Proceedings of ICRC 2001: 1013 © Copernicus Gesellschaft 2001

ICRC 2001

Muon flux simulation and comparison with Fréjus measurements

F. Schröder, W. Rhode, and H. Meyer

Fachbereich Physik, Universität Wuppertal, Gaußstr. 20, 42097 Wuppertal, Germany

Abstract. In order to test current air shower and muon propagation codes a simulation of the underground muon flux was performed. The atmospheric muon flux was computed with CORSIKA. To compare the results with Fréjus measurements different methods for the muon propagation through rock were applied. Results of the simulated atmospheric and underground muon flux are presented.

1 Introduction

Underground experiments like AMANDA with the goal to measure extraterrestrial neutrinos have to determine their background very precisely. Most of the measured signal is induced by high energetic atmospheric muons. To get a good data description simulations of the underground muon flux are performed by applying propagation codes for the atmospheric muon's passage through matter.

In order to test these tools simulations of the atmospheric and underground muon flux for the location of the Fréjus detector were done. Together with the measured Fréjus data set a very detailed comparison was possible.

The atmospheric muon flux was simulated with the CORSIKA air shower code. Calculating the energy loss in rock was done with four different Monte Carlo propagation codes (MUDEDX, PROP-MU, MUM and MMC) and a simple analytical approximation. The detector behaviour was taken into account by applying its acceptance and other geometric properties.

2 The Fréjus Data Set

The Fréjus detector was located under the Pte de Fréjus ($45^\circ\,8'\,32''$ N and $6^\circ\,41'\,21''$ E) in the Alps (France) 1260 m a.s.l. and at least 1450 m under the rocks surface. Geological examinations resulted in a homogeneous rock structure with a density of 2.74 g/cm^3.

Correspondence to: F. Schröder
(schroeder@physik.uni-wuppertal.de)

The detector with a physical dimension of $6 \cdot 6 \cdot 12.3 m^3$ was a high resolution calorimeter consisting of 114 modules. Each module was built up by eight layers of flash chambers (each with 1024) between two layers of Geiger tubes (each with 352).

The detector's lateral resolution was 0.7 cm and the angular resolution 0.4°. A detailed description of the detector is given in (Berger, 1987).

The measurements were made between February 1984 and September 1988 (1.03$\cdot 10^8$ s). The data set consists of 481817 single muon events above an energy threshold of 300 MeV at the detector location. Due to the good angular resolution a sky map of event rates with $1^\circ \times 1^\circ$ bins was constructed. Together with a depth map of the rock overburden, taken from stereographic pictures made during the D-1 mission space shuttle flight, the depth spectrum was measured. Using an analytical approximation for the energy loss in rock (see section 3.2) the surface energy spectrum could be extracted.

3 Computer Simulations

3.1 Atmospheric Muons

Computing the atmospheric muon flux was performed with CORSIKA (Heck, 1998) (version 5.9451) with its new *CURVED* (Heck and Schröder et al., 1999) and *VOLUMEDET* option. This allows to simulate an isotropic primary distribution up to zenith angles of 89°. The hadronic interactions are modelled by GHEISHA and QGSJET. Since all muons coming out of QGSJET are produced by pions or kaons, no prompt component had to be taken into account for later spectrum fits.

As input the chemical composition of cosmic rays as listed in (Wiebel and Biermann, 1998) was used. The fitted spectra of this compilation have uncertainties in the flux normalisations (at 1 TeV) in the order of a few percent and in the differential spectral indices smaller than one percent.

For simulating the composition a modification was implemented in the CORSIKA code. It handles primary spectra from H to Fe and mixes them according to their portion of the integral allparticle flux (Chirkin and Rhode, 1999).

