

## Estimation of shower parameters in wavefront sampling technique

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**Abstract.** Wavefront sampling experiments record arrival times of Čerenkov photons with high precision at various locations in Čerenkov pool using a distributed array of telescopes. It was shown earlier that this photon front can be fitted with a spherical surface traveling at a speed of light and originating from a single point on the shower axis. Radius of curvature of the spherical shower front ( $R$ ) is approximately equal to the height of shower maximum from observation level. For a given primary species, it is also found that  $R$  varies with the primary energy ( $E$ ) and this provides a method of estimating the primary energy. In general, one can estimate the arrival times at each telescope using the radius of curvature, arrival direction of the primary and the core location. This, when compared with the data enables us to estimate the above parameters for each shower. This method of obtaining the arrival direction alleviates the difficulty in the form of systematics arising out of the plane wavefront approximation for the Čerenkov front. Another outstanding problem in the field of atmospheric Čerenkov technique is the difficulty in locating the shower core. This method seems to solve both these problems and provides an elegant method to determine the arrival direction as well as the core location from timing information alone. In addition, using the Čerenkov photon density information and the core position we can estimate the energy of the primary if the nature of the primary is known. Combining these two independent estimates of the primary energy, the energy resolution can be further improved. Application of this methodology to simulated data and the results will be presented. The intrinsic uncertainties on the various estimated parameters also will be discussed.

experiment. The density measurements and core location information enable one to estimate the primary energy which in turn could be used to derive the energy spectrum of primary. On the other hand, the timing information which enables us to derive the arrival direction of the primary is essential for source search.

Estimation of shower parameters is rather easily done at PeV energies by measuring particle densities and their arrival times. However at TeV energies, where the charged particles do not reach the observation level, the measurement of shower parameters is rather a challenging task. This is primarily because a bulk of the recorded events have the core locations far away from the points of measurement. In the case of  $\gamma$ -ray primaries, the Čerenkov photon lateral distribution does not exhibit significant gradient which makes it even more difficult to estimate the core location. Further, in the case of Čerenkov telescope arrays, in most cases, only a part of the Čerenkov pool is intercepted by the array. As a result the estimate of the energy of the primary will be very uncertain without the core location information.

The arrival direction of the primary can be estimated from the timing measurements of the Čerenkov light front at spaced telescopes. The conventional plane front approximation leads to significant systematic errors in the estimated angles. These errors often undermine the otherwise excellent angular resolution of the system. In order to eliminate the interference due to systematic errors one has to use a spherical approximation to the light front. As a result, we can obtain important additional information like the radius of curvature of the Čerenkov front and the core location.

Here we present the methodology and results of such an effort based on simulation studies.

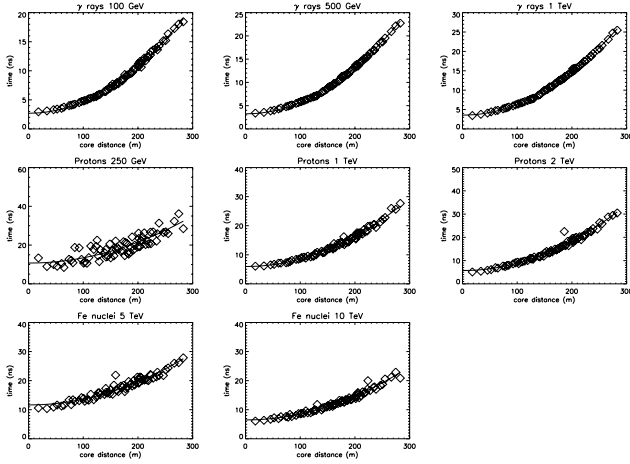
### 1 Introduction

Measurement of shower parameters like the primary energy, core location and direction of arrival are vital in any air shower

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### 2 Čerenkov shower front fitting for vertical showers

In a typical wavefront sampling experiment arrival time of Čerenkov shower front is recorded at several locations in Čerenkov pool with high precision. This information can be



**Fig. 1.** Variation of mean arrival time of Čerenkov shower front as a function of core distance for  $\gamma$ -rays, protons and Fe nuclei of various energies incident vertically at the top of the atmosphere. For each case, arrival time is averaged over typically 10 showers. The smooth curve corresponds to the best fit spherical wavefront.

used to reconstruct the shower direction. It is shown earlier that for vertically incident showers initiated by  $\gamma$ -rays and cosmic ray primaries, the relative arrival time delay [ $t(r)$ ] of Čerenkov shower front at a core distance  $r$  can be approximated by

$$t(r) = \frac{\sqrt{(R^2 + r^2)}}{c} - \frac{R}{c} \quad (1)$$

where  $R$  is the radius of curvature of the spherical front (Battistoni *et al.* (1998), Chitnis and Bhat (1999)).

