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The hypergeometric formalism for the lateral distributions of charged particles in EAS

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Abstract. We propose a new form of function describing lateral distribution of charged particles in extensive air showers (EAS). We had performed simulations of shower development using CORSIKA v.5.62 code for primary proton and iron nuclei with energies 10^{11} GeV/nucleon with thinning factor 10^{-6}. We obtained mean distributions of several EAS components for distances from the core up to 10^4 m. We fitted the lateral distribution with the function which has one more free parameter allowing to achieve consistency with simulations. The function is exactly normalized in terms of the hypergeometric formalism.

1 Radial electron distribution from cascade theory

The structure functions f(x) in 3-dimensional cascade theory (where $x = r/r_M$, r being the distance to the core in meters), generally so normalized that $\int_0^\infty 2\pi x f(x) dx = 1$, are related to the electron density $\Delta_e(r)$ by $\Delta_e(r) = N_e f(x)/r_M^2$. The analytical parameterizations of numerical results from the solutions of diffusion equations or from Monte Carlo calculations are commonly classified following the earliest forms proposed:

$$f(x) = 0.45(1/x + 4)\exp(-4x^{2/3}) \quad (1)$$

$$= c(s)x^{s-2}(x+1)^{s-4.5} \quad (2)$$

$$= g(s)x^{s-a}(x+1)^{s-b}(1+d\cdot x)^{-c} \quad (3)$$

The former approximation (Eq. 1) was derived by Bethe from Molière's theory for small values of the argument x and for $s = 1$. This form was generalized by Nishimura and Kamata following the numerical values of their solutions of transport equations via Mellin's and Hankel's transformations in the complex plane and saddle point approximation to get the final real solutions. The synthesis, so-called NKG formula (Greisen, 1960), contained in Eq. 2 under a pair of power laws representing respectively the asymptotic tendencies (near and far from the shower core), with a simple normalization in terms of Euler Beta function,

$$c(s) = \frac{\Gamma(4.5-s)}{2\pi\Gamma(s)\Gamma(4.5-2s)} \quad (4)$$

became one of the most widely used radial distributions. The comparison to experimental results suggested however a more complex situation and some corrections such as the introduction of a new argument $x = r/kr_M$, k being a factor reducing Molière radius, or a local age parameter $s(r)$, troublesome for the normalization of the structure function (Capdevielle and Gawin, 1982; Nagano et al., 1984). In order to provide a better skewness than the transition between two power laws, we proposed later (Capdevielle and Procureur, 1983) the more complex relation of Eq. 3; such structure function, which is also a general form containing Eq. 2 for a particular set of parameters, has the advantage to be exactly normalized in terms of Gaussian Hypergeometric function.

At large distances from axis, as emphasized by the Particle Data Group (Particle Data Group, 1996), the description of the 3D-cascade transport by diffusion equations fails (small angle approximation in multiple Coulomb scattering, effect of single scattering, Landau approximation is no more valid) and the analytical descriptions can be derived only from Monte Carlo or semi Monte Carlo calculations in order to restore some useful scaling properties. For instance, the condition $|ln(E_\gamma/\varepsilon)| \gg |ln(r/r_m)|$, where E_γ is the primary photon energy and ε the energy threshold for electrons, is no more fulfilled for a large number of subcascades in giant EAS at very large distance from shower axis, circumstance where both Approximation B and Landau's Approximation are no more valid.

2 Empirical distributions for giant EAS

The empirical structure functions were inspired by the theoretical functions quoted in section 1; as fitted to scintillator