To fit the differential atmospheric muon spectrum a formula given in (Gaisser, 1990) was used:

$$\frac{dN}{dE_\mu} \sim E_\mu^{-\gamma} \cdot \left(\frac{1}{1+\frac{1.1 E_\mu \cos\theta^*}{115 GeV}} + \frac{0.054}{1+\frac{1.1 E_\mu \cos\theta^*}{850 GeV}} \right)$$

Corrections for possible energy loss in the atmosphere were done with $\Delta E = (0.25 \frac{GeV}{mwe} + 3.5 \cdot 10^{-4} \frac{E_\mu}{mwe}) \cdot \Delta X$ and muon decay was taken into account by weighting with $P_\mu^{-1}(e_\mu) = \exp\left(\frac{l \cdot m_\mu}{c \cdot \tau_\mu \cdot E_\mu}\right)$. In these formulae θ^* is the zenith angle at production, ΔX is the slant depth and l the geometric path between the point of production and the rock's surface, c is the speed of light and m_μ, τ_μ and E_μ are the mass, lifetime and energy of the muon. The fit result for the differential energy index of the simulated atmospheric muon flux was γ = 3.73 $\pm$ 0.01, which is in good agreement with measurements.

3.2 Muon Propagation through Matter

3.2.1 Analytical Approximation

The differential energy loss of a muon with energy E crossing a depth X could be generally written as a sum of losses induced by basic interactions (ionisation, Bremsstrahlung, pair production, knock-on and inelastic interactions). The easiest method to obtain an analytical description is to fit a linear function to the general solution:

$$\frac{dE}{dX} = a + b \cdot E$$

The parameters a = 0.217 GeV/mwe and b = 4.12 $\cdot$ 10^{-4} mwe^{-1} used here were derived from Fréjus data(Rhode, 1993). Since the parameter a is mainly given by ionization and the Bethe-Bloch-formula, b describes a spectra weighted mean of the energy dependent interactions.

Using this analytic model the minimal energy a muon needs to reach the detector (cut energy) is

$$E_{cut} = \frac{a}{b} \cdot (exp(\rho \cdot t \cdot b) - 1)$$

with the geometric rock depth t and the rock density ρ. Since the muon flux follows a power law I = $I_0 \cdot E^{-\gamma}$ it is obvious that it exists a strong correlation between the rock's depth and density, the parameter b, which describes the physics input, and the spectral index.

3.2.2 Monte Carlo Propagation Codes

The muon energy loss Monte Carlo code MUDEDX uses cross sections published by (Lohmann et al., 1985). As external input the tool needs mainly a typical energy at which the energy loss parameters are fixed. Unfortunately the output is very sensitive to this typical energy so that the prediction on the muon spectrum made by this code in not better than the analytical approximation where one has to adjust the parameter b. Since the mean energy of muons reaching the Fréjus detector was 250 GeV, typical energies of 100 GeV and 500 GeV for all muons in rock were used.

Another attempt using mainly the Lohmann physics input is the muon propagation code PROP-MU(Lipari and Stanev, 1991) (this work uses version 2.0 (February 1993)). Since the program was designed and optimized for underground applications the muon propagation is much faster and no external typical energy is needed. Radiative processes above a relative energy threshold of 0.01 are treated stochastically. It exists the possibility to choose between different screening functions for Bremsstrahlung and implementations of the energy loss by pair production, but they have no significant impact on the reconstructed spectrum. A special feature of this program is the 3-dimensional simulation of the muon tracks, so that lateral and angular deviations of muons at propagation through matter could be obtained. Since all propagation codes were just used as point-like energy loss generators, this possibility was not used.

A recent Monte Carlo program for muons in media is MUM (version 1.2) (Sokalski et al., 2000)(Bugaev et al., 2000). This propagation tool uses the most recent improvements for cross sections, so that there is not just technically but also physically a difference to the former programs. One may choose between different parametrizations for the photonuclear interaction and give an absolute and relative lower threshold for the energy transfer to be handled stochastically. For the used simulations the cuts between stochastical and continues treatment were fixed to 0.01 GeV absolute and 20 % relative energy transfer.

