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Hadronic interactions, precocious unification, and air showers at Auger energies

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Abstract. At Auger energies only model predictions enable us to extract primary cosmic ray features. The simulation of the shower evolution depends sensitively on the first few interactions, necessarily related to the quality of our understanding of high energy hadronic collisions. Distortions of the standard “soft semi-hard” scenario include novel large compact dimensions and a string or quantum gravity scale not far above the electroweak scale. Naïvely, the additional degrees of freedom yield unification of all forces in the TeV range. In this article we study the influence of such precocious unification during atmospheric cascade developments by analyzing the most relevant observables in proton induced showers.

1 Introduction

Very recently, it has become evident that a promising route towards reconciling the apparent mismatch of the fundamental scales of particle physics and gravity is to modify the short distance behavior of gravity at scales much larger than the Planck length. Such modification can be most simply achieved by introducing extra dimensions (generally thought to be curled-up) in the sub-millimiter range (Arkani-Hamed, Dimopoulos and Dvali, 1998). Within this framework the fundamental scale of gravity M_* can be lowered all the way to $\mathcal{O}$ (TeV), and the observed Planck scale turns out to be just an effective scale valid for energies below the mass of Kaluza–Klein (KK) excitations. Clearly, while the gravitational force has not been directly measured far below the millimeter range, Standard Model (SM) interactions have been investigated well below this scale. Therefore, if large extra dimensions really exist, one needs some mechanism to prevent SM particles from feeling those extra dimensions. Remarkably, there are several possibilities to confine SM fields (and even gravity) to a 4 dimensional subspace (referred to as a brane-world) within the $(4+n)$ dimensional spacetime (Dvali and M. Shifman, 1997; Randall and Sundrum, 1999).

The extremely high center-of-mass (c.m.) energies attained in cosmic ray collisions at the top of the atmosphere are well above those necessary to excite the hypothetical KK modes which would reflect a change in spacetime dimensionality. Therefore, a natural question to ask is wether KK excitations could have a direct influence in the development of extensive air showers. In this communication we report on proton-induced showers.

2 Fifth dimension calling: do you accept the charges?

To illustrate the effect of extra dimensional gravity, we will estimate the effects of exchanging a tower of KK gravitons between the hadrons. As usual, the parton evolution of interacting hadrons a and b must be separated into: (i) the non-perturbative soft cascades, characterized by a small momentum transfer $q_t < q_0 \approx 2$ GeV and described by soft Pomeron exchange, (ii) the hard cascades, $q_t > q_0$, that should be described perturbatively (Kalmykov, Ostapchenko and Pavlov, 1997). To assess the contribution of KK graviton exchange, we omit interference effects and take for the cross section

$$\sigma_{\rm tot} = \sigma^{\rm KK} + \sigma^{4-\rm dim} \tag{1}$$

where $\sigma^{\rm KK}$ denotes the contribution from the virtual graviton exchange, and

$$\sigma_{ab}^{4-\rm dim} = \frac{1}{C_{ab}} \int d^2b \left\{1 - e^{-C_{ab}\,[\chi_{ab}^{\rm soft}(s,b)+\chi_{ab}^{\rm hard}(s,b)]}\right\}. \tag{2}$$

Here, $\chi_{ab}^{\rm soft}(s,b)$ stands for the soft eikonal defined by (Ter-Martirosyan, 1973),

$$\chi_{ab}^{\rm soft}(s,b) = \frac{\gamma_a\,\gamma_b}{R_{ab}^2}\,\exp\left(\Delta y - \frac{b^2}{4\,R_{ab}^2}\right), \tag{3}$$

where b is the impact parameter, $y = \ln s$, $\Delta = \alpha_P(0) - 1$, and $R_{ab}^2 = R_a^2 + R_b^2 + \alpha'_P(0) y$. The parameters of the

