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Cerenkov radiation of cosmic ray extensive air showers. Part 3. Longitudinal development of showers in the energy region of $10^{15} \div 10^{17}$ eV

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Abstract. The longitudinal development of showers in the energy region of $10^{15} \div 10^{17}$ eV has been reconstructed by using the lateral distribution of EAS Cherenkov light and zenith dependence of the parameter N_s at the fixed energy. Thereby, the adaptive method algorithm in solution of the inverse tasks is used. Taking into account it, the absorption path lengths of particles in a shower at a different stage of its development have been found. This parameter can be used in testing of the hadron interaction models at $E_0 > 10^{15}$ eV and the mutual calibration of all compact EAS arrays.

1 Introduction

Cosmic rays of ultrahigh energy, interacting with the atomic nuclei in air, generate a cascade of secondary particles, which is called the extensive air shower (EAS). Such a shower is accompanied by high – power coherent electromagnetic radiation, the most efficient among which is Cherenkov radiation generated by relativistic particles of EAS in the optical wavelength region.. The Cherenkov radiation generated through out the entire journey of shower particles in the atmosphere carries information in such a way about its longitudinal development (Chudakov et al.., 1960). Earlier in (Dyakonov and Knurenko, 1990; Dyakonov et al., 1991) we restored the cascade curve by the method (Dyakonov and Knurenko , 1986) using as additional information the measurements of total number of the charged particles at sea-level N_s.

The idea of the method supposed in the present work is that by LDF we can restore the cascade curve of the "average" shower knowing the attenuation of the Cherenkov light flux in different atmospheric layers. We have obtained this information using the method suggested by us in (Dyakonov and Knurenko 1999).

When considering the problem in detail, the ill-posed test of the type of the Fredholm integral equation of the fist kind arises. To solve it, one has to use modern methods for solution of the inverse tests. Taking into account the conditions of our experiments, we have selected, from the regularizing algorithms available, the adaptive method (Kochnev, 1985). In the mathematical statement of the test and *a priori* information available, this method suits our problem best of all.

2 Initial equation

Detectors usually measure the flux density Q(R) of Cherenkov radiation at a fixed distance R from the axis of a shower. A set of such measurement points represents the spatial distribution of radiation at the level of observation. The power of a radiation source at the altitude *z* in the atmosphere depends on the product of the total number of particles *N(z)* by the light yield function *g(R, z).* Here the *R* dependence of *g(R, z)* reflects the fact of angular distribution of these particles and the probability that radiation from them comes to this distance. The initial equation can be written in the form

$$Q(R)=\int_0^\infty A(R, z, T(z))\cdot N(z)dz \qquad (1)$$

Here *N(z)* is the cascade curve of the development of shower, *T(z)* is the transmittance of the atmosphere from the altitude z to the level of observations, and the number of photons emitted on a unit path length

$$A(R,z) = g(R, z, T(z)) \cdot \rho(z) , \qquad (2)$$

where $\rho(z)$ is the air density at the altitude z.

The function *g(R,z)* depends on the power spectrum of emitting particles and the energy threshold for Cherenkov effect. When an inverse test on the unknown *N(z)* is stated**,** *Eq.* (1) takes the form of the Fredholm integral equation of the first kind, which falls in the category of ill-posed inverse tests. Such tests are usually solved by introducing some *a priori* information about a solution sought, based on physical grounds of the problem.

3 Method of solution

Applying Chebyshev quadrature formula, represent *Eq.* (1) as a system of linear algebraic equation:

$$\sum_{j=1}^{m} A_{ij}\cdot N(z)_j = Q_i \quad i=1,\dots,n;\ j=1,\dots, m \qquad (3)$$

where n is the total number of detectors responded; m is the number of points at different altitudes in the atmosphere, at which the numbers of “cherenkov” electrons are being reconstructed.

The set of equations (3) has been solved using the adaptive method for solution of inverse testes. This method uses the vectors of initial approximations of the unknown $N_j^{(0)}$ and their certainties σ_{Nj}. as an *a priori* information. Having substituted $N_j^{(0)}$ into the i-th equation of the set (3), we obtain the prognostic value of $\overline{Q}_i$. The difference $\Delta Q_i = Q_i - \overline{Q}_i$ is referred to as the discrepancy. It can be presented as a sum:

$$\Delta Q_i = \sum_{j=0}^{m} u_j, \tag{4}$$

where it is meant that $u_0 = \xi_i$, and ξ_i are measurement errors.

