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Energy Flow in Extensive Air Showers

J. Matthews

Dept. of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803 USA
and
Dept. of Physics, Southern University, Baton Rouge, LA 70801 USA

Abstract. A simple, semi-empirical model illustrates the physical basis of a composition-independent EAS energy reconstruction recently given by the CASA-MIA experiment. This model develops the hadronic portion of air showers in a manner analogous to the well known Heitler splitting approximation of electromagnetic cascades. Various characteristics of EAS are plainly exhibited with numerical predictions in good accord with detailed monte carlo simulations and with data. Results for energy reconstruction, muon and electron sizes, the elongation rate, and for the effects of the primary's atomic number A are discussed.

1 Introduction

Properties of extensive air showers depend upon the type of primary particle which initiated the cascade. Experimental reconstruction of the primary energy therefore is subject to ambiguities if the primary composition is unknown. The CASA-MIA collaboration has recently presented an energy reconstruction which combines the measured muon and electron sizes of showers:

$$\log E \propto \log(N_e + 25N_\mu). \tag{1}$$

This relation was shown (see Figure 1) to be insensitive to the primary particle type (Glasmacher et al., 2000).

This method was based on full simulations of of air showers and of the detector. It is possible to understand the nature of the combination of N_e and N_μ using a simplified model of air showers. The purpose of constructing a simple model is to show plainly the physics underlying the effect. It cannot replace fully detailed simulations. We first give an outline of the model, then examine it in some detail.

Air showers have two components: an electromagnetic shower and a hadronic cascade. The two are coupled since π° decays feed energy into the electromagnetic component. Electromagnetic showers are modeled as a sequence of radiative or pair-production events. By analogy, hadron cascades are constructed as sequence of pion interactions, each producing more pions.

Heitler's model (Heitler, 1954) of electromagnetic showers has e^+, e^-, and photons undergoing repeated two-body splittings, either one-photon bremsstrahulung or e^+e^- production. Multiplication ceases when the individual $e^\pm$ energies drop below the *critical energy* ξ_c^e, where collisional energy losses exceed radiative losses. At this point, the energy $E_\circ$ of the single initiating particle has been divided among N secondaries - $e^\pm$ and γ's - such that $E_\circ = \xi_c^e N$.

We approximate hadronic interactions similarly. A collision produces some number of pions, of which the $\pi^\pm$ subsequently interact, producing more pions. The sequence continues until individual pion energies drop below a critical energy ξ_c^π, where a charged pion's interaction length exceeds its decay length. The decays of $\pi^\pm$ yield muons observed at the ground.

This hadronic cascade is different from the electromagnetic case: a third of the energy is "lost" to electromagnetic showers at each stage via π° production and decay. Thus the total energy of the initiating particle is divided into two channels, hadronic and electromagnetic,

$$E_\circ = \xi_c^e N + \xi_c^\pi N_\mu. \tag{2}$$

The shower size measured at the ground is smaller than the total size N. Experiments usually are more sensitive to charged particles than to photons; the shower also attenuates after reaching maximum. We express the *measured* size $N_e = N/g$, where $g \approx 10$. Then Eq. 2 becomes

$$E_\circ = g\xi_c^e(N_e + \frac{\xi_c^\pi}{g\xi_c^e}N_\mu) \approx 0.85 \text{ GeV } (N_e + 25N_\mu), \tag{3}$$

using $\xi_c^e = 85$ MeV and $\xi_c^\pi = 20$ GeV.

Equation 1 is thus seen as a consequence of energy conservation. The relative magnitude of the contributions from N_μ and from N_e is determined by their respective critical

