Proceedings of ICRC 2001: 1933 © Copernicus Gesellschaft 2001

ICRC 2001

Analytical solution of 3D cosmic-ray diffusion in boundaryless halo (I) – one component model –

T. Shibata

Department of Physics, Aoyama-Gakuin University, Chitosedai 6–16–1, Setagaya-ku, Tokyo 157–8572, Japan

Abstract. We show an analytical solution of three dimensional cosmic-ray diffusion, taking into account a rather realistic structure of our Galaxy, where we assume, 1) $D(r,z) = D_0 \exp[r/r_D + |z|/z_D]$, 2) $n(r,z) = n_0 \exp[-(r/r_n + |z|/z_n)]$, and 3) $s(r,z) = s_0 \exp[-(r/r_s + |z|/z_s)]$, i.e., three critical parameters, the diffusion coefficient D, gas density n and the cosmic-ray source density s depending on both radial distance r from the disk center and the perpendicular distance z from the galactic plane.

We also present the path-length distribution and its average value, based on the present analytical solution.

1 Introduction

Recently new data on cosmic-ray (CR) spectrum and composition (Apanasenko et al. 2001), 2-ry/1-ry ratio (Hareyama et al. 1999; Apanasenko et al. 2001), $\bar{p}$ (Orito et al. 2000), $e^{\pm}$ (Nishimura et al. 1996; Müller et al. 1995), the radioactive isotope (Yanasak et al. 2000) and so on, have become available, which are essentially important in understanding the origin and the propagation of CR. These components are closely related with each other, and the simultaneous study for all of them brings us a critical information of the structure of our Galaxy.

It is, however, not so straightforward to build an unified picture for the propagation of those in connection with the structure of Galaxy. Main reason originates in the uncertainty of the cross section in the fragmentation process, particularly for isotope products, apart from the limitation in statistics nowadays available.

Another reason is due to the mathematical complexity in the diffusion problem of CR taking account of various effects, such as the (r,z)-dependence in diffusion coefficient, the gas density and so on, though past ellaborate works, particularly by russian groups under Ginzburg (Berezinskii et al. 1990), have made clear already the essence of the CR propagation.

In the present paper, we show an analytical solution for primary components, taking account of the effects mentioned above, while the secondary components, including unstable nucleus, are presented in another paper (Shibata III 2001). In three papers presented in this conference, we don't touch the energy dependence in diffusion coefficient and the energy change coming from either ionization loss or the reacceleration, effective in the low energy region, due to limited space, which will be reported separately elsewhere.

Correspondence to: T.Shibata (shibata@cariocas.phys.aoyama.ac.jp)

2 Diffusion equation

Assuming a position of the cosmic-ray source is given by (r_0, z_0) in the cylindrical coordinate, the diffusion equation has a form

$$\left[\frac{1}{r}\frac{\partial}{\partial r} r D(r,z)\frac{\partial}{\partial r} + \frac{\partial}{\partial z} D(r,z)\frac{\partial}{\partial z} - n(r,z) v\sigma\right] \Phi(r,z) = -\delta(z-z_0)\delta(r-r_0)/2\pi r_0. \quad (1)$$

The boundary condition without the halo edges is

$$\Phi(r,\pm\infty) = 0, \quad \text{and} \quad \Phi(R,z) = 0. \quad (2)$$

Now, we assume the diffusion coefficient D, gas density n and CR source density s depend on the coordinate (r,z) in the following forms,

$$D(r,z) = D_0 \exp[r/r_D + |z|/z_D], \quad (3a)$$

$$n(r,z) = n_0 \exp[-(r/r_n + |z|/z_n)], \quad (3b)$$

$$s(r,z) = s_0 \exp[-(r/r_s + |z|/z_s)], \quad (3c)$$

where D_0, n_0 and s_0 correspond to the diffusion coefficient, gas density and CR source density at the galactic center $(0,0)$, respectively.