All terms are assumed independent random values distributed by the normal law. Then the joint probability density in the $(m+1)$ - dimensional space has the form:

$$\mu(u_{0,\ldots,}u_m) = \prod_{j=0}^{m} 1/(2\pi\sigma_{uj})\cdot exp(-u_j^2/2\sigma_{uj}^2), \tag{5}$$

where

$$\sigma_{uj}^2 = (A_{ij}\cdot\sigma_{Tj})^2;\ \sigma_{u0}^2 = \sigma_{Qi}^2;\ u_0 = (\Delta Q_i - \sum_{j=1}^{m} u_j), \tag{6}$$

σ_{Qi} is the rms measurement error.

The values of u_j are chosen so that the probability density is maximal. This can be achieved when the goal function

$$v(u_{1,\ldots,}u_m) = u_0^2/\sigma_{Qi}^2 + \sum_{j=1}^{m} u_j^2/\sigma_{uj}^2 \tag{7}$$

takes its minimum.

It is easy to see that the goal function is similar to the Tikhonov minimizing functional. If it is accepted that $\sigma_{Qi} = \sigma_Q$, $\sigma_{uj} = \sigma_u$ *and* $\alpha = \sigma_Q^2/\sigma_u^2$ then we have

$$min\ v(u_{1,\ldots,}u_m) = (\Delta Q_i - \sum_{j=1}^{m} u_j)^2 + \alpha\cdot\sum_{j=1}^{m} u_j^2 \tag{8}$$

What exactly corresponds to the Tikhonov reqularizing functional. Having differentiated *Eq.* (8) with respect to every unknown parameter, we obtain a set of n equations for m unknowns. This set has the following solution:

$$u_j = \Delta Q_i\cdot(\sigma_{uj}^2/(\sigma_{Qi}^2 + \sum_{j=1}^{m} \sigma_{uj}^2)) \tag{9}$$

Upon designating the second cofactor in *Eq.* (9) as β_{ij} and introducing the number of the next refining step k, we obtain the recursion expression

$$N_j^{(k+1)} = N_j^{(k)} + \Delta Q_i^{(k+1)}\cdot\beta_{ij}/A_{ij} \tag{10}$$

As the number of iteration increases, the r.m.s error of the obtained solution decreases:

$$(\sigma_{uj}^2)^{k+1} = (\sigma_{uj}^2)^k\cdot(1-\beta_{ij}) \tag{11}$$

This follows from the fact that the parameter β_{ij} varies from 0 to 1, and the r.m.s. error decreases, thus leading to the needed refinement of the solution sought.

4 Experimental data and discussion

The problem presented by *Eq.* (1) is solved using the initial approximation $N_j^{(0)} = const$ (the most neutral assumption on the sought solution) and vectors of certainties σ_{Nj}. Besides, in *Eq.* (3) the error in the right – hand side can be written as

$$\sigma_{Qi}^2 = \{0.04 + n^2*(\Delta R/R)^2\}\cdot Q_i^2\ , \tag{12}$$

where ΔR is the error in determination of the shower axis; the factor equal to 0.04 is due to the absolute calibration of radiation detectors, and n are the above – determined exponents at the exponential approximation of Q(R).

The results obtained are shown in Fig. 2. In Fig.3 the calculations by the QGSJET model are given (Knurenko et al., 1999).

We constructed experimental curves of the longitudinal EAS development for $\Delta E_0 = 10^{15} \div 10^{17}$ eV from X_{max} to sea level using method (Dyakonov and Knurenko, 1999) and by the reconstruction method of the cascade curve tail using dependences $N_s \approx f(Q(400))$ in the region of zenith angles $\Delta\theta = 0 \div 50°$. The transition to the dependence $N_s - f(E_0)$ was carried out by the formula

$$E_0 = (5.2\pm1.1)\cdot10^{16}\cdot Q(100)^{0.96\pm0.02} \tag{13}$$

and to the dependence $N(x) - f(X_o\cdot sec\theta)$ - by the section method of curves in Fig.1 for given fixed energies. To compare experimental and theoretical results the points were reduced to the single energy and the atmosphere depth. Fig. 2 demonstrates the cascade curves of EAS for the depths range of 1020 -1600 g/cm^2 and the curves reduced by the Cherenkov light LDF shape from the maximum to sea level. The calculated curves are shown by dotted lines on Fig.3. From Fig.3 the difference between experimental and calculated curves with the increase of the atmosphere depth is seen: the experimental curve is more sloping. We analyze absorption path lengths of charged particles for different parts by X(g/cm^2): ΔX_1(1020 - 1300), ΔX_2(1300 ÷1600). The results of analysis are shown in Fig.4. It is seen from Fig.3 and Fig.4 that there is a dependence of the absorption path length of particles λ on N_s and secθ. It is not in agreement with the calculation by the QGSJET and SIBYLL models. It may occur when the fraction of the second particles in the models and in experiment is different. For example, in calculations by the QGSJET and SIBYLL models the fraction of the second particles is equal to 363±98 and 263±89. In the experiment it is probably 396±110. It is not improbable that such a difference in the fraction of the second particles is connected with either the mass composition of primary particles or with the mass of the leading particle forming the consequent subcascades.