Correspondence to: matthews@rouge.phys.lsu.edu

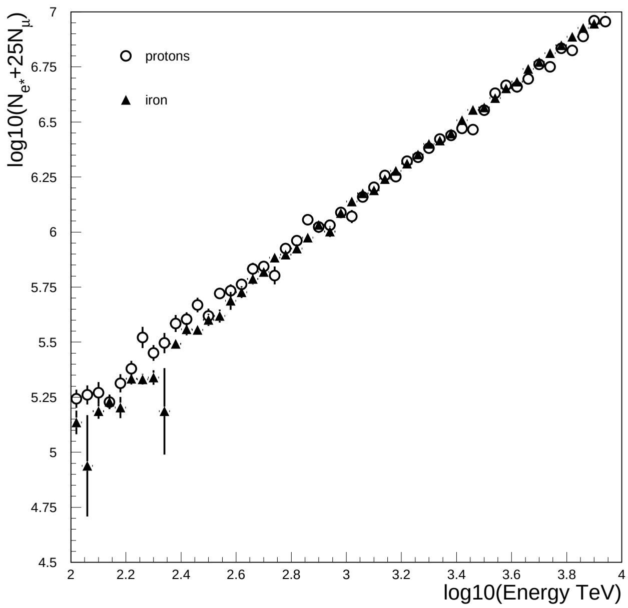

Fig. 1. Energy reconstruction from CASA-MIA (from (Glasmacher et al., 2000)).

energies - the energy scales at which electromagnetic and hadronic multiplication cease. Different primaries, as well as statistical fluctuations, allocate energy differently between the electromagnetic and hadronic components. Equation 1 implicitly accounts for these differences.

2 Electromagnetic showers

In Heitler's model, an electron or positron radiates a single photon after traveling one *splitting length* $\lambda_r \ln 2$, where λ_r is the radiation length in the medium. (Strictly, $\lambda_r \ln 2$ is the distance over which an electron has lost half its energy by radiation) After traveling the same distance, photons split into $e^{\pm}$ pairs. In either instance the energy of a particle (electron or photon) is equally divided between two outgoing particles. After n splitting lengths, the shower size is $N = 2^n$.

Multiplication ceases when the energies of the particles are too low for pair production or bremsstrahlung. Heitler takes this energy to be the critical energy ξ_c^e, below which radiative energy loss becomes less than collisional energy losses.

Consider a shower initiated by a single photon with energy $E_\circ$. The shower reaches maximum size $N = N_{max}$ when all particles have energy ξ_c^e, or

$$E_\circ = \xi_c^e N_{max}. \quad (4)$$

The penetration depth X_{max} at which the shower reaches maximum size is obtained by determining the number n_c of splitting lengths required for the energy per particle to be reduced to ξ_c^e. Since $N_{max} = 2^{n_c}$, we obtain from Eq. 4

$$n_c = \ln(E_\circ/\xi_c^e)/\ln 2,$$

so that

$$X_{max}^{\gamma} \equiv n_c \lambda_r \ln 2 = \lambda_r \ln(E_\circ/\xi_c^e). \quad (5)$$

The superscript "γ" emphasizes that this expression is appropriate for purely electromagnetic showers; the case for an air shower with hadronic components is considered below. The *elongation rate* Λ is the rate of increase of X_{max} with $E_\circ$. Equation 5 yields $\Lambda^{\gamma} = 2.3\lambda_r = 85$ g/cm^2 per decade of primary energy.

Typically the electrons and positrons in a shower dominate experimental measurements. In Heitler's model, at shower maximum $N_{e,max} = \frac{2}{3}N$, where N_e is the sum of e^+ and e^-. However (as noted by Heitler), when compared to real showers, this number is overestimated for several reasons, mainly that multiple photons are often radiated during bremsstrahlung. Moreover, many $e^{\pm}$ range out in the air. The development of the shower beyond its maximum is beyond the scope of the model, requiring detailed treatment of particle production and energy loss. We instead adopt an approximate correction factor

$$N_e = N/g, \quad (6)$$

with a constant value of $g = 10$. This is really only an order of magnitude estimate; a better value requires detailed knowledge of a specific experiment's altitude and its relative sensitivity to photons versus charged particles.

Despite its limitations, the Heitler model produces two basic features of electromagnetic shower development which are confirmed by detailed simulations and by experiments:

• The maximum size of the shower is proportional $E_\circ$,

• The depth of maximum increases logarithmically with energy.

