

# Non-homogeneous charge-consistent model for regular acceleration of iron in gradual SEP events

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**Abstract.** Acceleration of iron ions by a spherical shock wave moving through the solar corona is considered. The energy dependence of the mean charge,  $\bar{q}_{Fe}(E)$ , is determined by the characteristic acceleration time,  $T_a$ , and time for charge changes,  $T_q$ . The latter varies along with plasma number density during the propagation of the shock wave. An account of adiabatic energy changes and shock broadening is shown to insufficiently influence the dependence  $\bar{q}_{Fe}(E)$ . According to our estimations the photoionizing processes do not affect the ionic states of the accelerated iron in gradual events in most cases.

by thermal protons and electrons (including autoionization) along with radiative and dielectronic recombination. In addition, we have also analyzed the contribution of photoionization. Thus we have studied the differences in energy dependence of the mean charge of iron,  $\bar{q}_{Fe}(E)$ , arising in a more complicated model compared to the earlier and simpler ones.

## 1 Introduction

For the last two decades there has been a significant progress in accumulating experimental data on the charge states of cosmic rays of different origins. Particularly, those data (Luhn et al., 1984; 1985; Mason et al., 1995; Oetliker et al., 1997; Mazur et al., 1999; Möbius et al., 1999) have allowed one to discover the increase of the mean charge of heavy elements with energy. To account for this dependence the charge-consistent models for particle acceleration have to be used (Ostryakov and Stovskyuk 1999a; Barghouty and Mewaldt, 1999). This in turn enables to evaluate plasma parameters in the acceleration site from the experimental data fits (Kartavykh et al., 1998; Ostryakov and Stovskyuk, 1999b; Stovskyuk and Ostryakov, 2001).

In a series of our previous works on this problem we have considered regular acceleration of multicharged ions by a parallel shock in planar geometry. These simulations were performed under assumptions of homogeneous plasma and spatial independence of the diffusion coefficient. The model represented below is free from these simplifications. Namely, it takes into account spherical geometry of the shock moving through inhomogeneous solar corona and adiabatic energy changes of ions. Similar to previous papers (e.g., Stovskyuk and Ostryakov, 2001), we have included into consideration the processes of ionization of a projectile

## 2 Acceleration of iron by a propagating spherical shock

As an example we consider Fe because this multicharged and abundant element is especially convenient to demonstrate the energy and charge spectra alterations during acceleration. In general, the ion energy gain is accompanied both by electron loss ( $q \rightarrow q+1$ ) and electron gain ( $q \rightarrow q-1$ ) processes. The characteristic rates of these are correspondingly as follows:

$$S_q(V_i) = N \int \sigma_{qq+1}(V) f(V) V dV, \quad (1)$$

$$\alpha_q(V_i) = N \int \sigma_{qq-1}(V) f(V) V dV. \quad (2)$$

Here  $V$  is the velocity of electrons or protons with respect to the accelerated Fe ion;  $\sigma_{qq\pm 1}(V)$  are the relevant cross sections dependent on  $V$ ;  $f(V)$  is the distribution function of plasma particles in the rest frame of a moving ion of velocity  $V_i$  (Luhn and Hovestadt, 1987; Kocharov et al., 2000);  $N$  is the number density of plasma with equal number of electrons and protons,  $N_e = N_p = N$ . Let  $f_q$  be the distribution function of the ions of charge  $q$ , where we omit for brevity the energy,  $E$ , and heliocentric,  $r$ , variables. The system of diffusion equations accounting charge transitions in a spherical symmetric case can be written:

$$\begin{aligned} \frac{\partial f_q}{\partial t} = & \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 D_q(r) \frac{\partial f_q}{\partial r} \right) - u(r) \frac{\partial f_q}{\partial r} + \\ & + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u(r)) \cdot \left( \frac{2E}{3} \frac{\partial f_q}{\partial E} - \frac{f_q}{3} \right) + \\ & + N(r) (f_{q-1} S_{q-1} - f_q (S_q + \alpha_q) + f_{q+1} \alpha_{q+1}), \end{aligned} \quad (3)$$

where the charge index runs from  $q_{min}=+1$  up to  $q_{max}=+Z$ ,  $Z$  - is the nucleus charge. In the rest frame of the shock front propagating upward with the velocity  $u_1$  the inflow velocity can be chosen as:

$$u(r) = \begin{cases} -\frac{u_1 - u_2}{2} - \frac{u_1 + u_2}{2} \tanh\left(\frac{r-R}{\delta}\right), & r > R, \\ -u_2, & r < R, \end{cases} \quad (4)$$

where  $u_2 = u_1/\sigma$ ,  $\sigma = \frac{(\kappa-1)M_1^2 + 2}{(\kappa+1)M_1^2}$  is the compression ratio,