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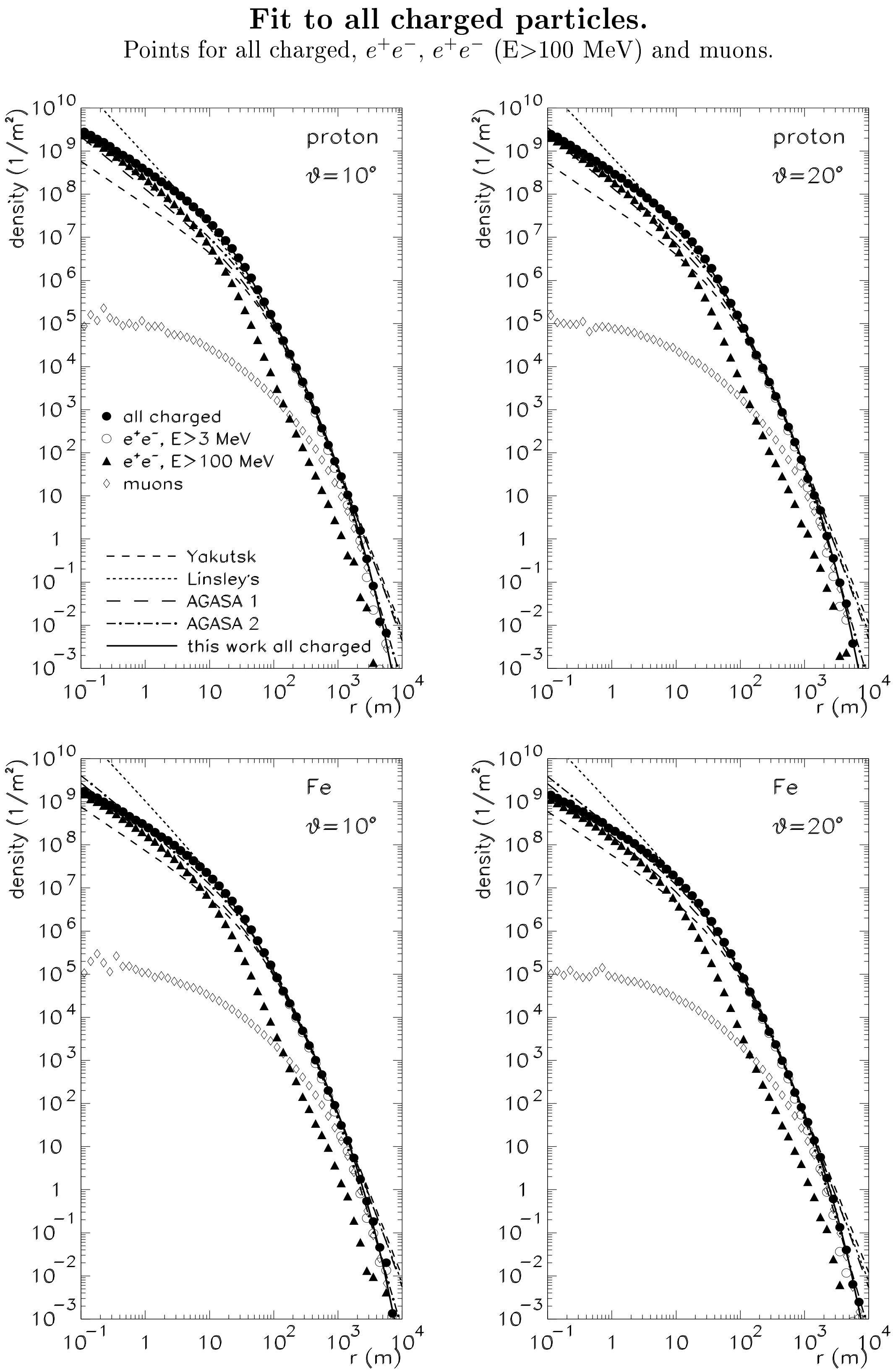

Fig. 1. Fits to all charged particles lateral distribution from simulations (average from 10 EAS). Primary particle energy 10^{11} GeV. Lines are normalized to $\varrho(600\text{ m})$. For solid line (this work) see formula 5 and parameters from the table 1.

Fit to e^+e^-.

Points for all charged, e^+e^-, e^+e^- (E>100 MeV) and muons.

