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Distribution in energies and acceleration times in DSA, and their effect on the cut-off

A. Brooks and R. J. Protheroe

Department of Physics and Mathematical Physics, The University of Adelaide, Adelaide, SA 5005, Australia

Abstract. We have conducted Monte Carlo simulations of diffusive shock acceleration (DSA) to determine the distribution of times since injection taken to reach energy $E > E_0$. This distribution of acceleration times for the case of momentum dependent diffusion is compared with that given by Drury and Forman (1983) based on extrapolation of the exact result (Toptygin 1980) for the case of the diffusion coefficient being independent of momentum.

As a result of this distribution we find, as suggested by Drury et al. (1999), that Monte Carlo simulations result in smoother cut-offs and pile-ups in spectra of accelerated particles than expected from simple "box model" treatments of shock acceleration (e.g., Protheroe and Stanev 1999, Drury et al. 1999). This is particularly so for the case synchrotron pile-ups, which we find are replaced by a small bump at an energy about a factor of 2 below the expected cut-off, followed by a smooth cut-off with particles extending to energies well beyond the expected cut-off energy.

1 Introduction

The spectrum of cosmic rays and the observation of non-thermal radiation with power-law spectra from many astrophysical objects clearly demonstrates the existence of power-law spectra of ultra-relativistic particles in nature. Diffusive shock acceleration at astrophysical shocks is currently the favoured mechanism for accelerating these particles (see e.g. Drury 1983 for a review).

Clues to the locations and physical conditions of the acceleration region may come from studying the shape of the cut-off in the spectrum of accelerated particles, both directly (if possible) or indirectly through the observation of the non-thermal radiation produced by the accelerated particles. The shape of the cut-off in the spectrum of accelerated particles also influences the shape of the cut off in the spectrum of radiation produced. The cut-off in the spectrum of accelerated particles may be produced by the finite size of the acceleration region, by the time available for acceleration, or by interactions of the particles with the environment of the acceleration region. For example, the spectrum could be cut-off due to synchrotron losses (a continuous process) or by inverse-Compton scattering in the case of electrons (a discontinuous process in the Klein-Nishina regime) or in the case of protons by Bethe-Heitler pair production (approximately continuous) or pion photoproduction (discontinuous).

Correspondence to: rprother@physics.adelaide.edu.au

To accurately treat discontinuous energy losses (interactions), i.e. taking into account fluctuations, requires a Monte Carlo treatment. Full Monte Carlo simulations of shock acceleration are, however, very time-consuming and so including a Monte Carlo of particle interactions compounds this problem. This motivated Protheroe and Stanev (1999) to construct a simple "box model" for acceleration that enabled a very quick and easy Monte Carlo treatment of the shock acceleration. Using this approach, Protheroe and Stanev (1999) showed that for the case of inverse-Compton scattering in the Klein-Nishina regime the discontinuous energy losses resulted in a smooth rather than a sharp cut-off with a pile-up that would occur if inverse-Compton scattering were treated as a continuous process.

Drury et al. (1999) pointed out an inaccuracy in the proposed box model for the case where the diffusion coefficient was not independent of energy, and this correction slightly reduces the pile-up in the case of continuous energy losses. Drury et al. also suggested that the non-continuous nature of the acceleration would also have a role in shaping the cut-off in the spectrum of accelerated particles. In this paper, we investigate this effect and show to what extent the shape of the cut-off is changed.

2 Simulation description and other models

We use the Monte Carlo method to simulate individual relativistic particles undergoing diffusion in the region of a non-relativistic astrophysical shock (we adopt a shock velocity of $0.033c$, and a ratio of specific heats of 5/3). The treatment of propagation and scattering was carried out in the rest frame of the local plasma. The position of the shock in both frames was known at all times, and when the particle crossed the shock its 4-position and 4-momentum were Lorentz transformed to the new plasma rest frame. Particles of some initial energy E_0 were injected slightly upstream of the shock and each particle was propagated along a random walk. After travelling a distance sampled from an exponential distribution with a given mean free path, particles were scattered elastically and isotropically in the frame of reference of whichever region they were in. The scattering mean free path was determined by some dependence on the particle's energy and the average magnetic field strength. For example, if a highly turbulent magnetic field was assumed, the scattering mean free path could be set to the gyroradius (Bohm diffusion); in this paper we adopt Bohm diffusion. If, at any time, a particle crossed the shock from upstream to downstream, or vice versa, it's position and momentum would be Lorentz transformed to the new frame of reference, so that it could still scatter elastically and isotropically.