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Pomeron trajectory (Δ and $\alpha'_P(0)$) as well as those describing the Pomeron-hadron vertices (γ and R^2) are set to their values in QGSJET in the air shower simulation (Kalmykov, Ostapchenko and Pavlov, 1997). The semi-hard interaction is treated as the soft Pomeron emission (soft pre-evolution) followed by the hard interaction of partons

$$\begin{aligned}\chi_{ab}^{\rm hard}(s,b) &= \frac{1}{2} r^2 \int dy_1 \int dy_2 \; \chi_{ab}^{\rm soft}(e^{y_a+y_b}, b) \\ &\times \; \sigma_{\rm hard}(e^{y-y_a-y_b}, q_0), \qquad (4)\end{aligned}$$

where $y_{1(2)}$ are the rapidities of the Pomeron end, $\sigma_{\rm hard}$ is the parton interaction cross section, r^2 is an adjustable parameter associated with parton density and C_{ab} is the shower enhancement coefficient (Kaidalov, 1982). The latter is also fixed to the value of QGSJET in the simulations.

A complete theory of massive KK graviton modes is not yet available, making it impossible to know the exact cross section at asymptotic energies. A simple Born approximation to the elastic cross section leads, without modification, to $\sigma^{\rm KK} \sim s^2$ (Jain et al. (a), 2000). Unmodified, this behavior by itself eventually violates unitarity. This may be seen either by examining the partial waves of this amplitude, or by noting the high energy Regge behavior of an amplitude with exchange of the graviton spin-2 Regge pole: with intercept $\alpha(0) = 2$, the elastic cross section

$$\frac{d\sigma}{dt} \sim \frac{|A_R(s,t)|^2}{s^2} \sim s^{2\alpha(0)-2} \sim s^2, \qquad (5)$$

whereas the total cross section

$$\sigma^{\rm KK} \sim \frac{{\rm Im}[A_R(0)]}{s} \sim s^{\alpha(0)-1} \sim s, \qquad (6)$$

so that eventually $\sigma_{\rm el}^{\rm KK} > \sigma^{\rm KK}$. Eikonal unitarization schemes modify these behaviors: in the case of the tree amplitudes (Nussinov and Shrock, 1999,2001) the resulting (unitarized) cross section $\sigma^{\rm KK} \sim s$, whereas for the single Regge pole exchange amplitude, (Kachelriess and Plumacher, 2000) $\sigma^{\rm KK} \sim \ln^2(s/s_0)$. However, the Regge picture of graviton exchange is not yet entirely established: both the (apparently) increasing dominance assumed by successive Regge cuts due to multiple Regge pole exchange (Muzinich and Soldate, 1988), as well as the presence of the zero mass graviton can introduce considerable uncertainty in the eventual energy behavior of the cross section. Hereafter, we work within the unitarization framework and adopt as our cross section (Tyler, Olinto, Sigl, 2001; Anchordoqui et al. (a), 2001)

$$\sigma^{\rm KK} \approx \frac{4\pi s}{M_*^4} \approx 10^{-28} \left(\frac{M_*}{\rm TeV}\right)^{-4} \left(\frac{E}{10^{19}\,{\rm eV}}\right) {\rm cm}^2. \qquad (7)$$

The experimental information obtained at ground level is only indirectly connected to the first few generations of hadrons. Consequently, the study of the influence of KK-modes on hadronic interactions with c.m. energies $s^{1/2} > 100$ TeV, requires correctly simulating the intrinsic fluctuations in the air showers.

Let us first discuss in a very general way the possible effects introduced by virtual graviton exchange. The survival probability N at atmospheric depth X of a particle a with mean free path

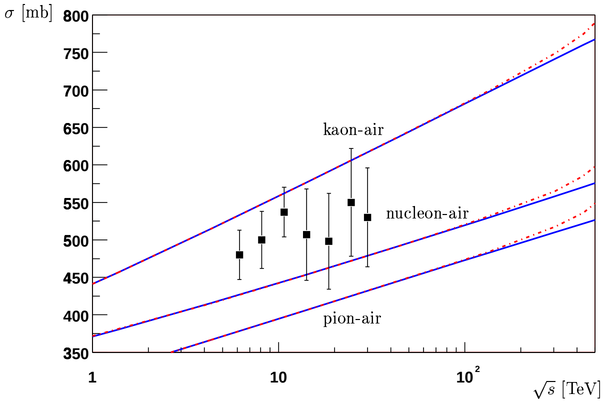

Fig. 1. Inelastic cross sections as a function of the c.m. energy. The solid line stands for the usual 4-dimensional cross section of QGSJET, whereas the dashed line represents corrections coming from the virtual graviton exchange. We also show in the figure experimental points of the inelastic p-air cross section as observed by different cosmic ray experiments (Baltrusaitis et al., 1984; Honda et al., 1993).