$M_1$  is the Mach number,  $\kappa$  is the adiabatic index and  $\delta$  is the shock front width. The negative sign in the definition (4) indicates that the plasma flow is directed towards  $r$  decrease. This formula also describes in a simplest way a broadening of the shock front caused by energetic protons (see, e.g., Topygin, 1985; Berezhko et al., 1988). Here and below the indices  $i=1,2$  refer to the upstream ( $r>R$ ) and downstream ( $r<R$ ) regions, respectively. Because the shock front radius  $R$  increases with time  $t$ ,  $R(t) = R_0 + \int_0^t u_1(t') dt'$ , the

spatial non-homogeneity of the ambient coronal plasma is equivalent to the temporal non-homogeneity of the inflow flux. The  $u_1(t)$ , or  $u_1(R)$ , dependence is apparently not a universal function. In the present paper we apply both analytical results by Kaplan (1967),  $u_1(R) \propto (R^2 N)^{1/4}$ , and numerical simulations of shock propagation in the solar corona carried out by Wu (1982).

The acceleration time being sufficiently long, the ion faces inhomogeneous plasma during shock wave propagation. Hence, this global property of matter could influence the charge state formation of heavy ions. For  $N \equiv N(r)$  in solar corona we have utilized the formula which is valid within several solar radii if shock still is not in the interplanetary space (Lang, 1974):

$$N(r) \approx N_0 \left( \frac{r}{R_s} \right)^{-6} \left[ 1 + 1.93 \left( \frac{r}{R_s} \right)^{-10} \right], \quad (5)$$

where  $R_s = 6.96 \times 10^{10}$  cm is the solar radius. Such a dependence agrees well with the measurements of the  $K$ -emission of solar corona if one supposes  $N_0 = 1.55 \times 10^8$  cm $^{-3}$ .

The spatial diffusion coefficient in (3) can be chosen as dependent on the heliocentric distance:

$$D_q(r) = \begin{cases} D_{q1}(E) \frac{r}{R}, & r > R, \\ D_{q2}(E) \frac{r}{R}, & r < R, \end{cases} \quad (6)$$

where the multiplicand

$$D_{qi}(E) = D_{oi} \left( \frac{q}{A} \right)^{S-2} \times E^{(3-S)/2}, \quad S < 2, \quad (7)$$

is the function of energy, charge, atomic mass number  $A$  and spectral index of turbulence  $S$  (Hasselmann and Wibberenz, 1968; Topygin, 1985);  $D_{oi}$  being some constants, which are different for both regions (usually  $D_{o1} \gg D_{o2}$ ).

Note that taking into account in Eq. (3) the only coordinate  $r$  means virtually the coincidence of the center of our spherical coordinate system with the center of the Sun. This

is a reasonable simplification at least for some shocks observed in solar corona (Maxwell and Dryer, 1982; Pinter, 1982). Besides, such shock fronts are segments of a sphere. This in turn means the necessity to include edge effects into consideration. However, the area of these edge regions being small, we assume their insignificant contribution to the total number of escaping particles.

To find a numerical solution of the system (3) we apply the Monte-Carlo approach. The random character of the charge change reactions is easy to notice in (3). In fact, the last three terms of the left hand side of Eqs. (3) may be formally presented as a three-point deconvolution of the ‘‘convection’’ and ‘‘diffusion’’ terms in charge space (Ivanov et al., 1987):

$$(S_q - \alpha_q) \frac{\partial f_q}{\partial q} + \frac{1}{2} (S_q + \alpha_q) \frac{\partial^2 f_q}{\partial q^2}. \quad (8)$$

$S_q$  and  $\alpha_q$  coefficients shall be construed here as continuous functions of  $q$ , the latter representing continuous variable itself. If the relative charge changes are small enough,  $1/q \ll 1$  (multicharged ions), such a substitution allows one to study the behavior of the function  $\bar{q}(E)$  analytically at high accuracy (Kurganov and Ostryakov, 1991; Ostryakov and Stovpyuk, 1997). In our simulations we consider, however, a more general kinetic approach based on Eqs. (3).

