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Angular resolution of Pachmarhi Array of Cerenkov Telescopes

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Abstract. Pachmarhi Array of Čerenkov Telescopes(PACT), consisting of a distributed array of 25 telescopes is used to sample the atmospheric Čerenkov Photon showers. The shower front is fitted to a plane and the direction of arrival of primary particle is obtained. The accuracy in the estimation of the arrival direction of showers has been estimated to be $\sim 0^\circ.1$ using 'split' array method. The angular resolution is expected to be even better when a spherical front is used for direction reconstruction or correction for the curvature of the front is applied. This is the best angular resolution among all the currently operating atmospheric Čerenkov telescopes in the world.

1 Introduction

Atmospheric Čerenkov Technique (ACT) is a well established and unique method for the investigation of celestial TeV $\gamma-$ rays. It is mainly based on the effective detection of Čerenkov light emitted by the secondary particles produced in the extensive air showers initiated by a primary $\gamma-$ ray[Cronin (1993)] and reconstructing its direction of arrival in space accurately. The signal to noise ratio of such an experiment[Acharya (1993)] is given by

$$S/N \sim \sqrt{\frac{AT}{F_c\phi}}F_\gamma \tag{1}$$

where A is the physical area of the telescope, T is the time of observations, ϕ is the field of view of the telescope, F_c is the background cosmic ray flux and F_γ the flux of $\gamma-$ rays from the source. In order to achieve high S/N one can either increase the numerator or decrease the denominator in equation(1). For a given exposure and resources one can possibly increase S/N by optimising ϕ, the telescope aperture, keeping in mind the finite opening angle of the Čerenkov cone. It is possible if the direction of arrival of the shower is determined accurately, i.e. the error in the estimation of arrival angle has to be very small or angular resolution has to be high. The arrival direction of a shower is determined by measuring the relative arrival time of Čerenkov photon front at each of the spatially sparated telescopes accurately and reconstructing the shower front. The angular resolution [Sinha (1987)] is given by

$$\delta\theta = (c\delta t)/(D\ cos\theta) \tag{2}$$

where θ is the zenith angle, D the average distance between the telescopes and δt the accuracy in timing measurement. The two factors which contribute to $\delta\theta$ are the average distance, D, between the telescopes and the uncertainty in the measurement of arrival time of photons. So, if we have a large number of telescopes separated by large distances, which measure the relative arrival time of photons, then the shower front could be reconstructed and the direction of the arrival of the shower could be estimated fairly accurately. Here we describe the method of analysis adopted for the estimation of arrival direction of the incident primary using Pachmarhi Array of Čerenkov Telescopes (PACT), which is currently in operation.

2 Pachmarhi Array of Čerenkov Telescopes

The experimental set-up of PACT has been explained in detail elsewhere [Bhat (2000)]. Briefly, it consists of a 5 x 5 array of atmosphereic Čerenkov telescopes deployed in the form of a rectangular matrix with a separation of 25 m in the N-S direction and 20 m in the E-W direction. Each telescope consists of 7 parabolic mirrors of 0.9 m diameter with a focal length of 90 cm. Each mirror is viewed by a fast EMI 9807B photomultiplier tube behind a 3° circular mask. The movement of the telescopes is remotely controlled by a low cost control system called Automatic Computerized Telescope Orientation System (ACTOS) [Gothe (2000)]. The alignment of the mirrors is checked with a bright star (typically of $m_v \sim 2$ to 3) scan. Using this method it is ensured that the optic axes of all the 7 mirrors(labelled A to G) in a

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telescope are parallel to each other within an error of about $0^\circ.2$ The system can orient to the putative source with an accuracy of $0^\circ.003 \pm 0^\circ.2$. The source tracking is monitored with an accuracy of $0^\circ.05$ and corrected in real time.