Naturally, we expect

$$r_D \gg z_D, \quad r_n \gg z_n, \quad r_s \gg z_s, \tag{4}$$

and probably r_D, r_n, r_s ($\simeq$ a few tens kpc or more) might be two order of magnitude larger than z_D, z_n, z_s ($\simeq$ a few hundred pc).

Substituting Eqs. (3a) and (3b) into Eq. (1), we get,

$$\left[\left(\frac{1}{r}\frac{\partial}{\partial r}r+\frac{1}{r_D}\right)\frac{\partial}{\partial r}+\left(\frac{\partial}{\partial z}+\frac{1}{z_D}\right)\frac{\partial}{\partial z}-\frac{n_0 v\sigma}{D_0}\mathrm{e}^{-2u}\right]\Phi(r,z)$$

$$= \frac{-\delta(z-z_0)}{D(r_0,z_0)}\frac{\delta(r-r_0)}{2\pi r_0}, \tag{5}$$

here

$$u \ = \ \frac{r}{\bar{r}} \ + \ \frac{|z|}{\bar{z}}, \tag{6}$$

$$\frac{1}{\bar{r}} = \frac{1}{2}\left(\frac{1}{r_n}+\frac{1}{r_D}\right), \quad \frac{1}{\bar{z}} = \frac{1}{2}\left(\frac{1}{z_n}+\frac{1}{z_D}\right). \tag{7}$$

Now, changing the variables (r,z) into (r,u), we search for a solution in the form of a Fourier-Bessel expansion, satisfying automatically the boundary condition appeared in the second relation of Eq. (2)

$$\Phi(r,u;r_0,u_0) = \sum_{k=1}^{\infty}\varphi_k(r,u;r_0,u_0)J_0(\xi_k r/R), \tag{8}$$

$$\text{with} \ \ u_0 = r_0/\bar{r} + |z_0|/\bar{z}, \tag{6'}$$

here J_0 is the Bessel function of zeroth order, and ξ_k $(k=1,2,\ldots)$ are the roots of $J_0(\xi_k)=0$.

Remembering the fact shown by Eq. (4), we expect (more qualitative discussion will be presented elsewhere)

$$\varphi_k(r,u;r_0,u_0) \simeq \varphi_k(u;r_0,u_0), \tag{9}$$

and applying the Fourier-Bessel expansion for $\delta(r-r_0)$ appeared in the righthand side of Eq. (5), we obtain

$$\left[\frac{d^2}{du^2}+2\nu\frac{d}{du}-(\lambda_0^2\mathrm{e}^{-2u}+\lambda_{1,k}^2)\right]\varphi_k = -q_k\delta(u-u_0), \tag{10}$$

$$\nu = \frac{\bar{z}}{2z_D} = 1\Big/\left(1+\frac{z_D}{z_n}\right), \tag{11}$$

$$\lambda_0 = \sqrt{\frac{n_0 v\sigma}{D_0}}\,\bar{z}, \quad \lambda_{1,k} = \frac{\xi_k\bar{z}}{R}, \tag{12}$$

$$q_k(r_0,z_0) = \frac{\bar{z}}{D(r_0,z_0)}\frac{J_0(\xi_k r_0/R)}{\pi R^2 J_1^2(\xi_k)}. \tag{13}$$

We should note $0<\nu<1$, and $\nu=1/2$ for $z_D=z_n$.