In our model the condition of the ion escape from region ‘‘2’’ was its rather distant position in regard to the shock surface, from where the probability to return back to the shock front is very low. This distance was chosen to be about  $2 \times 10^9$  cm. According to numerical simulations of Wu (1982) there are density and magnetic peculiarities in the downstream region. We assume that the accelerated particles could be accumulated and stored there. Subsequent opening of these ‘‘traps’’ could result in the particle leakage into the interplanetary space where they can be detected.

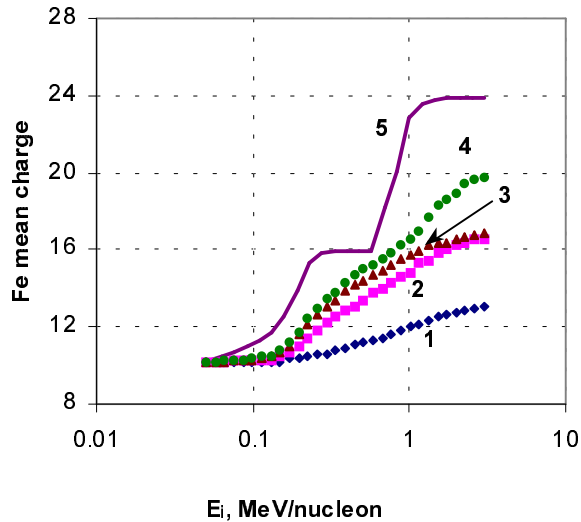
### 3 Results and discussion

A numerical scheme described in the previous Section was used to research the influence of various model parameters on the energy and charge spectra of accelerated ions. Let us first discuss plasma inhomogeneity. As shown by Kurganov and Ostryakov (1991), the principal criterion for the importance of charge change reactions during acceleration is the ratio of the characteristic acceleration time,  $T_a$ , and charge change time,  $T_q$  (including  $\tau_{ion}$  and  $\tau_{rec}$ ). The characteristic time  $T_a$  is construed here as follows (see, e.g., Berezhko et al., 1988):

$$T_a = \frac{3}{u_1 - u_2} \left( \frac{D_{q1}}{u_1} + \frac{D_{q2}}{u_2} \right). \quad (9)$$

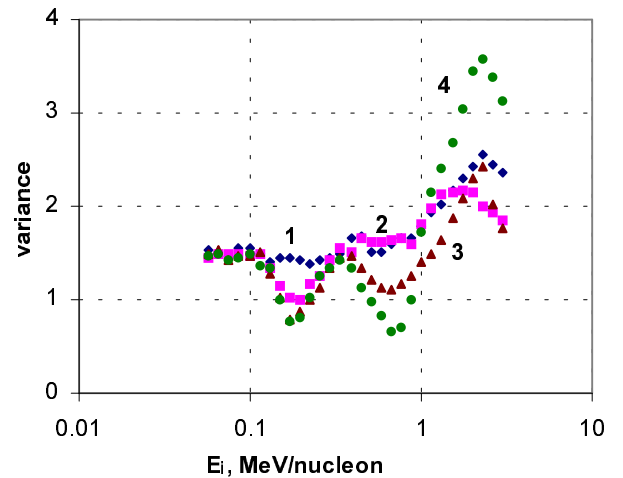
Since both  $\tau_{ion}$  and  $\tau_{rec}$  are inversely proportional to the number density of reactants (electrons and protons) of a surrounding plasma, the product  $T_a \cdot N(r)$  becomes the main parameter of the problem (Ostryakov and Stovpyuk, 1999b; Stovpyuk and Ostryakov, 2001). Clearly, the higher is the value of this product, the sharper is the growth of the mean charge of heavy element with energy. If, however, the