The array has been divided into 4 sectors with six telescopes in each [1]and the data are acquired in each sector as well as in central master signal processing centre separately. The pulses from 7 PMTs in a telescope are added linearly to form a telescope trigger pulse called 'royal-sum' pulse. Each 'royal-sum' pulse from all the 6 telescopes in a sector are suitably discrminated to yield a count rate of $\sim$ 30-40 kHz. An event trigger is generated by a coincidence of any 4 of the 6 telescope triggers in a sector which gives an event trigger rate of $\sim$ 2-5 Hz. In each sector the timing and density information of Čerenkov photons incident on the 6 peripheral mirrors of six telescopes as well as the timing information on six 'royal-sum' telescope pulses are recorded. Also, in the central control room the relative arrival times of all 24 telescope trigger pulses and sectorwise house-keeping information are recorded.

2.1 Estimation of Timing Resolution

The accuracy in timing measurement, δt, is estimated as follows. To determine the arrival time of photons accurately a fast low noise photomultiplier with high gain and minimum timing jitter is used. The intrinsic timing jitter of the signals from the PMTs limit the resolution of timing measurements(0.8 ns). The event trigger is used as a start pulse to the fast time to digital convertors(TDC). The individual PMT and 'royal-sum' pulses are delayed using ECL based delay generetors and then fed as TDC stops. The TDC modules (LeCroy and Philips Scientific make) were operated with a full scale setting of 500 ns which means a delay of 0.25 and 0.2 ns per count respectively. Data were collected with all telescopes in the vertical direction. The variance σ_{ij} or the width of the distribution of difference in relative arrival times of signals (δt_{ij}) of respective TDC channels is an indication of the limiting accuracy of timing measurement, provided the signals originate from PMT's located nearby.[Chitnis (1999)] To minimise the effects due to fluctuations in the arrival time of Čerenkov photons, which depends upon the core distance [Chitnis (1999)] only those combinations corresponding to neighbouring PMT's are considered. Using this method the limiting accuracy of timing measurement ($< \sigma_{ij} > /\sqrt{2}$) is estimated to be 1 ns.

3 Estimation of Arrival Direction of a Shower

The arrival direction of a shower is determined by measuring the relative arrival time of Čerenkov photon front at each telescopes accurately and reconstructing the shower front. A spherical shape represents the Čerenkov photon shower front fairly accurately, as also demonstrated from Monte Carlo simulations [Chitnis (1999)]. However, the algorithm and the analysis technique to determine the shower core in our experiment is under development[Chitnis (2001)]. In the absence of the knowledge on the shower core, we assume the front to be a plane and fit the measured relative arrival time of Čerenkov photons to a plane, normal to which gives the direction of shower axis. Such an assumption introduces a systematic error in the estimation of arrival direction.

3.0.1 Calculation of Time-offsets

The relative arrival time of pulses as measured in the experiment is not the relative arrival time of Čerenkov photons at the PMT, which is needed for reconstructing the shower front. A finite but constant delay between pulses from different PMT's (Channels) arise due to unequal cable lengths, differences in electronic propagation delays and differences in photomultiplier transit time etc. These are termed as T0 or Time-offsets. Thus the measured relative arrival times have to be corrected for this time-offsets to get the relative arrival time of Čerenkov front at the PMT. The average relative time delays between two PMT's(or telescopes), from a large sample of data, is entirely due to difference between the two time-offsets. In our experiment, the average separation between PMT's in a telescope is of the order of a metre but the separation between the telescopes in a sector is about 35 m small enough to be ignored. The difference in RMS fluctuations in arrival time of photons is negligible at this distance separation for any core distance.[Chitnis (1999)] If $T0_i$ and $T0_j$ are the time offsets for the PMTs i and j, we can write an equation of the form

$$(T0_i - T0_j) = C_{ij} \tag{3}$$

where C_{ij} is the mean delay between a pair of PMT's i and j after correcting for the time difference due to difference in height (Z-coordinates) of PMT's if any.

For each pair of PMT's an equation of the form

$$\chi^2 = \Sigma W_{ij}(T0_i - T0_j - C_{ij})^2 \tag{4}$$

can be written, where W_{ij}s are the statistical weight factors, (W_{ij}=1/${\sigma_{ij}}^2$), where σ_{ij} is the uncertainity in determining C_{ij}. Using χ^2 minimisation one gets an estimate of these Time-offsets.

