Proceedings of ICRC 2001: 3334 © Copernicus Gesellschaft 2001

ICRC 2001

Solar CR kinetics in the radial regular IMF

Yu. Fedorov[1] **and M. Stehlik**[2]

[1]MAO NASU, Kiev, Ukraine
[2]IEP SAS, Košice, Slovakia

Abstract. Theory of cosmic ray propagation in the radial regular magnetic field with superimposed magnetic irregularities is advanced. The particles can be divided into two parts which describe the unscattered and scattered populations. In result, the unscattered particles appear as a strongly anisotropic beam with a small angular width due to the magnetic focusing of charged particles in radial regular magnetic field. Evolution of the angular distribution of both populations and the temporal profiles of CR intensity are analyzed. The obtained results can be used for description of energetic particle streams in the turbulent interplanetary magnetic field.

1 Introduction

The rigorous treatment of cosmic ray (CR) propagation in the interplanetary medium is based on kinetic equation describing energetic charged particle scattering by magnetic irregularities and their focusing by regular interplanetary magnetic field (IMF). Larmor radius of energetic particles accelerated in solar flares is small compared to typical scale of regular magnetic field variations, so we can use the drift approximation of kinetic equation, (see, for example, Toptygin, 1985). In this paper we consider the propagation of CR in the radial regular IMF. In this approximation the focusing legth is inversely proportional to heliocentric distance and kinetic equation takes the following form:

$$\frac{\partial f}{\partial t} + v\mu\frac{\partial f}{\partial r} + \frac{v(1-\mu^2)}{r}\frac{\partial f}{\partial \mu} + \nu_s f - \frac{\nu_s}{2}\int_{-1}^{1} d\mu\, f = Q, \quad (1)$$

where f is CR distribution function, μ is particle pitch angle cosine, ν_s is a collision frequency of particles with magnetic irregularities. Two last term in the left hand side of (1) present the collision integral describing distribution function variation caused by particle scattering. The right hand side of the kinetic equation stands for particle source which is supposed to be instantaneous isotropic and placed at heliocentric distance r_0. It is useful to introduce the dimensionless variables

$$\rho = r/\Lambda, \tau = vt/\Lambda\,, \quad (2)$$

where $\Lambda = v/\nu_s$ is the particle mean free path. In these variables we can rewrite Eq. (1) as follows

$$\frac{\partial f}{\partial \tau} + v\mu\frac{\partial f}{\partial \rho} + \frac{1-\mu^2}{\rho}\frac{\partial f}{\partial \mu} + f - \frac{1}{2}\int_{-1}^{1} d\mu\, f = \frac{\delta(\rho-\rho_0)\delta(\tau)}{16\pi^2\Lambda^3\rho^2}\,. \quad (3)$$

The solution of kinetic equation (1) can be obtained by using Fourier - Laplace transform (Fedorov et al., 1995; Fedorov and Stehlik, 1999). The distribution function appears as a superposition of unscattered and scattered particles (f_0 and f_s, respectively):

$$f(\rho,\mu,\tau) = f_0(\rho,\mu,\tau) + f_s(\rho,\mu,\tau)\,. \quad (4)$$

The distribution function of unscattered particles has the form

$$f_0(\rho,\mu,\tau) = \frac{\delta\{\zeta(\tau)-\rho_0\}\exp(-\tau)}{16\pi^2\Lambda^3\rho_0\zeta(\tau)}\,, \quad (5)$$

with

$$\zeta(\tau) = \sqrt{\rho^2+\tau^2-2\rho\mu\tau}\,. \quad (6)$$

According to Eq. (5) the unscattered particles are present in position ρ only in the time interval $\rho-\rho_0 < \tau < \rho+\rho_0$, and their angular distribution is drastically anisotropic, so in the instant of time τ the particle pitch angle cosine is given by the expression

$$\mu = \frac{\rho^2+\tau^2-\rho_0^2}{2\rho\tau}\,. \quad (7)$$

Note that in the position ρ the unscattered particles can move only with $\mu > \mu_{min} = \sqrt{1-(\rho_0/\rho)^2}$.