When a particle diffused far enough downstream away from the shock such that return to the shock was very unlikely, it was considered to have escaped. The distance downstream to the point of escape was proportional to the mean free path of the particle. The final energy spectrum for shock acceleration was determined from these escaping particles. In addition to this basic simulation, synchrotron energy losses were simulated by subtracting the appropriate amount of energy from a particle after each step of the random walk.

A "box model" for acceleration (e.g. Protheroe and Stanev 1999) examines shock acceleration by injecting particles of some initial energy E_0 into a box representing the shock region. The box is characterised by the rate of acceleration whilst inside, $r_{acc}(E)$, and the rate of escape of particles from the box, $r_{esc}(E)$. Inside the box then, particles gain energy at a rate $dE/dt = Er_{acc}(E)$, and have a probability of escape in a time interval Δt of $\Delta t r_{esc}(E)$. A Monte Carlo simulation can be performed for many particles using these two rates. Note that the time taken to reach a particular energy will be the same for all particles, and so box models do not simulate the fluctuations in acceleration time that would occur in nature.

When synchrotron emission is also considered in a box model, it has a rate of energy loss given by bE^2 and the model predicts a precise cut-off energy E_{cut}. This is the energy at which the rate of energy gain, $Er_{acc}(E)$ is equal to the rate of energy loss. The rate of energy gain is given by $Er_{acc}(E) = aE^{1-\delta}$ where δ determines the energy dependence of the mean free path and also the diffusion coefficient, $\kappa \propto E^{\delta}$. This gives

$$E_{\rm cut} = \left(\frac{a}{b}\right)^{\frac{1}{1+\delta}} \tag{1}$$

where a, b and δ are constants. This precise cut-off energy leads to a final energy spectrum with a pile-up in the form of a large spike at the cut-off energy, see Protheroe and Stanev (1999).

Drury et al. (1999), suggested a similar model which could be solved analytically using momentum, p, and momentum distribution, $f(p)$, in a diffusion advection equation. The acceleration due to the shock gives an upward flux in momentum space. The advection of particles away from the shock and out of the "region of interest", or the box, represents a loss, with the box size depending on energy. The introduction of synchrotron emission caused a downward flux in momentum space and also introduces an additional loss process. Particles can now be lost by falling out of the end of the box. This will happen if the size of the box, which decreases with decreasing momentum, becomes so small that its edge moves upstream of the particle. This loss is in addition to the usual advection losses.

Drury et al. (1999) obtained a relatively simple result for the momentum distribution which could be used to find the final energy spectrum of escaping particles and also provided a condition necessary for a pile-up to occur, namely if

$$U_1 - 4U_2 + \delta(U_1 - U_2)\frac{L_1}{L_1 + L_2} > 0 \tag{2}$$

where U_1 and U_2 are the speeds of the upstream and downstream flows, respectively, in the shock's frame of reference, L_1 and L_2 are the sizes of the box on either side of the shock, and δ determines the energy dependence of the diffusion coefficient κ on momentum, $\kappa \propto p^{\delta}$. They, like Protheroe and Stanev (1999), also find that there is a precise cut-off energy which leads to a sharp spike in the final energy spectrum. They acknowledge that this is due to the assumption that all particles gain energy at the same rate, when, in fact, the rate at which particles gain energy fluctuates considerably and they predict that these fluctuations will have the effect of smoothing the spike in the final energy spectrum. However, they predict the spectrum should still show some local enhancements when compared to a spectrum in which no energy losses are considered.

Individual particles are accelerated at different and non-constant rates in the region of a shock. The times that particles take to be accelerated to a particular energy can be shown in an acceleration time distribution, $\zeta(t, E_0, E_1)$. The distribution is implicitly defined using a non-trivial Laplace transform which depends on the diffusion coefficients, Drury (1991). If the diffusion coefficients are independent of momentum, an exact solution for the distribution, Eq. 3 below, can be found, Toptygin (1980). Forman and Drury (1983) suggested using this exact solution

$$\zeta(t) = \frac{1}{\sqrt{2\pi c_2}}\left(\frac{t}{c_1}\right)^{-3/2} \exp\left[\frac{-c_1(t-c_1)^2}{2tc_2}\right] \tag{3}$$

as an approximation to a general acceleration time distribution with the mean, c_1 and the variance, c_2, determined by

$$c_1 = \int_{p_0}^{p_1} \frac{dp}{p}\frac{3}{U_1 - U_2}\left[\frac{\kappa_1(p)}{U_1} + \frac{\kappa_2(p)}{U_2}\right] \tag{4}$$