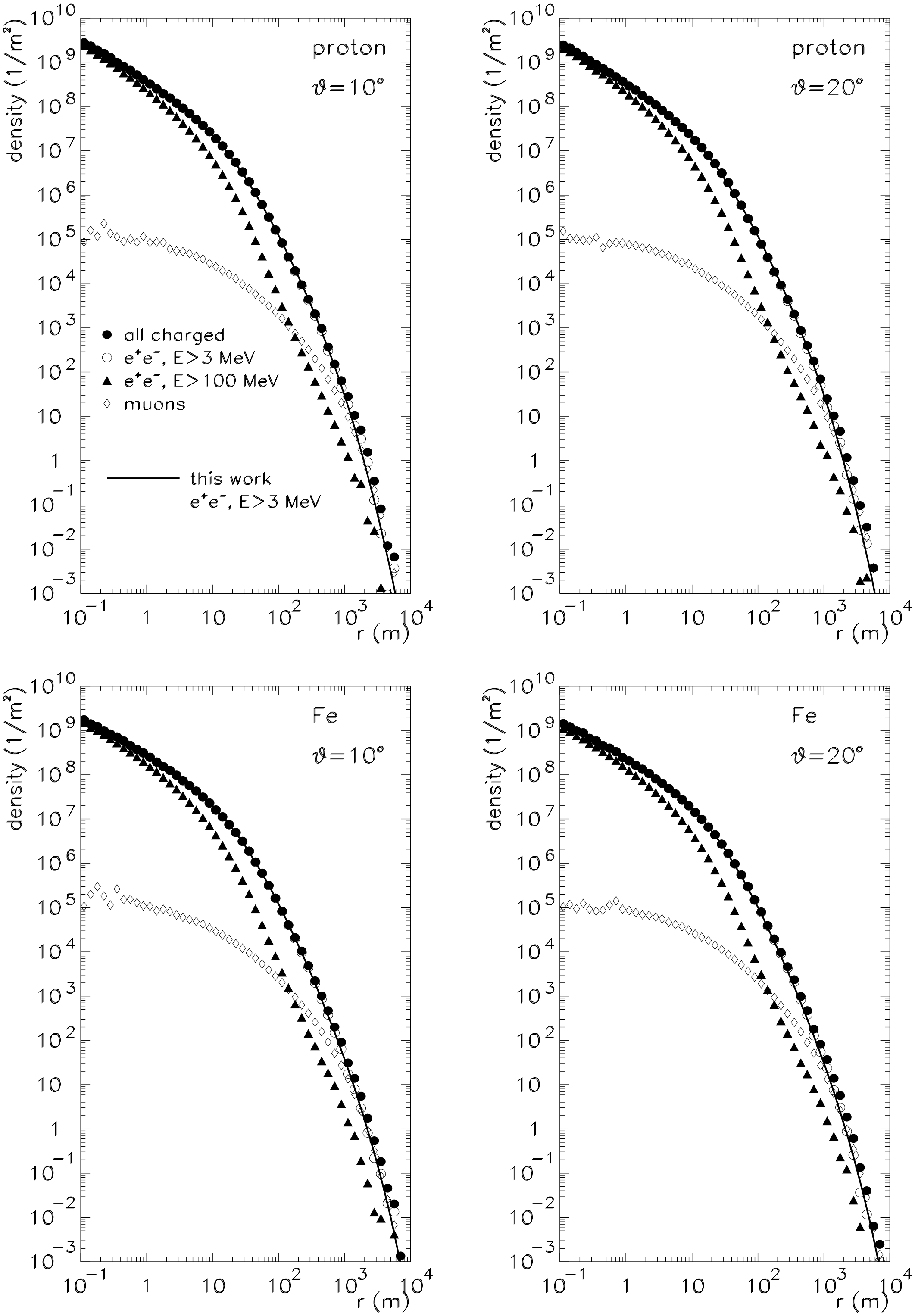

Fig. 2. Fits to electrons and positons (E $>$ 3 MeV) using formula 5 and parameters from the Table 2.

experiments, they deal generally with densities of charged particles $\varrho(r)$ (and not with pure electrons). We will refer to the following functions used for GAS: AGASA #1 (Nagano et al., 1992), AGASA #2 (Yoshida et al., 1995), Linsley's