The parameter λ can be estimated by a zenith-angular distribution of showers at the fixed N_s and the known spectrum by the number of particles (Afanasiev and Knurenko, 1994). Then the comparison of our result with data obtained at other compact arrays can serve to the aims of the mutual calibration and more precise definition of physical results.

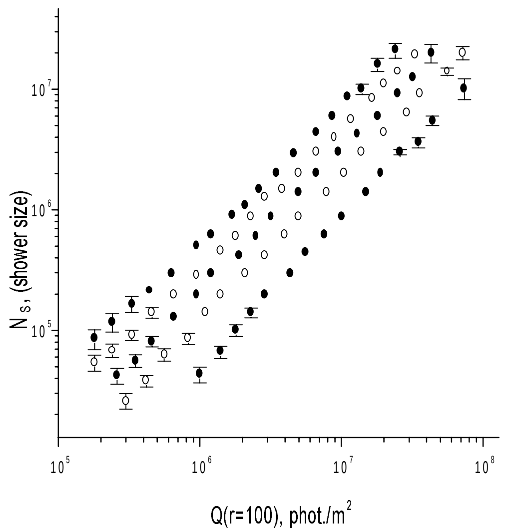

Fig.1. Shower size N_s versus the density of the Cherenkov light flux at a distance of 100 m from the shower core.

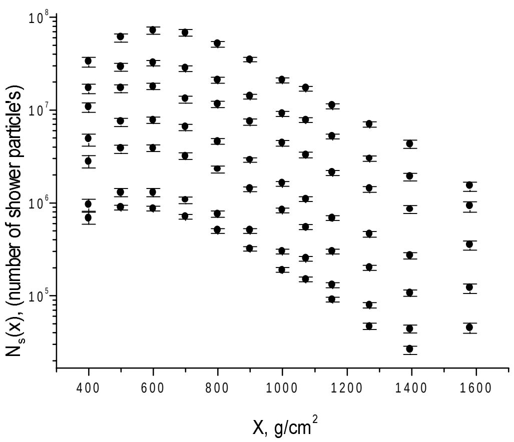

Fig.2. The experimental cascade curves of development of EAS for E_0 = 1,7·10^{15}; 3,0·10^{15}; 5,8·10^{15}; 1,1·10^{16}; 2,7·10^{16}; 5,2·10^{16}; 1,0·10^{17} eV.

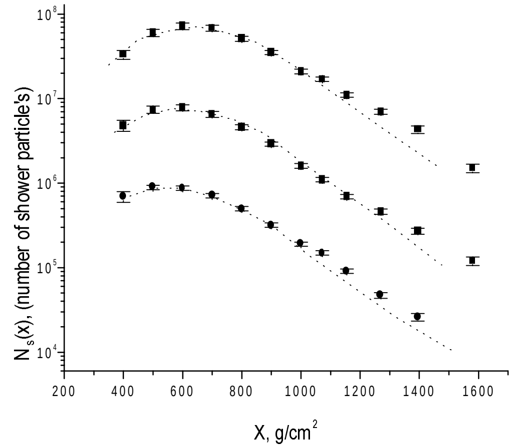

Fig.3. The experimental cascade curves for $E_0 \approx 10^{15}$; 10^{16} and 10^{17} eV. Dotted lines are calculations by the QGSJET model.

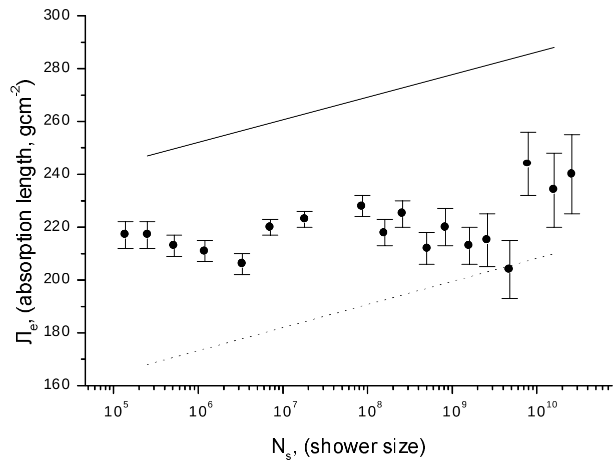

Fig.4. Dependence of the absorption path length of charged particles $\lambda_{abs.}$ on a shower size. (---) - calculations by the SIBYLL model and (....) - calculations by the QGSJET model.

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