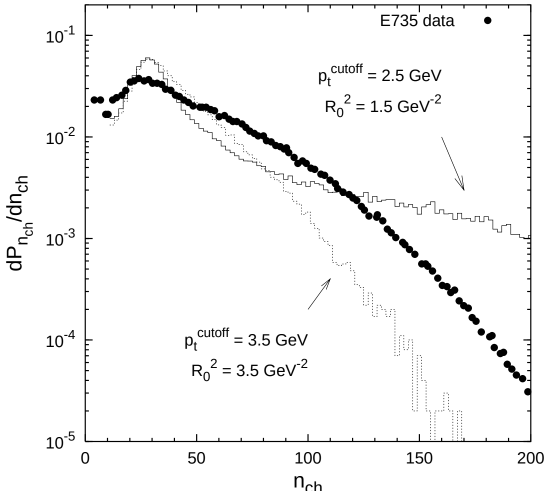

Fig. 4. Charged particle multiplicity distribution as measured by E735 (Alexopolous *et al.*, 1998). The data are compared to eikonal model predictions for two different minijet cross sections and density profiles.

The clustering of partons will naturally lead to large event-by-event fluctuations in the minijet multiplicity. The more the partons are grouped in clusters the wider will be the distribution of the number of minijets per hadron-hadron collision. Such fluctuations can be investigated by studying the charged particle multiplicity distribution. In Fig. 4 we show E735 multiplicity data (Alexopolous *et al.*, 1998) for $p\bar{p}$ collisions at Tevatron. The data are compared to simulations done with a modified version of SIBYLL, using the amplitude (3). It is obvious from this comparison that the multiplicity distribution is another very important constraint complementary to both fully inclusive distributions and total and elastic cross sections. The peak at low multiplicities is due to peripheral collisions with mainly soft interactions. The high-multiplicity tail is entirely determined by the hard part of the eikonal function. The different slopes of the distributions at low and high multiplicities reflect the different density profiles used for soft and hard interactions. In addition Fig. 4 shows that a simple two-component structure of the eikonal with two independent profile functions is not an adequate approximation at very high energy. A successful model has to incorporate a smooth transition between soft and hard interactions in the transverse momentum distribution as well as in the impact parameter density of the partons.

4 Discussion

If QCD factorization is realized then the very basic relation (2) between inclusive and exclusive cross sections imposes severe constraints on the structure of these models.

It has been shown that a transverse momentum cutoff of the order of 2 GeV, as used in SIBYLL 1.7 and QGSJET, is not compatible with Tevatron data and the low-x parton densities found at HERA. In particular this means that the "minijet model", in its original formulation (Gaisser & Halzen, 1985), is ruled out. It seems to be impossible to construct a model on the basis of an energy-independent $p_\perp$-cutoff and without substantial low-x parton shadowing corrections, in which the rise of the total cross section is entirely due to the increase of the minijet contribution.

A precursor to a reliable extrapolation to ultra-high energy is the understanding of the Tevatron data using modern parton densities. Models such as SIBYLL 1.7 and QGSJET describe $p\bar{p}$ collider data rather well although it is now clear that they do not implement the correct low-x extrapolation of parton densities. Developing a modern model with up-to-date parton densities cannot be done without changing the structure of these models. It will only be possible if features of low-x shadowing or saturation are taken into account. First attempts in this direction are the introduction of an energy-dependent transverse momentum cutoff (DPMJET (Bopp *et al.*, 1994), SIBYLL 2.1 (Engel *et al.*, 1999a)). However, the energy-dependent transverse momentum does not account for the different possible shadowing or saturation scenarios in nuclei since it is by construction the same in hadron-hadron, hadron-nucleus and nucleus-nucleus collisions.

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