Table 1. Best parameters to simulated e^+e^-+ muons (all charged) lateral distribution fit using JNC formula 5. Labels p10, p20, Fe10 and Fe20 in the first row refer to primary particle (proton or iron) and to the zenith angle (10° or 20°).

	p10	p20	Fe10	Fe20
$log_{10}N_e$	10.75	10.72	10.70	10.65
r_{M}	21.26	21.26	19.18	19.18
r_0	8785.	8785.	9536.	9536.
a	1.91	1.91	1.82	1.82
s	1.03	1.04	1.03	1.04
b	3.32	3.32	3.31	3.31
β	10.0	10.0	10.0	10.0

function (Linsley, 1980) and Yakutsk function for electrons (Efimov et al., 1988).

3 Hypergeometric formalism

In the case of GAS, we notice on Fig. 1, that in all cases the lateral muon densities become dominant for $r \geq 1.5$ km ($r \geq 200$ m for electrons with E $>$ 100 MeV) in the lateral distribution of charged particles; the lateral muon distribution flattens for showers initiated at higher altitude, just when the lateral electron distribution corresponds also to older profile. This circumstance, and the opposite situation for showers initiated deeper in the atmosphere, suggest the extension of the concept of lateral age parameter to the total lateral charged particles distribution of GAS, using the skewness of the profiles described by our gaussian hypergeometric distribution (Eq. 3); the parameters have been adjusted with MINUIT from our Monte Carlo simulation, giving the JNC functions:

$$\varrho(r) = N_e \cdot C \cdot x^{-\alpha} \cdot (1+x)^{(\alpha - \eta)} \cdot (1 + d \cdot x)^{-\beta} \quad (5)$$

where $x = \frac{r}{r_{\mathrm{M}}}$, $d = \frac{r_{\mathrm{M}}}{r_0}$, $s = 1.03$, $\alpha = a - s$, $\eta = b - s + \alpha$,

$$C = \frac{1}{2\pi \cdot r_{\mathrm{M}}^2} \cdot \frac{\Gamma(\beta + \eta - \alpha)}{\Gamma(2-\alpha) \cdot \Gamma(\beta + \eta - 2)} \cdot \frac{1}{F_{HG}}$$

where $F_{HG} = F_{HG}(\beta, 2-\alpha; \beta+\eta-\alpha; 1-d)$
with the conditions $2 - \alpha > 0$ and $\beta + \eta - 2 > 0$.

4 Adjustment to simulated lateral distributions

In order to appreciate the advantages of the hypergeometric gaussian approach compared to the classical Eulerian description, we have fitted for example the average radial distributions derived from groups of 10 showers simulated with CORSIKA for $E_0 = 10^{20}$eV, for zenith angles 10^0 and 20^0, respectively for protons and iron nuclei primaries. All those showers have been simulated with the Quark Gluon String Model (Kalmykov et al., 1997); the respective energy thresholds are here 3 MeV for electrons and photons and 300 MeV for muons.
This operation was performed for all charged particles (electrons + muons) (see Fig.1 and Table 1 for parameter values), and repeated separately for electrons only (E $>$ 3 MeV) using the same form of JNC function, but with different parameter values, listed in the table 2 (see Fig.2), and also for high energy electrons (E $>$ 100 MeV).

5 Conclusion

It appears that at distances lower than 200 m only the hypergeometric approach by reason of its skewness provides a correct adjustment to the lateral distribution function of charged particles, when systematically Linsley's function overestimates the densities and Yakutsk or AGASA functions underestimate the densities. In the consequence, as a majority of particles are contained in this area, the JNC function is the only one suitable to recover the size.

Table 2. Best parameters to e^+e^- (E $>$ 3 MeV) lateral distribution fit using JNC formula 5.

	p10	p20	Fe10	Fe20
$log_{10}N_e$	10.75	10.72	10.71	10.65
r_{M}	20.65	20.65	28.48	28.48
r_0	2698.	2698.	11423	11423
a	1.91	1.91	1.86	1.86
s	1.03	1.05	1.03	1.02
b	3.22	3.22	3.63	3.63
β	5.82	5.82	10.0	10.0

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