The muon Monte Carlo MMC was developed in 2000 by (Chirkin and Rhode, 2001). It has basically the same physics input (Rhode and Cârloganu, 1999)as the new MUM code, but has a total different approximation and tracking algorithm and was written in an object-oriented programming language (JAVA). These enables easy physical or technical additions and changes. Since the program was mainly designed for the use in the Massively Parallel Network Computing (SYMPHONY) (Winterer, 1999) computational speed was no primary issue and the original cross-section integrals were used. In order to speed up the tool parametrization and interpolation routines were implemented with an error precision smaller than the use of the original cross-section-formulae with an uncertainty of $\approx$ 1 %. This propagation code offers also the possibility to choose the lower threshold of the relative energy transfer to be handled stochastically. It was set to 0.2 to be consistent with MUM.

3.3 Detector simulation

In order to get a good agreement with Fréjus data an geometric detector simulation was applied. This included stochastical acceptance weighting for the muon direction and cuts on the quality of the used rock map. The lateral event scattering was done with the use of a matrix of independent detectors. Just single muon events per detector were accepted. Considering the multi-use of air showers each muon was weighted with the inverse number of used single muon subevents.

4 Results

In order to discuss the results, data and simulated data samples were fitted with a solution given in (Gaisser, 1990):

$$I_{normal}(> E_\mu) \ \sim \ \sec\theta^* \ \frac{E_\mu^{-\gamma}}{\gamma(\gamma+1)}$$

$$I_{prompt}(> E_\mu) \ \sim \ \frac{E_\mu^{-\gamma+1}}{\gamma-1}$$

$$I(> E_\mu) \ = \ I_{normal}(> E_\mu) \ + \ I_{prompt}(> E_\mu)$$

In this formulae E_μ is the cut energy computed with the analytical approximation, γ is the integral spectral index and θ^* is the zenith angle at production. The fit was done between 4 kmwe and 6 kmwe (where $\theta^* \le 60°$ and this solution is valid). Since QGSJET does not produce prompt muons, the simulated samples were just fitted with the normal flux component originated from pions and kaons. It should be mentioned, that E_μ, γ and b are just parameters, which were defined to compare simulations with experimental data.

In Figure 1 the measured depth spectrum is shown. As an example a simulated spectrum using the analytical approximation is plotted in the same frame. Figure 2 gives an overview of depth spectra deviations for different energy loss methods.

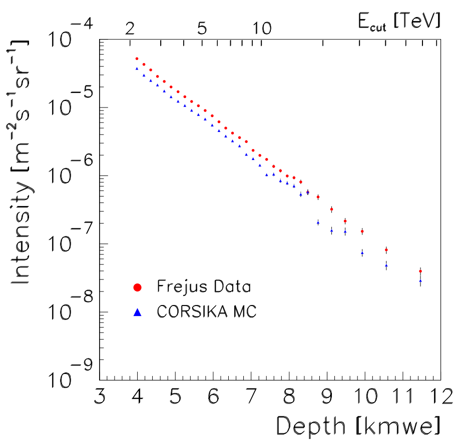

Fig. 1. Reconstructed integral muon spectrum for experimental data and simulation. Calculating the energy loss in the Fréjus rock was done by applying the simple analytical approximation.

Comparisons between different propagation codes are made in Table 1. All fitted integral spectral indices and their relative deviation to the experimental data are shown. Also fit results of the relative deviation are presented.

All simulated samples have a large shift of at least 30 % relative to the experimental data set. Estimating the systematic error for the simulated flux amplitude one has to regard the uncertainties given in (Wiebel and Biermann, 1998):

– Absolute normalization of the allparticle flux at 1 TeV: $\Delta F_0 = 1.63 \cdot 10^{-2}$ (m^2 s sr TeV / nucleus)$^{-1}$ ($\sim$ 6.34%)

Sample	Integral Flux: Spectral Index	Integral Flux: Rel. Dev. %	Deviation: shift %	Deviation: slope % / kmwe
Exp. Data	-2.75 $\pm$ 0.01	–	–	–
Approx.	-2.67 $\pm$ 0.01	-2.9	-29.7	2.8
MUDEDX	-2.39 $\pm$ 0.01	-13.1	-30.2	13.7
MUDEDX2	-2.76 $\pm$ 0.01	+0.4	-62.2	-0.2
PROP-MU	-2.54 $\pm$ 0.01	-7.6	-33.4	7.1
MUM	-2.50 $\pm$ 0.01	-9.1	-24.7	9.9
MUM2	-2.61 $\pm$ 0.01	-5.1	-47.9	3.6
MMC	-2.47 $\pm$ 0.01	-10.2	-29.2	10.4

Table 1. Results of different energy loss methods. The integral spectral indices are determined for depths $\le$ 6000 mwe. Fitting the deviation with a linear function between 4000 mwe and 6000 mwe delivers a shift (at 4000 mwe) and a slope for this depth range.