3 Solution of the diffusion equation

Fundamental solution of Eq. (10) is given with use of the Modified Bessel functions, taking account of two cases, $z\geq 0$ and $z\leq 0$,

$$\varphi_k^{(\pm)}(u;r_0,u_0) = a^{(\pm)}A_k(\Lambda)+b^{(\pm)}B_k(\Lambda), \tag{14}$$

$$\Lambda(u) = \lambda_0 U(u), \quad \text{with} \ \ U(u) = \mathrm{e}^{-u}, \tag{15}$$

where

$$A_k(\Lambda) = \Lambda^{\nu}I_{\nu_k}(\Lambda), \quad B_k(\Lambda) = \Lambda^{\nu}K_{\nu_k}(\Lambda), \tag{16}$$

$$\nu_k^2 \ = \ \nu^2 \ + \ \lambda_{1,k}^2. \tag{17}$$

In Eq. (14), $(\pm)$ correspond to two solutions for $z\geq 0$ and $z\leq 0$ respectively. Four coefficients, $a^{(\pm)}$ and $b^{(\pm)}$, are determined so that the boundary condition Eq. (2) as well as the smooth contination of $\varphi_k^{(+)}$ and $\varphi_k^{(-)}$ at $z=0$ are satisfied, taking into account the source term, $q_k(r_0,z_0)$.

Now the final solution is given by

$$\Phi^{(\pm)}(r,u;r_0,u_0) = \left(\frac{U}{U_0}\right)^{\nu}\sum_{k=1}^{\infty}q_k(r_0,z_0)J_0(\xi_k r/R)\times$$

$$\left[\frac{(I_{\nu_k}(\Lambda_r)I_{\nu_k}(\Lambda),\mathcal{L}_k(\Lambda_r,\Lambda_0))_{\pm}}{(I_{\nu_k}(\Lambda_r),I_{\nu_k}(\Lambda_r))_{+}}-\mathcal{L}_k(\Lambda,\Lambda_0)\theta(u_0-u)\right] \tag{18}$$

where

$$\Lambda_0 = \lambda_0 U_0 \ \ \text{with} \ \ U_0 = \mathrm{e}^{-u_0}, \tag{19a}$$

$$\Lambda_r = \lambda_0 U_r \ \ \text{with} \ \ U_r = \mathrm{e}^{-r/\bar{r}}, \tag{19b}$$

and we introduced a round bracket

$$(\mathcal{A},\ \mathcal{B})_{\pm} \ = \ \mathcal{A}\times\mathcal{B}^{\dagger} \ \pm \ \mathcal{A}^{\dagger}\times\mathcal{B}\,\Big|_{z=0} \tag{20}$$

which is quite useful for the two component model (Shibata II 2001) as presented in this volume. $\mathcal{L}_k(X,Y)$ and the meaning of $\dagger$ are summarized in Appendix A.

4 Source integration

Integrating over (r_0,z_0) with use of the source distribution function $s(r_0,z_0)$ given by Eq. (3c), we get

$$N(r,u) = \int_0^R 2\pi r_0 dr_0\int_{-\infty}^{+\infty}dz_0 s(r_0,z_0)\,\Phi^{(\pm)}(r,u;r_0,u_0). \tag{21}$$

Remarking two terms, $D(r_0,z_0)$ in q_k (see Eq. (13)) and $s(r_0,z_0)$ in Eq. (3c), we find

$$\mathrm{e}^{-(r_0/r_D+z_0/z_D)}\mathrm{e}^{-(r_0/r_s+z_0/z_s)} = U_0^{2\omega_\perp}\mathrm{e}^{-r_0/\hat{r}},$$

where we introduced following variables,

$$1/\hat{r} = 2(\omega_{/\!/}-\omega_\perp)/\bar{r}, \tag{22}$$

and

$$\omega_{/\!/} = \left(1+\frac{r_D}{r_s}\right)\Big/\left(1+\frac{r_D}{r_n}\right), \tag{23a}$$

$$\omega_\perp = \left(1+\frac{z_D}{z_s}\right)\Big/\left(1+\frac{z_D}{z_n}\right). \tag{23b}$$

Then we obtain

$$\int_0^R \mathrm{e}^{-r_0/\hat{r}} J_0(\xi_k r_0/R) 2\pi r_0 dr_0 = \pi R^2 \Xi(\xi_k, R/\hat{r}), \quad (24)$$

$$\text{with} \quad \Xi(\xi, a) = \int_0^1 \mathrm{e}^{-at} J_0(\xi t) 2t dt. \quad (25)$$