3.1 Reconstruction of arrival direction

Using the plane front approximation the arrival direction of the shower is estimated as follows [Sinha (1987)][Acharya (1993)]. If x_i,y_i,z_i are the coordinates of the i^{th} PMT, (l, m, n) the direction cosines of the shower axis and t_i the arrival time of the photons at this PMT then the equation relating them is given by

$$lx_i + my_i + nz_i + c(t_i - t_0) = 0 \tag{5}$$

where t_0 is the time at which the shower front passes through the origin of the coordinate system. Then the arrival direction of the shower can be estimated by a χ^2 minimisation where

$$\chi^2 = \Sigma w_i(lx_i + my_i + nz_i + c(t_i - t_0))^2 \tag{6}$$

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[1] central telescope is presently not included

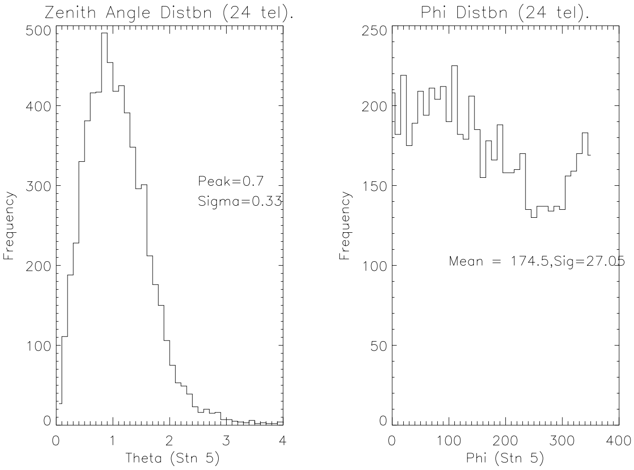

Fig. 1. Zenith and Azimuth Angle Distributions for 24 telescopes

where w_i is the statistical weight factor given to the i^{th} timing mesurement. The values of (l, m, n) and t_0 are calculated using the equations $\delta\chi^2/\delta l = 0$, $\delta\chi^2/\delta m = 0$, $\delta\chi^2/\delta t_0 = 0$ and $l^2 + m^2 + n^2 = 1$. Fig.1 shows the zenith and azimuthal angle distributions of the reconstructed arrival directions using the procedure explained above. The reconstruction of shower front was done using 24 telescopes for the data collected with telescopes in the vertical position.

4 Angular Resolution of PACT

4.1 Using Royal Sum TDC Information

The angular resolution of PACT has been estimated by using the *divided array* method. In the data recorded in the central control room one has information on 24 telescopes. The array is divided into two independent parts of 12 telescopes each, say Sectors 1 and 2 and Sectors 3 and 4. The arrival direction is estimated for each shower from these two independent arrays. The distribution of space angle between these two estimates is a measure of the accuracy with which one can estimate the arrival direction (Figs 2). Since one has two independent estimates of the direction the angular resolution will be given by the peak of the distribution of space angle between the two directions as $\frac{Peak}{\sqrt{2}}$. Similar analysis was performed by dividing the array using various combinations of telescopes to study the dependence of angular resolution as a function of max. separation between telescopes and number of telescopes used in the fit. The results are summarised in Table # 1. The term ‘Odd-Even’ refers to 3 telescopes each from Sector 3 and 4 grouped into one set and the remaining 6 grouped into another.

It is seen from Table # 1 that as the separation between the detectors increases there is a definite improvement in the angular resolution for the same number of degrees of freedom.

Table 1. Angular Resolution of PACT using Royal Sums From Control Room Data

No.of tel.	Combn of Detectors	Separation bet. Det.(mts)		Peak of Sp.Ang	Angl Resl(deg)
		Maxm	Avg		
6	Sec 3 v/s 4	53.85	31.77	0.875	0.618
6	Odd v/s Even	94.33	48.81	0.637	0.45
12	Sec 3 and 4 v/s Sec 1 and 2	94.33	44.2	0.586	0.395
12	Sec 1 and 3 v/s Sec 2 and 4	128.07	64.56	0.47	0.3