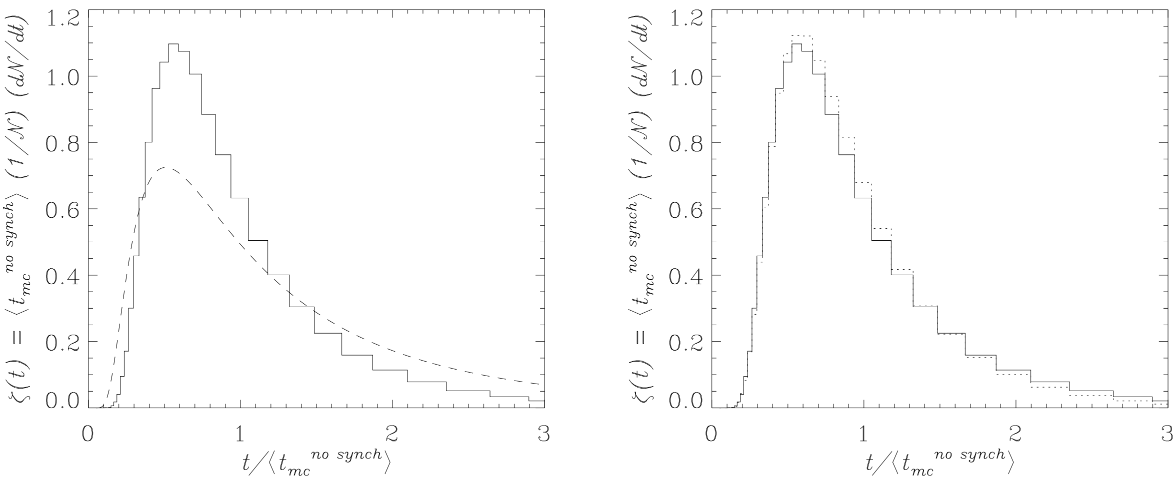

Fig. 1. Acceleration time distribution for particles remaining in the region of the shock, accelerated from the injection energy E_0 to $10E_0$. The left panel is for the case of no energy losses: the histogram is the result from the Monte Carlo simulation and the dashed line is the approximation provided by Eq. 3 from Forman and Drury (1983). The right panel shows the effect of including synchrotron losses. The solid histogram is for no losses (identical to histogram in left panel), and the dotted histogram is for the case where synchrotron losses are included and $E_{\rm cut} = 10E_0$.

$$c_2 = \int_{p_0}^{p_1} \frac{dp}{p} \frac{6}{U_1 - U_2} \left[\frac{\kappa_1(p)^2}{U_1^3} + \frac{\kappa_2(p)^2}{U_2^3} \right] \tag{5}$$

In our simulation particles spend varying times in the upstream and downstream regions between shock crossings and are accelerated at different and fluctuating rates. We should therefore expect the simulation to produce an acceleration time distribution in general agreement with the approximation above. Since a distribution of acceleration times is expected, we should also expect a smoothing of the spike in the escaping energy spectrum for which synchrotron losses are included, as suggested by Drury et al. (1999).

3 Results

We were able to obtain acceleration time distributions from our Monte Carlo simulations. Figure 1 shows the acceleration time distributions for particles accelerated to an energy equal to ten times the injection energy, using the mean free path proprtional to energy. The time is normalized to the mean acceleration time for the Monte Carlo simulation with no losses. In the left panel the results from the simulation are plotted as a histogram and compared with the approximations provided by Forman and Drury (1983), Eq. 3. While the shape of the distributions appear similar, the distribution from the Monte Carlo simulation is significantly narrower than given by the approximation. In the right panel we show the effect of synchrotron losses on the time distribution for the case where the expected cut-off energy is ten times the injection energy. The distribution is compared with that for no synchrotron losses given in the left panel, and found to be almost identical.

Figure 2 shows the final energy spectrum for escaping particles for the case of a synchrotron cut-off. The histogram is the result from the simulation. Plotted for comparison are the spectra predicted using box models by Protheroe and Stanev (1999) and by Drury et al. (1999) which give a pile-up in accordance with the condition provided by Drury et al. (1999) in Eq. 2. Energies of particles in the Monte Carlo simulation clearly extend beyond the nominal cut-off energy, and the spectrum does not exhibit the expected sharp spike. Instead there is a minor bump in the spectrum at roughly half the the nominal cut-off energy.