If $r_s \sim r_n$ and $z_s \sim z_n$ (equivalently $\omega_{/\!/} \sim \omega_\perp \sim 1$), which is quite likely, we obtain a simple result

$$\Xi(\xi_k, R/\hat{r}) = \frac{2}{\xi_k} J_1(\xi_k). \quad (25')$$

Remarking $(\cdots)_\pm$ in Eq. (18), one finds $\pm$-term appeared in the round bracket cancels each other due to the integration over z_0 in two ranges, $[-\infty, 0]$ and $[0, +\infty]$, and the integration over u_0 is much reduced. Introducing an exchange-function for $X \leftrightarrow Y$,

$$\Psi_k(X,Y,Z) = \left\{ \begin{array}{ll} \psi_k(X,Y,Z), & \text{for } X \le Y \\ \psi_k(Y,X,Z), & \text{for } X \ge Y \end{array} \right\} \quad (26)$$

$$\text{with} \quad \psi_k(X,Y,Z) = \mathcal{L}_k^\dagger(\lambda_0 Z, \lambda_0 Y) \frac{I_{\nu_k}(\lambda_0 X)}{I_{\nu_k}^\dagger(\lambda_0 Z)}, \quad (27)$$

finally we obtain a solution for the CR intensity

$$N(r,u) = \frac{s_0 \bar{z}^2}{D_0} U^\nu \sum_{k=1}^\infty \mathcal{M}_k(r) \mathcal{N}_k(U, U_r). \quad (28)$$

Here we define

$$\mathcal{M}_k(r) = \frac{\Xi(\xi_k, R/\hat{r}) J_0(\xi_k r/R)}{J_1^2(\xi_k)} \simeq \frac{2 J_0(\xi_k r/R)}{\xi_k J_1(\xi_k)}, \quad (29)$$

$$\mathcal{N}_k(X,Y) = \int_0^1 t^{\omega-1} \Psi_k(t,X,Y) dt, \quad (30)$$

$$\text{with} \quad \omega = 2\omega_\perp - \nu \simeq 2 - \nu. \quad (31)$$

Eq. (28) may be rewritten

$$N(r,u) = \frac{s_0 \bar{z}^2}{D(\kappa r/2, z/2)} \sum_{k=1}^\infty \mathcal{M}_k(r) \mathcal{N}_k(U, U_r), \quad (28')$$

$$\text{with} \quad \kappa = \left(1 + \frac{r_D}{r_n}\right) \Big/ \left(1 + \frac{z_D}{z_n}\right) \simeq 1, \quad (32)$$

the expression of which is useful for the two component model (Shibata II 2001).

In the case of the galactic plane ($z = 0$, or equivalently $u = u_r = r/\bar{r}$), we have a simple expression (see Appendix A),

$$\Psi_k(t, U_r, U_r) = \frac{I_{\nu_k}(\lambda_0 t)}{I_{\nu_k}^\dagger(\Lambda_r)}, \quad (33)$$

and Eq. (28) is reduced to

$$N(r, r/\bar{r}) \simeq \frac{s_0 \bar{z}^2}{D(r/2, 0)} \sum_{k=1}^\infty \frac{2 J_0(\xi_k r/R)}{\xi_k J_1(\xi_k)} \frac{\mathcal{I}_{\nu_k}(\lambda_0)}{I_{\nu_k}^\dagger(\Lambda_r)}, \quad (34)$$

$$\text{with} \quad \mathcal{I}_\nu(a) = \int_0^1 t^{\omega-1} I_\nu(at) dt. \quad (35)$$

5 Path length distribution

Path length distribution, $P(x; r, z)$, is obtained by the inverse Laplace transformation of $N(r, u)$ for the cross section σ, which is appeared in λ_0 (see the first relation of Eq. (12)). Here, we limit to the case of the solar system ($r = r_\odot, z = 0$) for the practical purpose.