3 Hadronic showers

Air showers initiated by hadrons are modeled using an approach similar to Heitler's. The atmosphere is imagined in layers of fixed thickness $\lambda_I \ln 2$, where λ_I is now the *interaction length* of strongly interacting particles. This thickness is assumed constant here, a fairly good approximation for interactions in the range $10 - 1000$ TeV. For pions in air, $\lambda_I \approx 120$ g/cm^2 (for protons $\lambda_I \approx 85$ g/cm^2).

Hadrons interact after traversing one layer, producing N_{ch} charged pions and $\frac{1}{2}N_{ch}$ neutral pions. A π° immediately decays to photons, initiating electromagnetic showers. Charged pions continue through another layer and interact. The process continues until the pions fall below the critical energy ξ_c^{π} where they then decay, yielding muons.

The charged multiplicity varies with interaction energy, but we adopt a constant value $N_{ch} = 10$ in the following. The validity of this approximation will be examined below.

3.1 Model parameters

For concreteness, we will consider numerical factors appropriate for a range of energies including the "knee" region of the primary spectrum - 10^{14} to 10^{17} eV. Consider a single cosmic ray proton entering the atmosphere with energy $E_\circ$. After n layers there are

$$N_\pi = (N_{ch})^n \tag{7}$$

total charged pions. Assuming equal division of energy during particle production, these pions carry a total energy of $(2/3)^n E_\circ$. The remainder of the primary energy $E_\circ$ has gone into electromagnetic showers from π° decays. The energy per charged pion in atmospheric layer n is therefore

$$E_\pi = \frac{E_\circ}{(\frac{3}{2}N_{ch})^n}. \tag{8}$$

After a certain number n_c of generations, E_π becomes less than ξ_c^π. Particle multiplication then ceases. We estimate ξ_c^π as the energy at which the decay length of a charged pion becomes less than the distance to the next interaction point.

For example, in a shower initiated by a 10^{15} eV primary, a pion's energy after 4 interaction layers is $E_\pi = 10^{15}/(\frac{3}{2}10)^4$ $= 20$ GeV, using $N_{ch} = 10$. The decay length of a pion with this energy is $\gamma c\tau = 1.1$ km. Assuming an exponential atmospheric density profile with scale height 8 km, the linear distance between the altitudes of the beginning and the end of the fourth interaction layer is 1.8 km. This is the first layer that pions have encountered where their probability of a decay exceeds that of arriving at the next interaction point. The critical energy in this shower is then $\xi_c^\pi = 20$ GeV.

If we repeat the above example using $E_\circ = 10^{17}$ eV, we find that there are $n_c = 6$ generations before pions reach the critical energy, which in this case is $\xi_c^\pi = 10$ GeV. It is evident that ξ_c^π slowly decreases with increasing primary energy. The weak energy dependence is partly offset by the slowly changing interaction cross section in this energy region, which we have neglected in this simplified treatment. We adopt a constant value $\xi_c^\pi = 20$ GeV hereafter.

The number of interactions needed to reach $E_\pi = \xi_c^\pi$ is

$$n_c = \frac{\ln(E_\circ/\xi_c^\pi)}{\ln(\frac{3}{2}N_{ch})} = 0.85\log_{10}(E_\circ/\xi_c^\pi), \tag{9}$$

from Eq.8, giving $n_c = 3, 4, 5, 6$ for $E_\circ = 10^{14}, 10^{15}, 10^{16}$, 10^{17} eV respectively. Equation 9 does not depend strongly on moderate variations of the value chosen for N_{ch}. Using a very large value, e.g., $N_{ch} = 20$, would change n_c only above $E_\circ = 10^{16}$ eV, reducing it by one layer.

We can check the consistency of the results for our choice $N_{ch} = 10$. The average energy of all the interacting pions in a shower is about 250 GeV, nearly independently of $E_\circ$. This energy corresponds to $\sqrt{s} = 22$ GeV for pions colliding with stationary nucleons. The mean pp charged multiplicity at this energy is about 8 (Particle Data Group, 2000). Allowing for multiple interactions inside target air nuclei, our selection of $N_{ch} = 10$ seems reasonable.