$$\lambda_a = \frac{m_{\rm air}}{\sigma_{a-{\rm air}}}, \qquad (8)$$

is given by

$$N(X) = e^{-X/\lambda_a}, \qquad (9)$$

where $m_{\rm air}$ is the mass of an average atom of air, and the cross sections $\sigma_{a-{\rm air}}$ inferred from Eq. (7) are shown in Fig. 1. It is straightforward to see that the total thickness of the atmosphere corresponds to more than 20 hadronic interaction lengths, depending on the primary zenith angle. The key feature in the evolution of the shower is the branching between decay and interaction of secondary hadrons along their path in the atmosphere. The latter strongly depends both on particle energy and target density.

Because of the low air density at the top of the atmosphere the point of the first interaction fluctuates considerably from shower to shower. However, KK-graviton exchange significantly reduces the nucleon attenuation length, e.g., at 3×10^{20} eV, $\lambda_p^4 \approx 41$ g/cm^2, whereas $\lambda_p^{(4+n)} \approx 38$ g/cm^2. Moreover, tiny deviations on the mean free path of non-leading secondaries yield a small change in the shower interaction length. Namely, the survival probability of a secondary pion (say $E = 5\times10^{19}$ eV) at $X = 40$ g/cm^2 is reduced from 43% to 41%, and that of a kaon with the same energy from 30% to 29%. Therefore, one can – perhaps naïvely – state that phenomenological models considering the virtual exchange of graviton towers would trigger, on average, earlier shower developments than a naked “soft semi-hard” scenario.

Test simulations runs of giant air shower evolution have been performed, choosing typical parameters for the experimental situation at the Fly’s Eye (Baltrusaitis et al., Nucl.

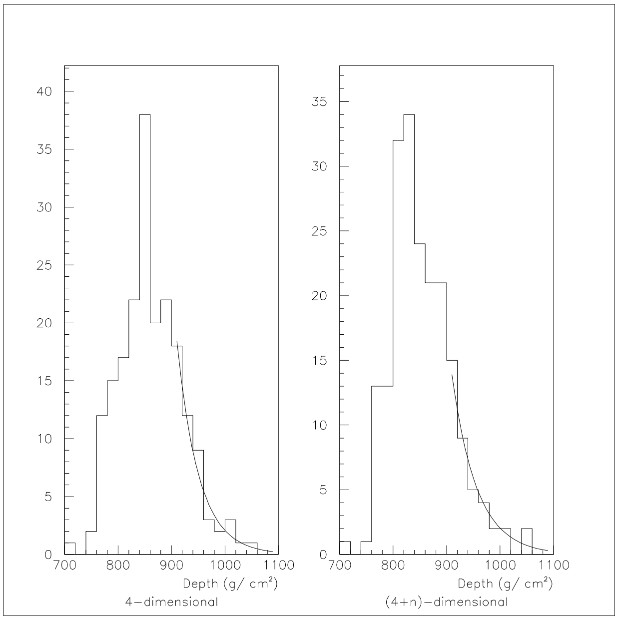

Fig. 2. Distributions of X_{max}.

1985) and Auger (Zavrtanik, 2000) experiments. The algorithms of AIRES (version 2.1.1) (Sciutto, 1999) were slightly modified so as to track the particles in the atmosphere via the standard 8 parameter function,

$$\lambda_a = P_1 \, \frac{1 + P_2 \; u + P_3 \; u^2 + P_4 \; u^3}{1 + P_5 \; u + P_6 \; u^2 + P_7 \; u^3 + P_8 \; u^4} \; \mathrm{g\,cm}^{-2}, \quad (10)$$

where $u = \ln E$ [GeV] and the coefficients P_i can be found in (Anchordoqui et al. (b), 2001). The hadronization algorithm that translates the parton strings produced during the scattering process into ordinary particles, remains the same.

