Proceedings of ICRC 2001: 460 © Copernicus Gesellschaft 2001

ICRC 2001

Efficient simulation of ultra-high energy air showers

J. Alvarez-Muñiz[1], R. Engel[1], P. Lipari[2], J. A. Ortiz[1,3], and T. Stanev[1]

[1]Bartol Research Institute, University of Delaware, Newark, DE 19716, U.S.A.
[2]INFN and Dipt. di Fisica,Univ. di Roma "La Sapienza", P. A. Moro 2, I-00185 Rome, ITALY
[3]Instituto de Física "Gleb Wataghin", Universidade Estadual de Campinas, 13083-970 Campinas-SP, BRAZIL

Abstract. We have developed a fast and efficient one dimensional method to calculate the development of extensive air showers. This method allows us to simulate ultra-high energy showers with very high statistics. It is based on precalculated pion induced showers and a bootstrap technique, accounting for fluctuations in the electromagnetic and muonic components. As a first application of this code we consider in detail the longitudinal shower development and the number of muons at observation level as predicted by different hadronic interaction models. The relation between the various assumptions in modeling hadron production, in particular its extrapolation to ultra-high energy, and extensive air shower observables is discussed.

1 Introduction

One of the main sources of uncertainty in understanding the behavior of extensive air showers (EAS) arises from the model of high energy hadronic interactions. Different hadronic models predict in fact different, and at times contradicting distributions for the values of important observable quantities (such as $X_{\max}$, the position of the maximum of the longitudinal development, or $S_{\max}$, the size at the maximum) for a set of showers generated by primary particles of a fixed type, energy and zenith angle. The prediction of the average values, but also of the fluctuations and the correlations between observable quantities is essential for the interpretation of the experimental data.

Once a model of the hadronic interactions has been formulated, it remains a technical but nonetheless very difficult problem: the calculation (according to the chosen model) of the gigantic showers that corresponds to the highest energy cosmic rays. The widely used solution is the simulation of EAS employing 'thinning' techniques as suggested by Hillas (Hillas, 1981, 1997), which provides an excellent way to estimate detectable signals and to compute the average value of the observables. To keep the complexity of the problem at an affordable level, interactions, propagation and decay are simulated only for a representative number of EAS particles. The ignored particles are taken into account by increasing the statistical weight of the former ones correspondingly. There may be, however, some problems with the representation of the shower fluctuations even at very high (10^{-5}, 10^{-6}) levels of thinning.

Another method for calculating EAS characteristics and their dependence on hadronic interactions is that of solving coupled cascade equations (see (Gaisser, 1990) and Refs. therein). Being based on parametrizations of mean particle yields and decay distributions, this method can only be used for computing average values of observables.

In this paper we explore a different way of calculating the air shower development - an efficient one-dimensional hybrid calculation of the electromagnetic and muon component of EAS. It involves a full Monte Carlo simulation of the high energy part of the cascade (above a certain energy cutoff) in combination with a library of presimulated pion-initiated showers (Gaisser *et al.*, 1997).

2 A hybrid simulation technique

The fundamental idea at the basis of our approach is that fluctuations in the properties of a shower are determined by fluctuations in the early part of its development. This can be understood noting that the fluctuations in the development of low energy subshowers have little impact because of the very large number of such subshowers. The distribution of shower properties can then be calculated studying in detail the initial part of the shower, tracing exactly all particles with $E > f\,E_0$ (where E_0 is the primary energy and f is an appropriate fraction of it: in the following we will use $f = 0.01$) and then superimposing sub–showers, corresponding to all the sub–cutoff particles, described according to appropriate parametrizations that give the correct average behavior also

Correspondence to: J. Alvarez-Muñiz
(alvarez@bartol.udel.edu)

including some appropriate fluctuations. This method allows naturally to "bootstrap" itself to higher and higher energy, because the results for showers of energy E_0 can then be used to calculate the development of showers of higher energy $E_1 > E_0$, and so on recursively. Although we do not need to account for the sub-PeV pion shower fluctuations for the calculation of 100 EeV showers, we parametrize the fluctuations in the whole energy range to be able to calculate correctly showers that could be simulated with direct Monte Carlo, and check and normalize the bootstrap procedure.