In effect, Čerenkov shower front can be approximated with a wavefront moving at a speed of light, originating from a single point on shower axis. Figure 1 shows the variation of mean arrival time of Čerenkov shower front as a function of core distance for  $\gamma$ -rays, protons and Fe nuclei of various energies incident vertically at the top of the atmosphere. These showers were simulated using a package known as CORSIKA (Heck *et al.*, 1998). For each case, arrival time is averaged over typically 10 showers. Best fit spherical wavefront is also indicated by smooth curve. Radius of the fitted wavefront ( $R$ ) approximately corresponds to the height of shower maximum ( $h_e$ ) from observation level. This is expected since large fraction of emission arises from the region in the vicinity of shower maximum.

### 3 Algorithm for spherical wavefront fitting for inclined showers

Here we extend this approach to the showers generated by primaries incident at an angle with respect to the vertical. Consider a distributed array of telescopes at observation level. Shower is incident at a zenith angle  $\theta$  and azimuth angle  $\phi$ . Here it is assumed that the shower axis passes through the

centre of the array. Coordinates of the telescopes in the array are given by  $(x_i, y_i)$ , with respect to the centre of the array.

The arrival time of the shower front at a telescope situated at  $(x_i, y_i)$ , with respect to that at the footprint of the front, is given by

$$t(r) = \frac{1}{c} \left[ \sqrt{\frac{R^2}{\cos^2 \theta} - 2R \tan \theta (x_i \cos \phi + y_i \sin \phi) + x_i^2 + y_i^2} - R \right] \quad (2)$$

where  $R$  is the vertical height of the emission point on shower axis from the observation level.

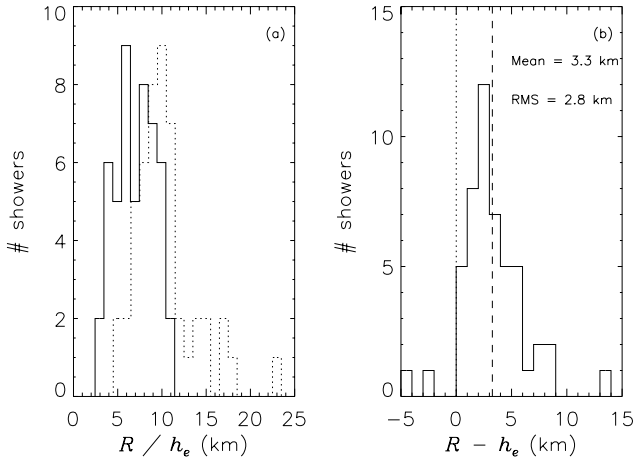
Using this algorithm one can fit the measured arrival time and estimate the arrival direction of shower front ( $\theta$  and  $\phi$ ) as well as the radius of curvature of the shower front ( $R'$ ).

### 4 Spherical wavefront fitting of simulated data

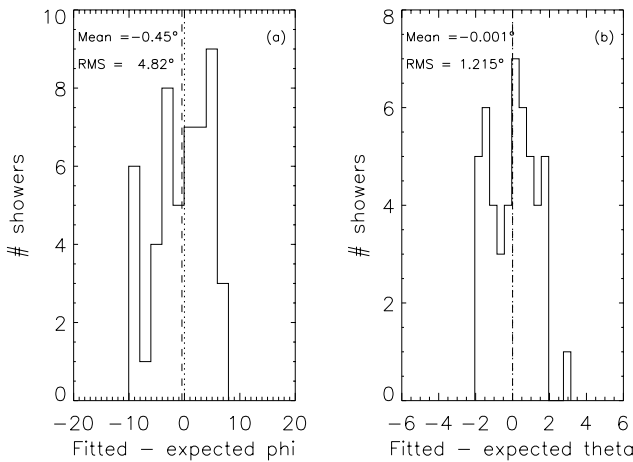
Efficacy of this algorithm is tested using simulated data. A large number of showers initiated by protons incident at an average angle of  $15^\circ$  with respect to vertical are simulated using CORSIKA (Heck *et al.*, 1998). Energies of these proton primaries are chosen randomly in the range 250 GeV - 20 TeV with the spectral index of -2.65. Zenith angles for showers are chosen randomly within  $2^\circ$  around the average value. Whereas azimuthal angles are randomly selected in the range  $0$ - $360^\circ$ . Axes of all the showers pass through the central telescope of the array. Arrival times of Čerenkov photons are recorded at 357 telescopes spread over an area of  $400\text{ m} \times 400\text{ m}$  at an altitude of  $1\text{ km}$ .