In the simulation, several sets of protons with $E = 3 \times 10^{20}$ eV were injected at 100 km above sea level (a.s.l.). The sample was uniformly spread in the interval of 0° to 50° zenith angle at the top of the atmosphere. All shower particles with energies above the following thresholds were tracked: 750 keV for gammas, 900 keV for electrons and positrons, 10 MeV for muons, 60 MeV for mesons and 120 MeV for nucleons. The results of these simulations were processed with the help of the AIRES analysis package.

The atmospheric depth X_{max} at which the shower reaches its maximum number of secondary particles is the standard observable to describe the speed of the shower development. The charged multiplicity, essentially electrons and positrons, is used to determine the number of charged particles and the location of the shower maximum by means of 4-parameter fits to the Gaisser-Hillas (1977) function

$$\begin{aligned} N^{\mathrm{ch}}(X) \quad &= \quad N^{\mathrm{ch}}_{\mathrm{max}} \left(\frac{X - X_0}{X_{\mathrm{max}} - X_0} \right)^{[(X_{\mathrm{max}} - X_0)/\lambda]} \\ &\times \quad \exp \left\{ \frac{X_{\mathrm{max}} - X}{\lambda} \right\}, \quad X \geq X_0, \qquad (11) \end{aligned}$$

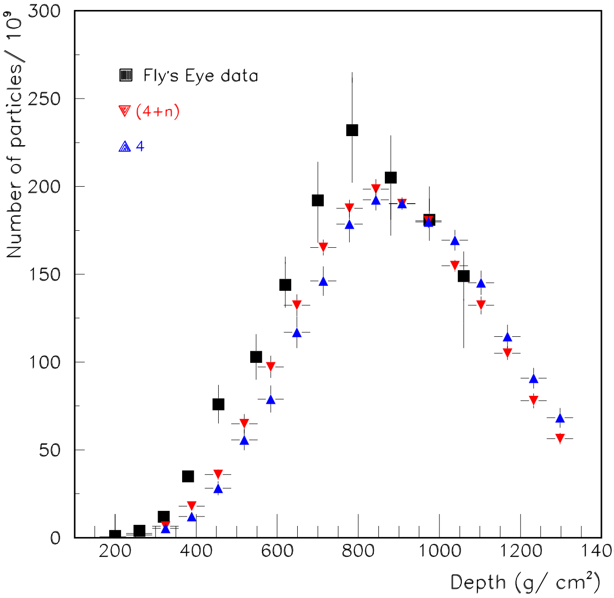

Fig. 3. Atmospheric cascade development of proton showers ($E = 3 \times 10^{20}$ eV), superimposed over the Fly's Eye data. The error bars in the simulated curves indicate RMS fluctuations of the means.

where X_{max}, $N^{\mathrm{ch}}_{\mathrm{max}}$, λ, and X_0 are the free parameters to be adjusted.[1] Shown in Fig. 2 are the resultant X_{max} distributions of proton showers with 3×10^{20} eV and primary zenith angle 43.9°.[2] The tails ($X_{\mathrm{max}} > 900$ g/cm^2) of these distributions were fitted with exponentials ($\alpha \, e^{-\beta \, X_{\mathrm{max}}}$), floating both the normalisation α and the exponent in the fit. The resulting parameters are: $\beta = 2.6 \pm 0.1 \times 10^{-2}$ cm^2/g for the 4-dimensional case, and $\beta = 2.9 \pm 0.1 \times 10^{-2}$ cm^2/g for the $(4 + n)$-dimensional case. A statistically significant difference between the two approaches arises in the tail of the distribution. This is because the depth of such penetrating showers increasingly reflects that of the first interaction (Gaisser et al., 1982; Ellsworth et al., 1982). Results of the fits to the X_{max} distributions generated by applying progressively less restricted data cuts (distances near the peak) lead to exponential slopes that within the error are consistent with one another.

In Fig. 3 we show the longitudinal developments of proton showers superimposed over the experimental data of the world's highest energy cosmic ray shower observed to date (Bird et al., 1995). We selected from our shower sample those with a primary zenith angle of 43.9°, setting the observation level at 850 m a.s.l and with geomagnetic field specific for the Fly's Eye site. Although at the same total energy a shower that takes into account the virtual graviton exchange develops faster than that modelled with unmodified QGSJET, as expected from our previous analysis, the differences in the

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[1] The parameter λ is externally fixed at 70 g/cm^2. This is the usual procedure in AIRES.