– Differential spectral index of the allparticle flux: $\Delta\gamma$ = 0.03 ($\sim$ 1.12 %)

Using error propagation one gets an relative uncertainty of $\approx$ 7.5%. Comparing this result with the observed shift there remains a large discrepancy. Its source might lie in the used cosmic ray composition, in CORSIKA respectively in QGSJET or in an unknown experiment specific error. Since (Desiati et al., 2001) got comparable shifts for AMANDA simulations, the main uncertainty might lie in CORSIKA or in the used chemical composition.

The best description of the experimental spectral index was reached by using the simple approximation. The deviation of 3 % might partly be explained by uncertainties concerning the composition input and the CORSIKA simulation. Another component could result from a small experiment specific error.

MUDEDX with 100 GeV typical energy gives an spectral index of -2.39 corresponding to a relative deviation of $\approx$ 13 %. In order to demonstrate the strong typical energy dependency MUDEDX was also used with a typical energy of 500 GeV (marked with MUDEDX2 in Table 1). There one gets a good agreement in the spectral index (0.4 % deviation), but a huge relative shift in the absolute muon flux (62 %). PROP-MU delivers the smallest difference in the spectral index (7.1 %). Using MUM one gets a spectral index (-2.50) lying between the results of MUDEDX and PROP-MU. To show the strong correlation between the rock's density and depth, the physics input and the spectral index there was also produced a sample with an unphysical rock density of 2.95 g/cm^3 (marked with MUM2 in Table 1). The reconstruction of the integral energy spectrum was done as usual with a density of 2.74 g/cm^3. The higher rock density results in a higher spectral index (-2.61) and thus in smaller deviation (5 %). Applying MMC to the atmospheric muon sample gives an spectral index (-2.47) nearby MUM .

Comparing these spectral indices one gets the following impression. PROP-MU and MUDEDX define the lower and upper edge of the interval. The results of MUM and MMC