4 Conclusion

Our simulation bears out the suggestion made by Drury et al. (1999) that box models are not capable of accurately describing cut-offs and pile-ups in shock acceleration spectra. The spread of acceleration times (Fig. 1) results in the smoothing of the sharp pile-up predicted by box models of shock acceleration.

This result is clearly understood, and perhaps is not surprising. Indeed, Protheroe and Stanev (1999) actually showed that the sharp pile-up disappeared when one of the two processes (acceleration and energy loss) became non-continuous. This was demonstrated for the case of continuous energy gains but discontinuous energy losses in the case of inverse-Compton scattering in the Klein-Nishina regime. The results we obtain here for the case of continuous synchrotron energy losses and discontinuous energy gains are qualitatively very similar.

We conclude that a full treatment of the acceleration pro-

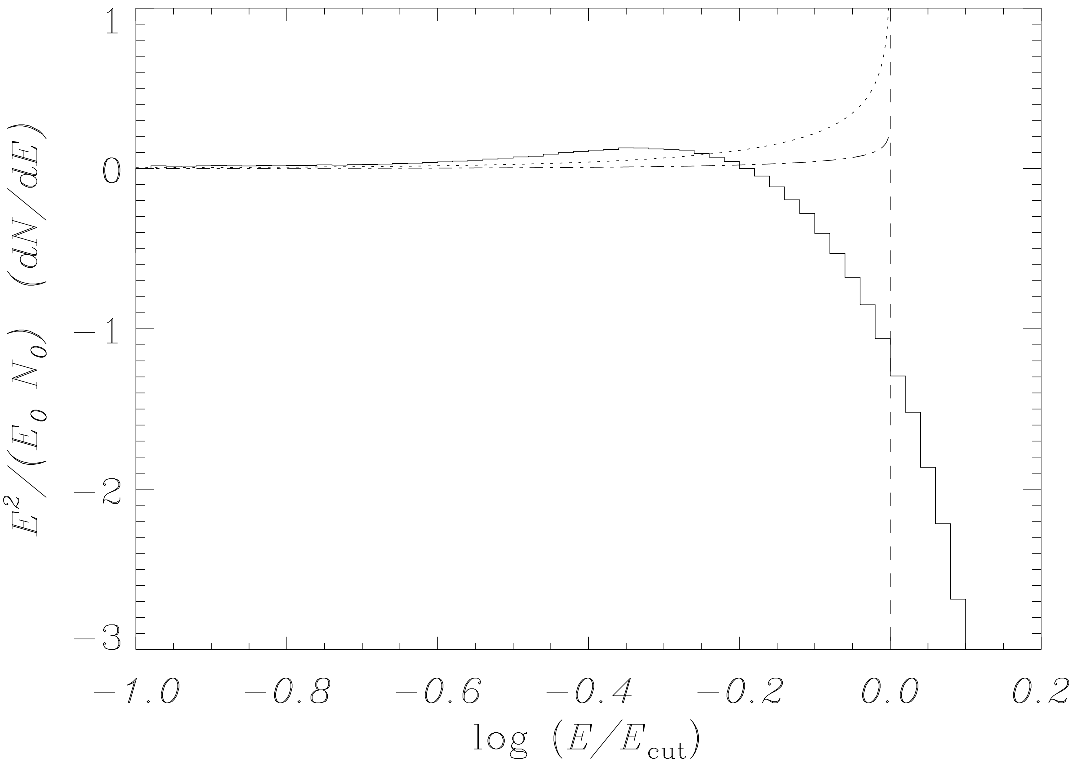

Fig. 2. Final energy spectrum for escaping particles from shock acceleration including synchrotron emissions. The histogram is the result from the simulation, the dotted line the prediction from Protheroe and Stanev (1999), and the dot-dash line the prediction from Drury et al. (1999). The vertical dashed line represents the theoretical cut-off energy from the latter two models.

cess, including effects of the distribution in acceleration times, is necessary to accurately predict the shape of the cut-off in the spectrum of accelerated particles. While such a treatment may be possible analytically for the case of continuous energy losses, in the case of discontinuous energy losses (interactions) a Monte Carlo method is probably necessary as shown by Protheroe and Stanev (1999). The problem of the time-consuming nature of accurate Monte Carlo simulations of the acceleration process is addressed in a separate paper in these proceedings (Protheroe 2001) in the case of relativistic shock acceleration, but the techniques discussed there can be applied also to non-relativistic shocks.

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