Remarking to the term related to σ only,

$$\frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} \frac{I_{\nu_k}(\lambda_0 t)}{I_{\nu_k}^\dagger(\Lambda_r)} \mathrm{e}^{\sigma x} d\sigma. \quad (36)$$

Since $I_{\nu_k}^\dagger(\Lambda_r)$ (see Eq. $(A1a)$) has zero points on negative imaginary axis on the complex Λ_r-plane (note that $\Lambda_r = 0$ is not a singular point in the integrand of Eq. (36)), putting the ℓ-th zero point for $I_{\nu_k}^\dagger(\Lambda_r) = 0$ as

$$\Lambda_r = -i \Lambda_{k,\ell},$$

we find the Bessel function appeared in Eq. (36) is now rewritten as

$$\frac{I_{\nu_k}(\lambda_0 t)}{I_{\nu_k}^\dagger(\Lambda_r)} = \frac{J_{\nu_k}(\lambda_{k,\ell} t)}{J_{\nu_k}^\dagger(\Lambda_{k,\ell})}, \quad (37)$$

$$\text{with} \quad \Lambda_{k,\ell} = \lambda_{k,\ell} U_r = \lambda_{k,\ell} \mathrm{e}^{-r/\bar{r}}, \quad (38)$$

where we defined a following function, corresponding to $I_{\nu_k}^\dagger(\Lambda)$,

$$J_{\nu_k}^\dagger(\Lambda) = 2\bar{\nu}_k J_{\nu_k}(\Lambda) - \Lambda J_{\nu_k+1}(\Lambda). \quad (39)$$

Then one can find simple poles $\Lambda_{k,\ell}$ ($\ell = 1, 2, \ldots$) on positive real axis on Λ_r-plane, i.e.,

$$\Lambda_{k,\ell} = 2\bar{\nu}_k \frac{J_{\nu_k}(\Lambda_{k,\ell})}{J_{\nu_k+1}(\Lambda_{k,\ell})}. \quad (40)$$

From Eqs. (12) and (38), the cross section corresponding to $\Lambda_{k,\ell}$ is given by

$$\frac{1}{\sigma_{k,\ell}} = -\frac{1}{\Lambda_{k,\ell}^2} \frac{n_0 v \bar{z}^2}{D_0} \mathrm{e}^{-2r/\bar{r}} \equiv -\bar{x}_{k,\ell}. \quad (41)$$

Finally we obtain

$$P(x) = P_0 \sum_{k=1}^\infty \mathcal{M}_k(r_\odot) \sum_{\ell=1}^\infty \frac{\lambda_{k,\ell} \mathcal{J}_{\nu_k}(\lambda_{k,\ell})}{J_{\nu_k+1}^\dagger(\Lambda_{k,\ell})} \mathrm{e}^{-x/\bar{x}_{k,\ell}}, \quad (42)$$

$$\text{with} \quad P_0 = \frac{2 s_0}{n_0 v} \mathrm{e}^{(1-\nu) r_\odot/\bar{r}}, \quad (43)$$

where from Eq. (41),

$$\bar{x}_{k,\ell} = \bar{x}_\odot / \Lambda_{k,\ell}^2 \quad \text{with} \quad \bar{x}_\odot = \frac{\bar{n}_\odot v \bar{z}^2}{\bar{D}_\odot}, \quad (44)$$

$$\bar{n}_\odot = n_0 \mathrm{e}^{-r_\odot/\bar{r}}, \quad \bar{D}_\odot = D_0 \mathrm{e}^{r_\odot/\bar{r}}, \quad (45)$$