4.2 Using Individual Mirror Information

The relative arrival times at individual PMT’s are available only within a sector. Data from Sector 3 and 4 and only those events with information in all 6 telescopes are used for the estimation of angular resolution. The angular resolution is obtained by dividing the data from the sectors into 2 subsets and obtaining the space angle between the two arrival directions corresponding to two subsets of data. Information for the 3 PMT’s(labelled A, C and E) are grouped into one set while those for the remaining 3 PMT’s of a telescope(B, D and F) are grouped into other set. The results are summarised in Table #2. The rows 1 and 4 correspond to the cases in which two subsets are obtained by demanding all A, all B etc. Rows 2 and 5 correspond to cases in which the 1st group consists of valid TDC information for any one of 3 A, C, E PMT’s and the second group from any one of 3 B, D, F PMT’s. Similarly the rows 3 and 6 correspond to two sets with at least 2 valid TDC’s in each telescope. Finally the row 7 refers to the case in which the arrival directions are obtained separately from Sector 3 and Sector 4 events and collating event arrival times to pick common events. Column ‘2’ shows the corresponding no. of detectors used in the fit for all cases. Many cases have been studied to understand the improvement in angular resolution with increasing degrees of freedom starting from at least 1 mirror in a telescope to greater than 25 in a sector.

Table 2. Angular Resolution of PACT using Individual PMT Information

Sector #	No. of Det. used	Combination of Detectors	Peak of Sp. Angl.	Ang. Resln
3	6	all A, all B, etc	0.46	0.325
3	≥ 6	at least 1 in each telescope	0.51	0.36
3	≥ 12	$\sim$2 in a telescope	0.43	0.3
4	6	all A, all B, etc	0.48	0.339
4	≥ 6	at least 1 in each telescope	0.39	0.275
4	≥ 12	$\sim$2 in a telescope	0.34	0.24
3 and 4	≥ 25	3,4 Collated by time	0.325	0.23

Table #2 shows that in both the sectors as the number of detectors used in the fit go up the angular resolution does improve. On an average 13 to 14 mirrors in a sector have valid TDC information available for fitting the angle and hence

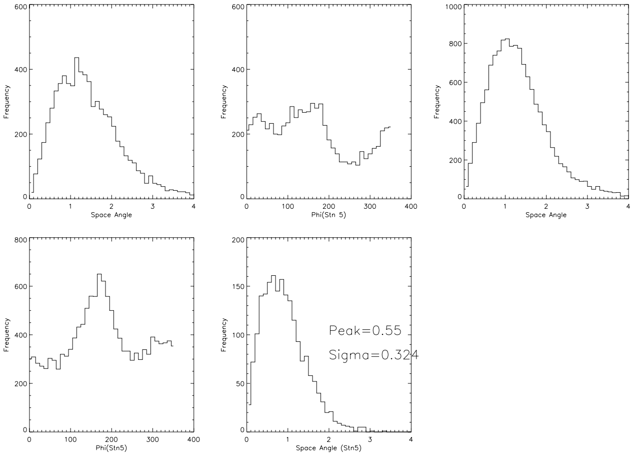

Fig. 2. Space Angle Distbn using Sector 5 data. The fit made with Sector 1,2 and Sector 3,4 telescopes.

increase in n, the number of detectors is ~ 2.2. From the angular resolution computed as one goes from 6 to 12 mirrors it is seen that the improvement in angular resolution goes as $\sim n^{0.3}$. A conservative estimate for the angular resolution(ψ) of the array is obtained from Table 2 which is ψ = $0.23°$/$2^{-0.75}$ $\times$ $4^{0.3}$ or $0.09°$.

5 Discussions and Conclusions

We have made a detailed analysis on the angular resolution of PACT using data collected with telescopes in vertical position. The improvement in angular resolution with longer baseline between detectors and with increase in the number of degrees of freedom has been established. The angular resolution of PACT has been estimated to be $0°.24$ using royal sum TDC information. The angular resolution from a sector has been estimated to be $0°.23$ using individual PMT information. So a conservative estimate of the array is $0.09°$. This is the best angular resolution achieved so far in the world among all the contemporary atmospheric Čerenkov telescopes. While the angular resolution of the imaging telescopes are limited by the PMT sizes, only the future imaging telescope arrays (like VERITAS [Bradbury (1999)] or HESS) claim a better angular resolution. PACT is able to achieve this because of the multiple sampling technique in a distributed array of ACTs.

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