Correspondence to: M. Stehlik (stehlik@saske.sk)

The distribution function of scattered particles in the radial regular IMF has been obtained by Fedorov and Stehlik (1999). In the present paper we use this function under condition $\tau > \rho + \rho_0$. Then only scattered particles in the position ρ are present and, the distribution function $f_s(\rho, \mu, \tau)$ has the form

$$f_s = \frac{1}{8\pi^3\Lambda^3\rho_0} \int_0^\tau \frac{d\xi}{\zeta(\xi)} \int_0^{\pi/2} dk\, \frac{k^2 \sin(k\rho_0)\sin(k\zeta)}{\sin^2 k}$$

$$\times\ \exp[-\xi k \cot k - \tau(1 - k\cot k)] + \frac{\exp(-\tau)}{64\pi^3\Lambda^3\rho_0} \tag{8}$$

$$\times \left\{ \int_0^{\zeta_1} d\xi \int_0^{\frac{|\zeta(\xi)-\rho_0|}{\tau-\xi}} d\eta\, \Phi_1(\xi,\eta) \ + \int_{\zeta_1}^{\tau} d\xi \int_0^1 d\eta\, \Phi_1(\xi,\eta) \right.$$

$$\left. + \int_0^{\zeta_2} d\xi \int_0^{\frac{|\zeta(\xi)+\rho_0|}{\tau-\xi}} d\eta\, \Phi_2(\xi,\eta) \ + \int_{\zeta_2}^{\tau} d\xi \int_0^1 d\eta\, \Phi_2(\xi,\eta) \right\},$$

where

$$\zeta_{1,2} = \frac{(\tau \pm \rho_0)^2 - \rho^2}{2(\tau - \rho\mu \pm \rho_0)}$$

$$\Phi_i(\xi,\eta) = \exp\left[\frac{\kappa z_i}{2}\right] \{2\pi\kappa\, \cos\frac{\pi z_i}{2} + (\kappa^2 - \pi^2)\sin\frac{\pi z_i}{2}\},$$

$$\kappa = \ln\frac{1-\eta}{1+\eta}; \qquad z_{2,1} = \mid \zeta(\xi) \pm \rho_0 \mid -(\tau - \xi)\eta\,.$$

2 Discussion

The dependence of CR distribution function (8) on particle pitch angle cosine μ is illustrated on Fig. 1. It corresponds to $\rho = 1$ and $\rho_0 = 0.02$ and numbers near the curves stand for dimensionless time τ. Note that all curves correspond to the time when only scattered particles are present only at position $\rho = 1$. Our calculation shows that at this position the particle density decreases monotonically for $\tau > \rho + \rho_0$. The distribution function shown in Fig. 1 is normalized to its maximal value $f(\mu = 1)$. One can see that at $\tau = 1.1$ the distribution function is drastically anisotropic and particles are moving preferable along IMF, and the CR intensity of backward moving particles is negligible. The CR distribution function become more and more isotropic for increased time due to particle scattering by magnetic irregularities.

Let one consider the angular distribution of particles at a later instant when the distribution function is near to isotropic. In this case ($\tau \gg 1$) the main contribution to (8) is given by the first term because the others are proportional to the exponentially decreasing multiplier. The Eq. (8) can be simplified in this limit case when the source size is extremely small

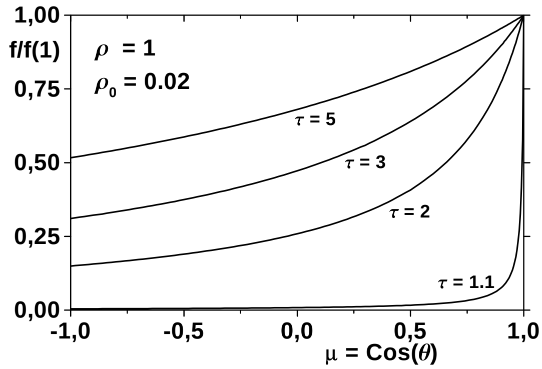

Fig. 1. The distribution function (8) normalized to $f(\mu = 1)$ on distance $\rho = 1$ at instant $\tau = 1.1, 2, 3,$ and 5.