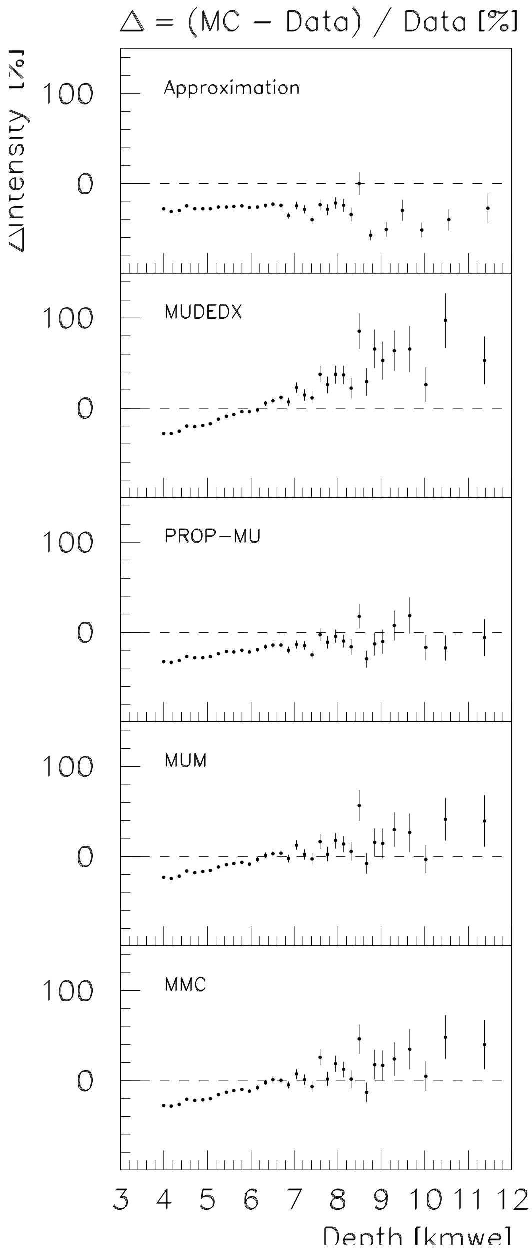

Fig. 2. Deviations in the depth spectra between experimental data and simulations applying different energy loss methods.

are between them. PROP-MU and MUDEDX use older parameterizations of cross sections than the recent codes MUM and MMC, but all Monte Carlo methods use different approximation and tracking algorithms. It seems that the systematic error in the physics implementation (parameterization, interpolation and tracking) became smaller and at least a part of the deviation which is given by MUM and MMC relative to the data sample is introduced by the physics input.

5 Conclusions

After comparing the results of different simulations with Fréjus data, two problems have to be solved. The first one affects the absolute muon flux. The remaining deviation of 30 % might stem from a larger uncertainty in the primary composition than given in (Wiebel and Biermann, 1998). Another possibility would be uncorrect primary interactions in CORSIKA respectively in QGSJET. The implemented primary interactions might simulate a wrong inelasticity in very forward particle direction. A possible experiment specific error should be small, since there exists a similar shift in AMANDA simulations relative to data.

The second problem concerns the reconstructed spectral index. The deviations of ≈ 10 % could not be explained with a wrong input spectrum, since the spectral index of the atmospheric muon flux simulated with CORSIKA is nearby the experimental result at the surface. The main uncertainty has to be in the used propagation codes or in experiment specific parts. Since AMANDA simulations show similar results, an experiment specific error would just contribute a small fraction. The substantial part was introduced by the propagation codes as shown by the wide spread of spectral indices. Since the two recent codes MUM and MMC produced similar results, it seems that now the main fraction of the deviation has its origin in the used cross section formulae. The energy loss of propagating muons with hundreds of interactions along their way may be sensitive to the claimed uncertainties of 1 % or to a systematic error in the formulae. The fact, that simulations in ice (AMANDA) and rock (Fréjus) give similar relative deviations, might be a clue for solving these problems.

Acknowledgements. Thanks to D. Chirkin for preparing the MMC code to the special needs of the Fréjus data simulation and to P. Desiati for the newest results concerning the AMANDA muon flux analysis.

References

Berger, Ch. et al., NIM A262, 463, 1987
Bugaev, E.V., Sokalski, I.A., Klimushin, S.I, *hep-ph/0010323*, 2000
Chirkin, D., Rhode, W., Proc. 26^{th} ICRC, Salt Lake City, 2, 16, 1999
Chirkin, D., Rhode, W., these proceedings, HE.2.2
Desiati, P., et al., these proceedings, HE.2.2
Gaisser, T.K., Cosmic Rays and Particle Physics, Cambridge 1990
Heck, D. et al., FZKA 6019, Forschungszentrum Karlsruhe, 1998
Heck, D., Schröder, F. et al., Proc. 26^{th} ICRC, Salt Lake City, 1, 498, 1999
Lipari, P., Stanev, T., Phys. Rev. D 41, No.11, 3543-3554, 1991
Lohmann, W., Kopp, R., Voss, R., CERN 85-03, 1985
Rhode, W., PhD thesis WUB-DIS 93-11, Wuppertal, 1993
Rhode, W., Cârloganu, C., DESY-PROC-1999-01, 1999
Sokalski, I.A., Bugaev, E.V., Klimushin, S.I, *hep-ph/0010322*, 2000
Wiebel-Sooth, B., Biermann, P.L., *Chapter 7.6: Cosmic Rays*, Landolt-Börnstein, New Series, vol. VI 3C, p. 37 - 90, 1998
Winterer, V.-H., SYMPHONY (talk)