3.2 Results of the model

The hadronic shower model above gives predictions for several observables. We adopt the following (constant) values for the parameters: $N_{ch} = 10$, $\xi_c^e = 85\,\mathrm{MeV}$, $\xi_c^\pi = 20\,\mathrm{GeV}$, $\lambda_I = 120\,\mathrm{g/cm^2}$, $\lambda_r = 37\,\mathrm{g/cm^2}$.

The primary energy is ultimately divided between N_π pions and N electromagnetic particles in subshowers. The number of muons is $N_\mu = N_\pi$. The total energy in the hadronic channel is $E_h = N_\mu \xi_c^\pi$, while there is $E_{em} = N\xi_c^e$ in the electromagnetic component. Scaling to the total electron size N_e in the usual way,

$$E_\circ = E_{em} + E_h = gN_e\xi_c^e + N_\mu\xi_c^\pi,$$

or

$$E_\circ = g\xi_c^e(N_e + \frac{\xi_c^\pi}{g\xi_c^e}N_\mu) \approx 0.85\mathrm{GeV}(N_e + 25N_\mu). \tag{10}$$

This expression accounts for all the energy of the shower and so is insensitive to fluctuations in the division of energy between the hadronic and electromagnetic channels. Such fluctuations may be statistical or they may be systematic, such as in showers initiated by heavy nuclei instead of protons.

• *The energy is given by a linear combination of measured muon and electron sizes. The weighting does not depend on the details of the model, only on the characteristic energy scales at which hadronic cascading and electromagnetic showering cease.*

The number of muons in the shower is obtained using $N_\mu = N_\pi = (N_{ch})^{n_c}$. Using Eq.9, the energy dependence of the muon size is obtained by casting N_μ in the form

$$\ln N_\mu = \ln N_\pi = n_c \ln N_{ch} = \beta\ln(E_\circ/\xi_c^\pi) \tag{11}$$

where

$$\beta = \frac{\ln(N_{ch})}{\ln(\frac{3}{2}N_{ch})} = 0.85.$$

Note that although N_{ch} in fact changes as the shower develops, β depends only logarithmically on its value - our assumption of a constant N_{ch} has little effect. The muon size of the shower is then

$$N_\mu = \left(\frac{E_\circ}{\xi_c^\pi}\right)^{0.85} \approx\ 9900\left(\frac{E_\circ}{10^{15}\mathrm{eV}}\right)^{0.85}, \tag{12}$$

in good agreement with more detailed simulations (Engel et al., 1999). The less-than-linear growth of N_μ with primary energy has important consequences for modeling showers initiated by nuclei heavier than protons, described in the next section.

This behavior of N_μ, along with Eq. 10, implies that the size N_e will increase slightly more quickly than linearly with $E_\circ$.

• *Muon size grows with primary energy more slowly than proportionally. The exponent depends on the division of energy between charged and neutral daughter particles in each interaction.*

The electromagnetic component of the shower is generated by photons from π° decays. The first interaction diverts

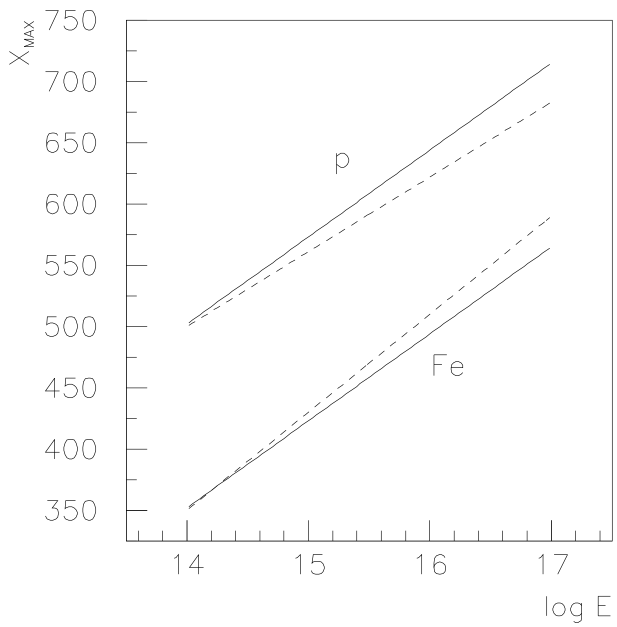

Fig. 2. Depth of maximum for proton and iron induced air showers. Dashed lines are from QGSJET simulations.