The calculation we present has been done for a single incident zenith angle of $45^{\rm o}$. Nucleon initiated subthreshold showers have not been presimulated and hence nucleons are followed explicitly in the Monte Carlo down to the particle production threshold. Electromagnetic showers are modeled with a full screening electromagnetic Monte Carlo in combination with a modified Greisen parametrization. Photoproduction of hadrons is included in the Monte Carlo but not the Landau-Pomeranchuk-Migdal effect. When building the library we have accounted for fluctuations in the main observables describing the behavior of the longitudinal development of the subthreshold showers namely: $S_{\rm max}$, the maximum number of $e^- + e^+$ in the shower; $X_{\rm max}$, the depth at which the maximum of the shower occurs and $N_\mu^{\rm obs}$, the number of muons at observation level with energies above 0.3, 1, 3, 10 and 30 GeV.

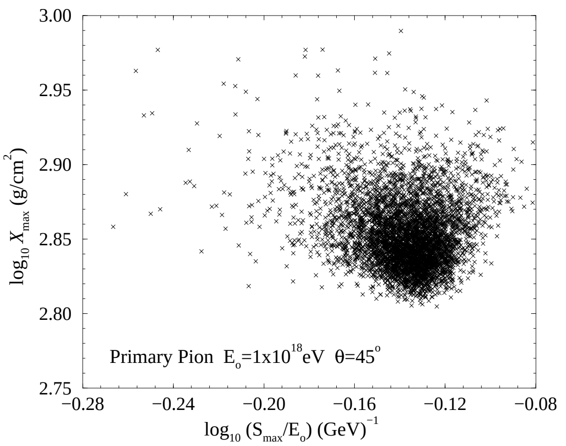

Fig. 1. The correlation between $\log_{10} X_{\rm max}$ and $\log_{10} S_{\rm max}$ for the generated library of pion induced showers of $E_\circ = 10^{18}$ eV initiated at $X_\circ = 5$ g/cm^2.

The first step in the calculation of hadron initiated showers at high energies is to create a library of pion induced showers. We have simulated pion interactions at atmospheric depths X_0 of 5, 50, 100, 200, 500 and 800 $\rm g/cm^2$. There is no noticeable difference between the longitudinal development of the electromagnetic and muonic longitudinal profiles of showers initiated deeper than 500 $\rm g/cm^2$, however we have simulated showers initiated at 800 $\rm g/cm^2$ to properly account for the number of muons at sea level, since we have chosen not to parametrize their longitudinal profile which is dependent on the muon energy threshold. 10,000 (5,000) pion showers were fully simulated for each interaction depth at energies E_0 from 10 GeV to 300 GeV (from 1 TeV to 100 TeV), 2 energies per decade. For each energy and depth we obtain the distributions of $X_{\rm max}$ and $S_{\rm max}$, the correlation between them, the distributions of $N_\mu^{\rm obs}$ with energies above the thresholds indicated above, and the slope of the muon longitudinal profile between 1000 $\rm g/cm^2$ and sea level. We parametrize the subthreshold meson induced showers using a slightly modified version of the well-known Gaisser-Hillas function that gives the number of $e^- + e^+$ at atmospheric depth X, Gaisser (1990):

$$S_{\rm GH}(X) = S_{\rm max} \left(\frac{X - X_0}{X_{\rm max} - X_0}\right)^{(X - X_{\rm max})/\lambda(X)} \times \exp\left[-\frac{(X - X_{\rm max})}{\lambda(X)}\right] \quad (1)$$

where $\lambda(X) = \lambda_0 + bX + cX^2$ and λ_0, b and c are taken as parameters. X_0 is the depth at which the first pion interaction occurs. Instead of using the average values of the different observables to generate subthreshold meson showers of a certain energy, we sample their values from their corresponding presimulated distributions. This procedure accounts for the fluctuations in the low energy subshowers.

Fig. 1 shows the correlation between $X_{\rm max}$ and $S_{\rm max}$ for 10^9 GeV pion showers initiated at atmospheric depth of $X_\circ$=5 g/cm^2. The correct representation of this correlation is crucial for the successful shower modeling.

We extend the library from 100 TeV to pion energies as high as 3 EeV using a recursive (bootstrap) technique with the following energy thresholds: $E_{\rm thr}^{\rm mes} = E_0/100$ for the meson initiated subthreshold showers; $E_{\rm thr}^{\rm em} = E_0/100$ for the electromagnetic subthreshold showers and $E_{\rm thr}^{\rm N} = m_{\rm N} \sim$ 1 GeV for protons and neutrons. We have created libraries for the interaction models SIBYLL, versions 1.7 (Fletcher *et al.*, 1994) and 2.1 (Engel *et al.*, 1999, 2001), and QGSJET (Kalmykov *et al.*, 1997).