and

$$\mathcal{J}_\nu(a) = \int_0^1 t^{\omega-1} J_\nu(at) dt. \quad (46)$$

Average path-length $\bar{x}$ is rather easily obtained by (Berezinskii et al. 1990),

$$\bar{x} = -\frac{1}{N}\left.\frac{\partial N}{\partial \sigma}\right|_{\sigma=0} = \bar{\chi}_\odot \frac{n_0 v \bar{z}^2}{D_0}, \quad (47)$$

$$\bar{\chi}_\odot = \frac{\sum_{k=1}^{\infty} \chi_k(r_\odot)\chi_{k,0}(r_\odot)\mathcal{M}_k(r_\odot)}{\sum_{k=1}^{\infty} \chi_{k,0}(r_\odot)\mathcal{M}_k(r_\odot)}, \quad (48)$$

$$\chi_{k,0}(r) = \frac{1}{\nu_k+\nu}\,\frac{1}{\nu_k+\omega}\,\mathrm{e}^{\nu_k r/\bar{r}}, \quad (49a)$$

$$\chi_k(r) = \frac{1}{4}\frac{1}{\nu_k+1}\left\{\frac{\nu_k+\nu+2}{\nu_k+\nu}\mathrm{e}^{-2r/\bar{r}} - \frac{\nu_k+\omega}{\nu_k+\omega+2}\right\}. \quad (49b)$$

6 Discussion

We obtained analytically the solution of 3D cosmic-ray diffusion, taking account of a rather realistic structure of our Galaxy, with the assumption that three critical parameters, D (diffusion coefficient), n (gas density) and s (CR source density), are all of the exponential type in the radial distance r from the disk center and the perpendicular ditance z from the galactic plane.

It should be remarked that the path-length distribution, Eq. (42), depends on the both gas density $\bar{n}_\odot$ and the diffusion coefficient $\bar{D}_\odot$ *at solar system*, while current diffusion model uses their *average* values for D. For instance, it is known that the diffusion model is equivalent to the leaky box model by putting (Berezinskii et al. 1990)

$$\bar{x} = n_g v h_g h_h / D, \quad (50)$$

where n_g is the gas density in the disk, D is the average diffusion coefficient in the Galaxy ($D = D_g = D_h$), and h_g, h_h are the thickness of the disk and the halo respectively. More discussion on the relation between Eq. (47) and Eq. (50) will be done elsewhere.

In this paper we focused to the mathematical procedure of the evaluation of the analytical solution, and the explicit numerical results and the comparison with the observed data will be reported in the conference.

Appendix A Summary of variables and functions

We summarize here variables and functions appeared in the text. Our results are basically expressed with the use of two independent Modified Bessel functtions, I_{ν_k} and K_{ν_k}, and their linear combinations.

$$I^\dagger_{\nu_k}(\Lambda) = 2\bar{\nu}_k I_{\nu_k}(\Lambda) + \Lambda I_{\nu_k+1}(\Lambda), \quad (A1a)$$

$$K^\dagger_{\nu_k}(\Lambda) = 2\bar{\nu}_k K_{\nu_k}(\Lambda) - \Lambda K_{\nu_k+1}(\Lambda), \quad (A1b)$$

$$2\bar{\nu}_k = \nu + \nu_k = \nu + \sqrt{\nu^2 + \lambda_{1,k}^2}. \quad (A2)$$

With use of these functions, we find immediately (see Eq. (16))

$$\frac{d}{du}A_k(\Lambda) = -\Lambda^\nu I^\dagger_{\nu_k}(\Lambda), \quad (A3a)$$

$$\frac{d}{du}B_k(\Lambda) = -\Lambda^\nu K^\dagger_{\nu_k}(\Lambda). \quad (A3b)$$