For each shower, arrival times at various core distances are fitted with a spherical wavefront. Angles  $\theta$  and  $\phi$  and vertical height of emission point from observation level ( $R$ ) are estimated using Marquardt routine. This exercise is carried out for a sample of 50 showers. Figure 2(a) shows the distribution of fitted radii of curvature ( $R$ ) of shower front. For comparison, distribution of the shower maxima  $h_e$  from observation level is also shown in the same figure. Distribution of difference in  $R$  and  $h_e$  is shown in Figure 2(b). Mean value of this difference distribution is  $3.3\text{ km}$  with an RMS spread of  $2.8\text{ km}$ , indicating that effective point of emission is above the shower maximum. Difference distribution of fitted azimuthal angle ( $\phi$ ) and the actual azimuthal angle is shown in Figure 3a. Similarly difference distribution of fitted zenith angle ( $\theta$ ) and actual zenith angle is shown in Figure 3b.

So far we have carried out spherical wavefront fitting assuming shower core to be exactly at the centre of the array, which is also the origin of our coordinate system. However, in an actual experiment shower core may lie anywhere in the telescope array and at times even outside the array. By introducing the core location as another variable, same algorithm can be further extended to get location of the shower core.



**Fig. 2.** (a) Distribution of the fitted radii of shower front  $R$  (continuous line) and distribution of the actual shower maxima  $h_e$  from observation level (shown by dotted line). (b) Difference distribution of  $R$  and  $h_e$ . Mean of the distribution and the expected value are marked by dashed and dotted lines. Mean and RMS of the difference distribution are indicated.



**Fig. 3.** (a) Difference distribution of fitted and actual  $\phi$ . Mean of the distribution and the expected value are marked by dashed and dotted lines. Mean and RMS of the difference distribution are indicated. (b) Difference distribution of fitted and actual  $\theta$ . Mean of the distribution and the expected value are marked by dashed and dotted lines. Mean and RMS of the difference distribution are indicated.

## 5 Discussion and conclusions

$\check{C}$ erenkov shower fronts from proton showers incident at an angle of  $15^\circ$  are fitted with a spherical wavefront. Fitted radius  $R$  is found to be less than the height of shower maximum  $h_e$  by about 3 kms. For vertical showers  $R$  is found to be about 1 km lower than  $h_e$  (Chitnis and Bhat, 2001). Larger difference between  $R$  and  $h_e$  for inclined showers could be intrinsic. Reduction in the relative contribution of the curvature to the arrival time differences at various telescopes compared to those due to propagation delay could result in a

larger error in fitted radius as well. One can subtract the relative time of arrival differences arising purely out of spatial separation of the detectors using plane front approximation. The residuals can then be fitted to a spherical front which could enhance the sensitivity of the curvature of the shower front which in turn will reduce the systematic error on the fitted radius of curvature. We plan to implement this in our future fitting procedures when we will be able to present a quantitative estimate of the degree of improvement in the fitted radius of curvature.

For a given species the height of shower maximum decreases as the logarithm of the primary energy (Rahman *et al.*, 2001). This offers us an alternate method of estimating the primary energy which in turn improves the primary energy estimate.

In addition to the above, spherical fit to the  $\check{C}$ erenkov light front offers arrival direction as well as core location information of the shower. As mentioned in the introduction the arrival direction information derived by this method is relatively free of systematic errors and hence one can fully exploit the actual angular resolution of the experiment limited by the system hardware. This, in turn offers a major advantage of rejecting a bulk of the off-axis showers which could be presumed to be due to cosmic rays with better confidence.

Finally, shower parameters estimated by this methodology like the radius of curvature and core location coupled with measured and simulated photon density distributions will greatly improve the estimation of the primary energy.

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