To ensure the consistency of our simulation approach, we have compared full simulations of pion showers to simulations within the hybrid approach for the same initial energy and depth using several energy thresholds. We find a good agreement between them in both the average values of the different observables and their fluctuations with differences which are typically less than 5%. We believe the discrepancies come mainly from the representation of the intrinsic fluctuations in the shower development and the interpolation in energy and depth that we have to do. These comparisons also show a remarkable stability of the code under changes of the energy threshold, from which we conclude that the primary to threshold energy ratios we have used are sufficient to achieve a good description of the nucleon initiated showers.

3 Results for proton-initiated showers

The $X_{\rm max}$ quantity is measured by fluorescence and Cherenkov light detectors in several experiments. It is a crucial shower parameter which statistically reveals the type of primary particle and which is of significant importance for the energy determination by air shower detectors. Fig. 2 shows the average $X_{\max}$ of proton showers calculated by our method (lines) and the well tested CORSIKA code (symbols, Heck *et al.* (1998)) as predicted by the QGSJET and SIBYLL interaction models. The depths are measured along the shower trajectory. The results from CORSIKA are from Heck (2001) and Pryke (2001). At each energy we simulated 5,000 proton showers, while all points generated by CORSIKA represent 500 showers using the thinning procedure. The energy dependence of

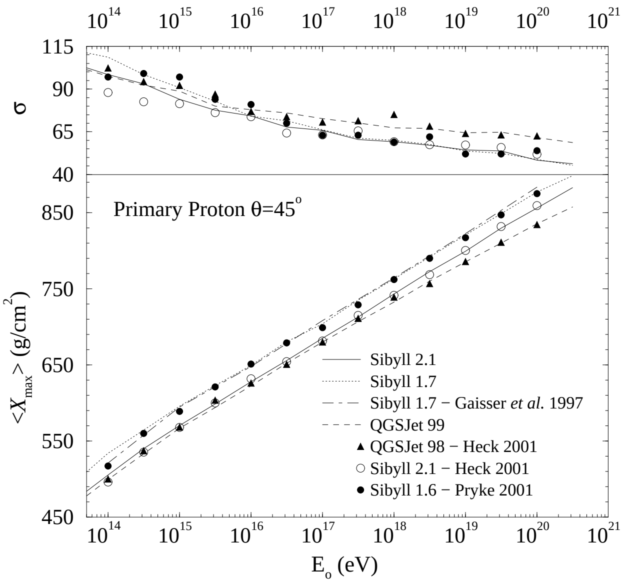

Fig. 2. Average depth of maximum $\langle X_{\max}\rangle$ of proton showers as a function of primary energy generated by our one dimensional method and CORSIKA code using different hadronic interaction models. The top panel shows the corresponding sigma values.

$X_{\max}$ reflects the features of the hadronic interaction model. The new SIBYLL 2.1 model predicts a decrease of $X_{\max}$ by ~ 25 g/cm^2 compared to the older 1.7 version. The QGSJET model produces still lower values for $X_{\max}$ than SIBYLL. The difference is evident, although it is not large, in comparison with the 1.6 and 1.7 versions of SIBYLL, being of the order of 29 g/cm^2 at 1 PeV and 36 g/cm^2 above 10 EeV. This is due to the much higher multiplicity of charged and neutral hadrons, and the lower elasticity (fractional energy of the most energetic secondary particle) produced by the QGSJET model. These features are responsible for the accelerated shower development in the atmosphere. The top panel of fig. 2 shows the standard deviation values of $X_{\max}$, varying from $\sim$ 19% at 10^{14}eV to $\sim$ 5% at $3{\times}10^{20}$eV. As expected the σ values decrease with increasing shower energy because of the decreasing proton interaction length and the increase of secondary particles produced in the first interactions. The values of $X_{\max}$ and σ calculated by both codes for the same models are in good agreement.

Fig. 3 shows the depth of maximum distribution for 5,000 primary proton showers at 1 PeV and 1 EeV calculated with our method using SIBYLL and QGSJET. At 1 PeV SIBYLL 2.1 and QGSJET are in good agreement: $\langle X^{\rm Sib21}_{\max}\rangle$=570, and $\langle X^{\rm QGS}_{\max}\rangle$=567 with SIBYLL 1.7 having $\langle X^{\rm Sib17}_{\max}\rangle = 596$, (all in g/cm^2). At 1 EeV, QGSJET shows a slight shift towards earlier shower development mainly when compared with the SIBYLL 1.7 distribution ($\langle X^{\rm Sib17}_{\max}\rangle = 763$, $\langle X^{\rm Sib21}_{\max}\rangle = 743$, and $\langle X^{\rm QGS}_{\max}\rangle = 732$, all in g/cm^2).