number density itself depends on time (coordinate) because of the shock propagation, the value of  $T_q$  is affected by this propagation speed. Thus, one more parameter (for example, the absolute value of  $T_a$ ) is to characterize this situation at a given shock velocity. According to Maxwell and Dryer (1982) the time necessary for shock front to move from  $r=R_0=R_S+H$  to  $r=2R_S$  is about 15 minutes ( $H\sim 10^{10}$  cm is the height above solar photosphere where the shock is produced). Its velocity for the first 5 minutes was shown to be  $700 \text{ km s}^{-1}$  increasing further up to  $1700 \text{ km s}^{-1}$  within two solar radii. Though the Mach number of the shock also varies together with flow velocity, it yields minor effect on  $\bar{q}_{Fe}(E)$  dependence, providing that  $T_a \cdot N_0$  is kept constant. Figure 1 depicts this dependence obtained at various acceleration times  $T_a$  and various parameters  $T_a \cdot N_0$ . It is obvious that the plasma non-homogeneity begins to affect this function if  $T_a$  is comparable to (or greater than) the characteristic time of density changes. For example, “3” and “4” curves refer to the same parameter  $T_a \cdot N_0$  but different  $T_a$ . For the case described by curve “4”, the acceleration occurs fast enough as compared with the shock propagation, therefore the results are very close to those for homogeneous plasma. Alternatively, at large values of the parameter  $T_a$  (curve “3”) the shock front reaches sufficiently lower density regions, where the charge transfer processes cease to play any role in prolonged acceleration. As a result, lower magnitudes of the mean charge of iron can be achieved. For comparison, also depicted is a curve of equilibrium Fe charge,  $\bar{q}_{Fe}^{eq}(E)$ , formally corresponding to the infinite time, which ions spend in a plasma (Kocharov et al., 2000; Ostryakov et al., 2000; Stovpyuk and Ostryakov, 2001).



**Fig. 1.** The mean charge of iron accelerated on a shock front propagating in the solar corona (see text) with a temperature  $T=10^6$  K. At energy  $E=1$  MeV/nucleon for curve “1”  $T_a(\text{Fe}^{+8}) \times N_0 = 2.3 \times 10^9 \text{ s} \cdot \text{cm}^{-3}$  and  $T_a(\text{Fe}^{+8}) = 45 \text{ s}$ ; for “2”  $T_a(\text{Fe}^{+8}) \times N_0 = 2.3 \times 10^{10} \text{ s} \cdot \text{cm}^{-3}$  and  $T_a(\text{Fe}^{+8}) = 450 \text{ s}$ ; for “3”  $T_a(\text{Fe}^{+8}) \times N_0 = 6.9 \times 10^{10} \text{ s} \cdot \text{cm}^{-3}$  and  $T_a(\text{Fe}^{+8}) = 1350 \text{ s}$ ; for “4”  $T_a(\text{Fe}^{+8}) \times N_0 = 6.9 \times 10^{10} \text{ s} \cdot \text{cm}^{-3}$  and  $T_a(\text{Fe}^{+8}) = 45 \text{ s}$ ; curve “5” is the iron equilibrium charge.

The next panel (Figure 2) shows the variance of the charge distribution function versus energy,  $\sigma_{Fe}(E)$ , corresponding to the mean charge from Figure 1. It manifests more pronounced structure at large values of  $T_a \cdot N_0$ , i.e., for the cases when charge transfer processes become more important. The mostly narrow distributions (local minima) in  $\sigma_{Fe}(E)$  occur for ion energies at which equilibrium charge is approximately constant. As pointed out by Ostryakov et al. (2000), these horizontal plateaus in  $\bar{q}_{Fe}^{eq}(E)$  are the consequence of the electron shell structure of the element. In fact, due to the sharp jumps in electron binding energy for K-, L-, ... shells the atomic reactions practically do not alter the mean ionic state within those energy intervals. This leads to the “concentration” of charges in the vicinity of  $\bar{q}_{Fe}(E) = \dots, 16, 24$  (closed electron shells ...,  $1s^2 2s^2 2p^6, 1s^2$ , respectively) and hence to a rapid decreasing in variance  $\sigma_{Fe}(E)$ . In contrast, very sharp increase in  $\bar{q}_{Fe}^{eq}(E)$  is a signature that the ionization does take place at these energies. This corresponds also to the growth in variance  $\sigma_{Fe}(E)$  because threshold energies for various electrons within a definite shell do not differ so dramatically as those for neighbouring shells. This is a good example of the effective diffusion in charge space (see Eq. (8)) displayed most noticeably for large values of  $T_a \cdot N_0$ .



**Fig. 2.** The variance of Fe charge distributions corresponding to the mean charge from Figure 1.

One should note that particle storage inside the “magnetic traps” of the downstream region could result in  $\bar{q}(E)$  rise. This effect is especially important if the time of shock wave propagation is much longer than the acceleration time. In this case  $\bar{q}_{Fe}(E)$  approaches  $\bar{q}_{Fe}^{eq}(E)$ .