[2] This is the primary zenith angle of the Fly's Eye event (Bird et al., 1995).

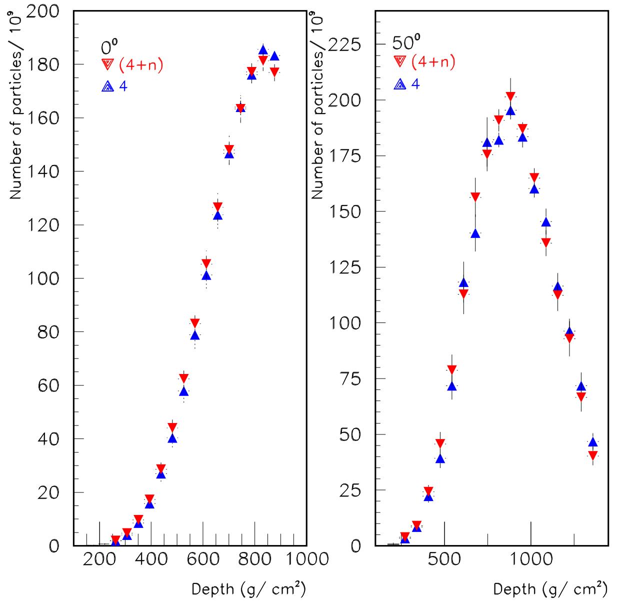

Fig. 4. Atmospheric cascade developments of proton showers for extreme primary zenith angles (0° and 50°) and $E = 3 \times 10^{20}$ eV. The error bars indicate RMS fluctuations of the means.

position of $X_{\rm max}$ fall within the errors. However, there are visible deviations in the evolution of the charged multiplicity. To estimate the amount of departure from the standard 4-dimensional scenario we analyzed the data by means of a χ^2 test. We assume that the set of measured values by Fly's Eye are uncorrelated (any depth measurement is independent of any other), and make use of the quantity

$$\chi^2 \equiv \sum_{j=1}^{q} \frac{|x_j - \alpha_j|^2}{\sigma^2_{x_j}}, \qquad (12)$$

where q is the total number of points in the analysis, σ_{x_j} is the error on the x_jth coordinate, x_j is the measured value of the coordinate, and α_j the (hypothetical) true value of the coordinate. The obtained results are $\chi^2_4/{\rm DOF} = 324.89/12$, $\chi^2_{(4+n)}/{\rm DOF} = 200.52/12$. If in the future the situation should arise that one can be confident that the hadronic interactions are correctly modeled, then it will be necessary to carry out a more sophisticated statistical analysis which, for example, accounts for the non-Gaussian distributions.

All in all, *KK-graviton exchange offers a viable mechanism to reduce by around 6% the mean free path of ultra high energy ($E > 5 \times 10^{19}$ eV) hadrons in the atmosphere*.

3 Outlook

Theories with large compact dimensions and TeV-scale quantum gravity represent a radical departure from previous fundamental particle physics. If these scenarii have some truth, the scattering phenomenology above collider energies would be quite distinct from SM expectations. In particular, the exchange of KK towers of gravitons leads to a modification of SM hadronic cross sections at $s^{1/2} > 100$ TeV. Extremely high energy cosmic rays that impinge on stationary nucleons at the top of the atmosphere start chain reactions where the c.m. energy can be as high as 500 TeV. It is therefore instructive to explore KK exchange sensitivity within the entire average profile of the air shower. In this paper we have contributed a few results to this topic. We have shown that the exchange of KK gravitons could affect the rate of development of atmospheric cascades initiated by protons. For primary energies above 3×10^{20} eV, the effects are statistically significant (they become dominant if the s^2 behavior is retained for $\sigma_{\rm KK}$ (Jain et al. (b), 2000)) and can thus be observed by fluorescence detectors. The details of our analysis should be treated with some caution since they may be sensitive to the hadronic interaction model used. The overall conclusion, however, should remain the same.

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