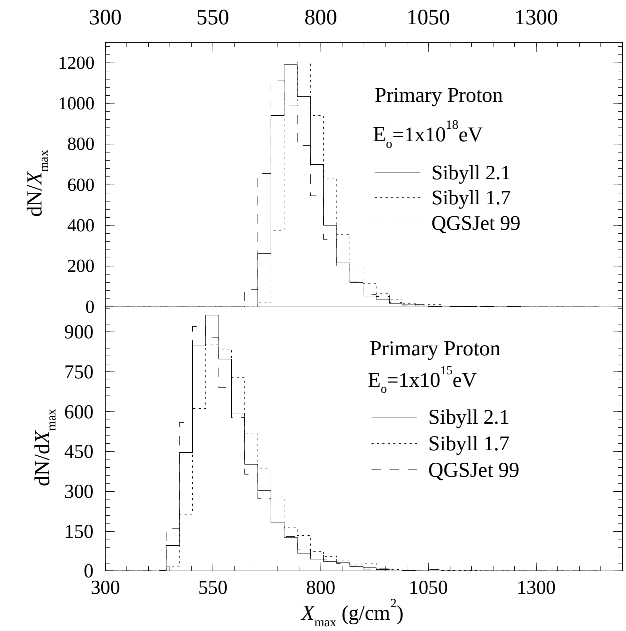

Fig. 3. Depth of maximum distribution $dN/dX_{\max}$. Results are for 5,000 primary proton showers of energies 1×10^{15}eV and 1×10^{18}eV calculated by our method using SIBYLL (1.7 and 2.1 versions) and QGSJET (1999 version) hadronic models.

Fig. 4 shows the distribution of the number of muons with energy greater than 3 GeV at sea level for the different hadronic models. Each histogram represents 5,000 showers generated with different meson energy threshold. The bottom panel of fig. 4 (a) shows the results predicted by SIBYLL 1.7; the middle panel (b) shows the results predicted by SIBYLL 2.1; and the top panel shows the results obtained using QGSJET99 model.

The hybrid simulation program is still not free of biases. In figure 4, the average number of muons at sea level, obtained when the meson energy threshold is 100 times smaller than the primary energy, is $\sim$ 4% bigger than the full simulated events for SIBYLL 1.7 (a), $\sim$ 7% for SIBYLL 2.1 (b), and $\sim$ 5% for QGSJET. Although this systematic shift is not very significant, it has been investigated and shown to be energy independent. The differences in fig. 4 come mostly from the interpolation procedure inherent to the hybrid approach.

Apart from these shifts, the fluctuations in the muon number in the air showers are fairly well represented. The width of each histogram in figure 4, with $E_{th} = E_\circ/100$ (full simulation), calculated with SIBYLL 1.7, SIBYLL 2.1, and QGSJET are, respectively, 91 (85), 94 (87), and 97 (93). One could see in fig. 4 that the average number of muons is higher, and its distribution wider, when calculated with QGSJET than in the distributions calculated with SIBYLL 1.7 and 2.1. This is a consequence of the higher multiplicity and multiplicity fluctuations in QGSJET. The difference in the averages between QGSJET and SIBYLL 2.1 is $\sim$ 7% while it becomes larger between QGSJET and SIBYLL 1.7, $\sim$ 17%.