($\rho_0 \to 0$) in comparison with heliocentric distance ρ:

$$f(\rho,\mu,\tau) = \frac{1}{8\pi^3\Lambda^3} \int_0^\tau \frac{d\xi}{\zeta(\xi)} \int_0^{\pi/2} dk\, \frac{k^3 \exp[-\xi k\cot k]}{\sin^2 k}$$

$$\times\quad \sin[k\zeta(\xi)]\ \exp[\tau(k\cot k - 1)]\,. \tag{9}$$

At late time ($\tau \gg 1$) the last exponential multiplier in integrand of (9) has a strong maximum at $k = 0$, so one can replace it by $\exp[-k^2\tau/3]$, and use sunstitution $k = 0$ in the first multiplier with subsequent integration over k up to the infinity (see Fedorov et al., 1995). Thus we can obtain the following expression for distribution function in the large time limits,

$$f(\rho,\mu,\tau) = \tag{10}$$
$$\frac{3}{32\pi\Lambda^3\tau} \exp\left[\frac{\tau}{3}\left(1 - \frac{3\rho\mu}{2\tau}\right)^2\right] \mathrm{erfc}\left[\sqrt{\frac{\tau}{3}}\left(1 - \frac{3\rho\mu}{2\tau}\right)\right].$$

Here one can use asymptotic expansion of $\mathrm{erfc}[z]$ under condition $z \gg 1$ in Eq.(10) resulting

$$f(\rho,\mu,\tau) = \tag{11}$$
$$\frac{3\sqrt{3}}{32\pi^{3/2}\Lambda^3\tau^{3/2}} \exp\left[-\frac{3\rho^2}{4\tau}\right]\left(1 - \frac{3\rho\mu}{2\tau}\right)^{-1}.$$

It is evident that well known diffusion approximation relationship for particle density and anisotropy follows from (10) for $\rho \ll \tau$.

Note interesting fact that in the case of multiple small angle scattering the angular distribution of particles at late times ($\tau \gg 1$) differs essentially from Eqs. (10), (11). The dependence of CRdistribution function on particle pitch angle is determined under small angle scattering by exponential function of μ because distribution function contains the term proportional to $\exp[\Lambda\mu/L]$ (so called focusing anisotropy of CR), (Earl, 1981; Beeck and Wibberenz, 1986; Bieber et al., 1986). The quantity L is the focusing length which is equal to $r/2$ in the considered approximation of radial magnetic field. Thus one can see that the angular distribution of solar CR contains valuable information about features of energetic particle scattering by magnetic field irregularities.

Acknowledgements. This work was partially supported by Slovak Academy of Sciences within Project No. 7232 and 1147.

References

Beeck, J. and Wibberenz, G., Pitch angle distribution of solar energetic particles and the local scattering properties of the interplanetary medium, Astrophys. J., 311, 437-450, 1986.

Bieber, J. W., Evenson, P. A. and Pomerantz, M. A., Focusing anisotropy of solar cosmic rays, J. Geophys. Res., 91, 8713-8724, 1986.

Earl, J. A., Analytical description of charged particle transport along arbitrary guiding field configurations, Astrophys. J., 251, 739-755, 1981.

Fedorov, Yu. I., Shakhov, B. A. and Stehlik, M., Non-diffusive transport of cosmic rays in homogeneous regular magnetic fields, Astron. Astrophys., 302, 623-634, 1995.

Fedorov, Yu. I. and Stehlik, M., Non-diffusive charged particle propagation in the radial IMF, Proc. 26th Intern. Cosmic Ray Conf., Salt Lake City, v. 6, 348-350, 1999.

Toptygin, I. N., Cosmic rays in the interplanetary magnetic fields, D. Reidel Publ. Co., Dordrecht, Holland, 1985.