Now we briefly discuss the influence of photoionizing processes. Our estimations of the characteristic photoionization time allow us to assert that the process under discussion does not noticeably contribute to the formation of the charge states of iron at least in gradual solar energetic parti-

cle events. In this procedure we have relied on the approximation formulae for partial cross sections given by Verner and Yakovlev (1995):

$$\sigma_{nl}^{ph}(\nu) = \sigma_0 F(h\nu/E_0), \quad \text{Mb}, \quad (10)$$

where

$$F(y) = \left[ (y-1)^2 + y_w^2 \right] y^{-Q} \left( 1 + \sqrt{(y/y_a)} \right)^{-P}, \quad y = h\nu/E_0. \quad (11)$$

Here  $n$ ,  $l$  are the principal and orbital quantum numbers, respectively;  $\sigma_0$ ,  $E_0$ ,  $Q$ ,  $y_a$  and  $y_w$  are the fitting parameters. Relevant X-ray data for solar flares are currently available from patrol measurements of GOES-6, 7 satellites being regularly published in Solar Geophysical Data. A typical value for the fluence at the Earth's orbit is apparently around  $0.1 \text{ erg cm}^{-2} \text{ s}^{-1}$  (in the energy range of photons  $h\nu = 1.5 \div 12 \text{ keV}$ ). If we choose for a spatial scale of gradual event to be  $\sim 10^{10} \text{ cm}$ , then this fluence is  $J \sim 2.5 \times 10^5 \text{ erg cm}^{-2} \text{ s}^{-1}$  inside the active region. Soft X-rays have typically power law distribution,  $J_\nu \sim \nu^{-\gamma+1}$ , with the spectral indices  $\gamma$  ranging from 3 to 7. Thus, the photoionization rate, represented by the formula

$$S^{ph} = \sum_{nl} \int_{\nu_{th}}^{\infty} \frac{J_\nu}{h\nu} \sigma_{nl}^{ph}(\nu) d\nu, \quad (12)$$

for  $\text{Fe}^{+8}$  ion and  $\gamma=3$  yields  $S^{ph} = 2 \times 10^{-3} \text{ s}^{-1}$ , while for  $\text{Fe}^{+15}$  ion it proves to be smaller by two orders of magnitude:  $\sim 3 \times 10^{-5} \text{ s}^{-1}$ . This value is the sum of partial ionization cross sections of a subshell  $nl$  characterized also by the ionization threshold frequency  $\nu_{th}$ . For comparison, the total collisional ionization rates for those ions at injection energy  $E_{inj} = 50 \text{ keV nucleon}^{-1}$  and  $N_0 \sim 10^8 \text{ cm}^{-3}$  are  $3 \times 10^{-2} \text{ s}^{-1}$  and  $1.4 \times 10^{-3} \text{ s}^{-1}$ , respectively. Numerical calculations indicate that the account of photoionization alters  $\bar{q}_{Fe}(E)$  by  $\sim 1\%$ . That is within the accuracy of our Monte-Carlo method. One should note that the magnitudes for all the above parameters have been chosen to obtain a typical estimate for  $S^{ph}$ . At the same time, the process considered could be much more significant for a compact impulsive event with the characteristic spatial scale of the order of  $\sim 10^8 \text{ cm}$  or less. Then, the photoionization rates increase by a factor of  $10^4$ . It is worth noting that the above estimates have been made assuming the existence of the ionizing photons during the whole phase of a flare event. If heavy particle acceleration (and particle escape) occurs faster than the global plasma heating and/or acceleration of electrons, then the photoionization may be negligible even for impulsive events.

#### 4 Conclusions

In the present paper the recently proposed charge-consistent acceleration model has been further developed. New simulations have been applied to solar cosmic rays and take into account i) propagation of a spherical shock wave through non-homogeneous solar corona; ii) shock broadening and adiabatic energy changes of ions; iii) photoionization by soft X-rays emitted by active region. Finally, our analysis has shown that

- the mean charge of iron depends on the shock wave velocity if the acceleration is not fast enough;
- the influence of such effects as shock front broadening and adiabatic energy changes on the function  $\bar{q}_{Fe}(E)$  is insufficient;
- photoionization by soft X-rays cannot dominate for gradual solar energetic particle events and might be important for impulsive ones.

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