Further we define following functions,

$$\mathcal{L}_k(X,Y) = I_{\nu_k}(X)K_{\nu_k}(Y) - I_{\nu_k}(Y)K_{\nu_k}(X), \quad (A4a)$$

$$\mathcal{L}^\dagger_k(X,Y) = I^\dagger_{\nu_k}(X)K_{\nu_k}(Y) - I_{\nu_k}(Y)K^\dagger_{\nu_k}(X). \quad (A4b)$$

Remarking a well-known relation

$$I_\nu(\Lambda)K_{\nu+1}(\Lambda) + I_{\nu+1}(\Lambda)K_\nu(\Lambda) = 1/\Lambda, \quad (A5)$$

we find

$$\mathcal{L}_k(\Lambda_r,\Lambda_r) = 0, \quad \mathcal{L}^\dagger_k(\Lambda_r,\Lambda_r) = 1, \quad (A6)$$

the result of which is used to obtain Eq. (33) in the text.

The round bracket defined by Eq. (20) is often appeared in the present work, which comes from the smooth continuation condition of $\varphi_k^{(+)}$ and $\varphi_k^{(-)}$ at $z = 0$. Then we should note that this bracket is applied only for the variable Λ_r, and other variables, Λ, Λ_0, ... , are freely moved in and out the bracket. We show typical examples appeared in Eq. (18) in the following.

$$\begin{aligned}(I_{\nu_k}(\Lambda_r), I_{\nu_k}(\Lambda_r))_+ &= \\ I_{\nu_k}(\Lambda_r)I^\dagger_{\nu_k}(\Lambda_r) &+ I^\dagger_{\nu_k}(\Lambda_r)I_{\nu_k}(\Lambda_r) \\ &= 2I_{\nu_k}(\Lambda_r)I^\dagger_{\nu_k}(\Lambda_r), \quad (A7)\end{aligned}$$

$$\begin{aligned}&(I_{\nu_k}(\Lambda_r)I_{\nu_k}(\Lambda), \mathcal{L}_k(\Lambda_r,\Lambda_0))_- = \\ &I_{\nu_k}(\Lambda_r)I_{\nu_k}(\Lambda)\mathcal{L}^\dagger_k(\Lambda_r,\Lambda_0) - I^\dagger_{\nu_k}(\Lambda_r)I_{\nu_k}(\Lambda)\mathcal{L}_k(\Lambda_r,\Lambda_0) \\ &= I_{\nu_k}(\Lambda)I_{\nu_k}(\Lambda_0)\mathcal{L}^\dagger_k(\Lambda_r,\Lambda). \quad (A8)\end{aligned}$$

References

A.V. Apanasenko et al. (RUNJOB collaboration), Composition and energy spectra for light elements by RUNJOB; Composition and energy spectra for heavy elements by RUNJOB; All-particle spectrum and mass average by RUNJOB, in OG session.

V.S. Berezinskii et al., Astrophysics of Cosmic Rays (edited by V.L. Ginzburg), North Holland, 1990.

M. Hareyama et al., Sub-Fe/Fe ratio obtained by Sanriku balloon experiment, 26th ICRC (Salt Lake City) 3, 105-108, 1999.

T. Shibata (III), Analytical solution of 3D cosmic-ray diffusion in boundaryless halo (III) — secondary components —, in this volume.

D. Müller et al., Electron and positron energy spectra: HEAT magnet spectrometer results, 26th ICRC (Salt Lake City) 3, 49-52, 1999.

J. Nishimura et al., Astrophysical significance of the confinement time of primary electrons in the Galaxy, 24th ICRC (Rome) 3, 29-32, 1995.

S. Orito et al. (BESS collaboration), Precision measurement of cosmic-ray antiproton spectrum, Phys. Rev. Lett. 84, 1078-1081, 2000.

T. Shibata (II), Analytical solution of 3D cosmic-ray diffusion in boundaryless halo (II) — two component model —, in this volume.

N.E. Yanasak et al., Abundances of the cosmic ray β-decay secondaries and implications for cosmic ray transport, AIP Conf. Proc. 528, 402-405, 2000.