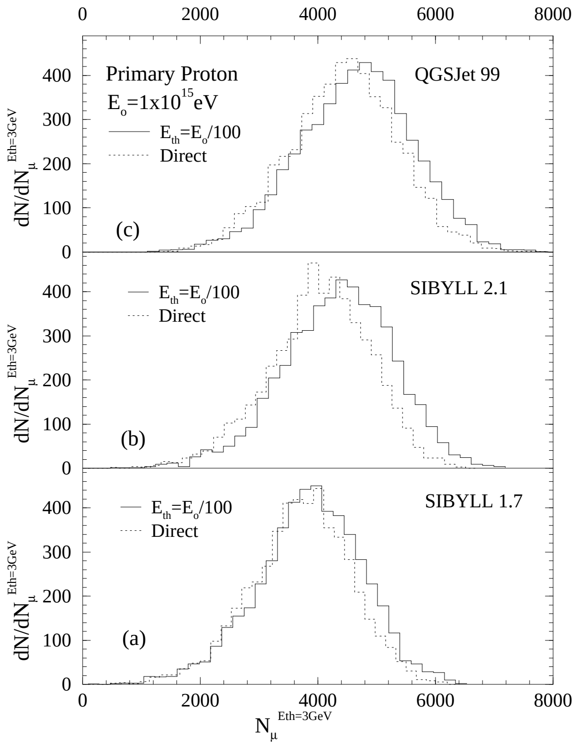

Fig. 4. Number of muons distribution dN/dN_μ. Results are shown for 5,000 primary proton showers of energy 1×10^{15} eV calculated with our method. The solid line represents showers with meson energy threshold 100 times below the total primary energy. The dotted line represents fully simulated showers.

Fig. 5 illustrates the difference in the average number of muons at sea level predicted by the three models. The values in fig. 5 have been normalized to the primary energy. The difference in the bottom panel between the QGSJET and the SIBYLL 2.1 is $\sim$ 6% at 10 PeV and $\sim$ 18% at 10 EeV while the difference between the QGSJET and the SIBYLL 1.7 is $\sim$ 20% at 10 PeV and $\sim$ 35% at 10 EeV.

4 Discussion

We discussed a hybrid shower simulation technique that allows the creation of big enough shower libraries to study the shower fluctuations predicted by different hadronic interaction models. We have demonstrated that in most cases this technique represents well the average shower parameters and their fluctuations. The current code introduces shifts of order 5% in the calculated number of muons, when compared to a direct simulation. Such shifts will be further investigated and

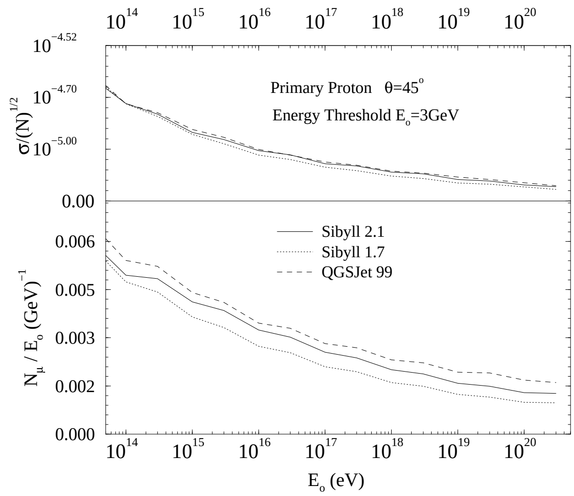

Fig. 5. Number of muons at sea level with energy above 3 GeV. The numbers have been normalized to primary energy. The top panel shows the statistical errors.

corrected in the near future.

We plan to build shower libraries that will lets us simulate air showers in a wide angular range for several hadronic interaction models. We will then be able to help the interpretation of the current and future experimental data on ultra high energy cosmic ray showers.

5 Acknowledgments

This research is supported in part by the US DOE contract DE-FG02 91ER 40626 and NASA grant NAG5-7009. Jeferson A. Ortiz would like to thank to CAPES “Bolsista da CAPES - Brasília/Brasil”. One of the authors (R.E.) thanks D. Heck for providing him with $X_{\max}$ tables from CORSIKA simulations.

References

Engel, R., Gaisser, T. K., Lipari, P. and Stanev, T., Proc. 26th ICRC **1**, 415, Salt Lake City, 1999.

Engel, R. *et al.*, these proceedings 2001.

Fletcher, R.S., Gaisser, T.K., Lipari, P. and Stanev, T., Phys. Rev. **D50**, 5710, 1994.

Gaisser, T.K., Cosmic Rays and Particle Physics, Cambridge University Press, 1990.

Gaisser, T.K., Lipari, P. and Stanev, T. Proc. 25th ICRC **6**, 281, Durban, 1997.

Heck, D., Knapp, J., Capdevielle, J.N., Schatz, G. and Thouw, T., report FZKA 6019, 1998.

Heck, D., astro-ph/0103073 and private communication, 2001.

Hillas, A.M., Proc. 17th ICRC **8**, 193, Paris, 1981.

Hillas, A.M., Nucl. Phys. B (Proc. Suppl.), 52B, 29, 1997.

Kalmykov, N.N., Ostapchenko, S.S. and Pavlov,A.I., Nucl. Phys. B (Proc. Suppl.), 52B, 17, 1997.

Pryke, C.L., Astropart. Phys., 14, 4, 2001.