$\frac{1}{3}E_\circ$ into these channels. This is followed by separate showers from each subsequent interaction point. As a first estimate of the depth of maximum, we use as that arising from first generation γ showers.

The first interaction occurs at an atmospheric depth $X_\circ = \lambda_I \ln 2 = 59$ g/cm^2, yielding $\frac{1}{2}N_{ch}\pi^\circ \rightarrow N_{ch}\gamma$'s. Each γ initiates an electromagnetic shower of energy $E_\circ/3N_{ch}$, developing in parallel with the others. Using Eq. 5,

$$X^p_{max} = X_\circ + \lambda_r \ln[E_\circ/(3N_{ch}\xi^e_c)] \quad (13)$$

$$= X^\gamma_{max} + 18 - 85\log_{10}[N_{ch}] \ \text{g/cm}^2 \quad (14)$$

Here X^γ_{max} is the depth of maximum of an electromagnetic shower from a γ-ray with energy $E_\circ$. The values of X^p_{max} are far too low, from neglecting the contributions of later generations of π° production. As mentioned previously, proper inclusion of this is beyond the scope of the model; however, the elongation rate will not be strongly changed.

If, as before, we use a constant N_{ch} in Eq. 14, then proton showers will have the same elongation rate as pure electromagnetic ones. However, this approximation is inappropriate here. The factor N_{ch} in Eq.14 actually requires the multiplicity of charged pions in the *first* interaction. For $E_\circ >$ 100 TeV, $N_{ch} \approx 40(E_\circ/1\text{PeV})^{1/6}$ (Particle Data Group, 2000), giving an elongation rate from Eq. 14 of $(5/6)85 =$ 71 g/cm^2. Figure 2 displays the result with an (arbitrary) 3.5 splitting length offset, compared with detailed simulations using QGSJET(Fowler et al., 2000). Elongation rates are in very good agreement. Inclusion of energy-dependent inelasticity could account for the small discrepancies.

This estimation of X^p_{max} illustrates Linsley's *elongation rate theorem* (Linsley, 1977), which pointed out that electromagnetic showers represent an upper limit on the elongation rate for hadron showers.

• *The elongation rate is less than that from purely electromagnetic showers. The amount of difference depends mainly on the rate of increase of multiplicity with energy in hadronic interactions.*

3.3 Nuclear primaries

The *superposition model* is a simplified view of the interaction of a cosmic ray nucleus with the atmosphere. A nucleus with atomic number A and total energy $E_\circ$ is taken to be A individual single nucleons, each with energy $E_\circ/A$, and each acting independently. We treat the resulting air shower as the sum of A separate air showers all starting at the same point.

We can produce observable shower features by substituting the lower primary energy into the various expressions derived previously for proton showers and summing A such showers where appropriate. The resulting nuclear-initiated shower properties are easily expressed in terms of the corresponding quantitites of a proton shower with the same total energy $E_\circ$:

$$N^A_\mu = N^p_\mu A^{0.15}, \quad (15)$$

$$X^A_{max} = X^p_{max} - \lambda_r \ln A, \quad (16)$$

$$E_\circ = 0.85\text{GeV}(N_e + 25N_\mu). \quad (17)$$

One consequence is that nuclear showers have more muons than proton showers, at the same total primary energy. This results from the less-than-linear growth of the muon number with energy. The lower energy individual nucleons in a nuclear shower generate fewer interaction generations, and so lose less energy to electromagnetic components. An iron shower will have $(56)^{0.15} = 1.8$ times as many muons as a proton shower of the same energy.

X_{max} of iron showers is higher than proton showers by $\lambda_r \ln(56) = 150$ g/cm^2 at all energies (see Fig. 2). This is confirmed by detailed simulations (Fowler et al., 2000).

The energy assignment (Eq.17 or 10) is unaffected by A because it intrinsically accounts for all of the primary energy being distributed into a hadronic channel (seen as muons) and into electromagnetic showers.

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