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# High energy nucleus-nucleus collisions in view of the multi-peripheral model

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**Abstract.** The hypothesis of the multi peripheral model is extended to the hadron-nucleus interactions and then generalized to the nucleus-nucleus case. The processing depends on input data that is extracted from the features of the experiments in this field. The number of encountered nucleons from both target and projectile are estimated according to the eikonal scattering approach. The screening effect due to the interaction of the projectile nucleons in successive manner with the target nucleus is considered. The rapidity distributions of fast particles are reproduced at the successive collisions in p-S and  ${}^{32}S{}^{-32}S$  interactions at 200 A GeV. A global fair agreement is found in comparison with data of the experiment CERN-NA-035.

## 1. Introduction

One of the many factors that lead to an optimistic assessment that matter at high density and high temperature may be produced with nucleus-nucleus collisions is the occurrence of multiple collisions. By this mean, a nucleon of one nucleus may collide with many nucleons in the other and in the process deposit a large amount of energy in the collision region. Therefore when the projectile nucleon collides with many target nucleons, particles production arising from the first N-N collision is not finished before the collision of the projectile with another target nucleon begins. There are models [1-4] that describe how is the second collision is affected by the first one. In the present work we shall investigate the particle production mechanism in heavy ion collision by extending the multi-peripheral model [5-7] to the nucleon-Nucleus and then generalize to the nucleus-nucleus case.

**2.** Experimental Features of High Energy Events The rapidity range is classified into regions that characterizes the type of the interaction. The target and the projectile fragmentation regions contain the fast particles characterizing the forward and backward production from the fragmentation of both the target and the projectile nuclei in their center of mass system. The central rapidity interval is the hot region of the reaction. It sends global information about the strong interactions inside the nuclear bowl. Moreover, signals about (QGP) may be extracted from the study of this region. The nucleus-nucleus (A-A) collision may be seen as N-N base or N-A base. To estimate the number of base-collisions in each reaction we have to renormalize the rapidity distribution to get a scaled function that is independent on the projectile and target size. The average number of encountered nucleons from the target by an incident hadron [8-10] is a good measure of the number of collisions inside a target nucleus. This is defined as.

$$\nu = A_t \frac{\sigma_{hN}}{\sigma_{hA}} \tag{1}$$

Where,  $\sigma_{hN}$  and  $\sigma_{hA}$  are the total inelastic cross section of (N-N) and (N-A) respectively. By analogy, the average number of binary (N-N) collisions inside the (A-A) collision may be defined as,

$$v_{NN} = A_{p}A_{t}\frac{\sigma_{hN}}{\sigma_{AA}}$$
(2)

 $\sigma_{AA}$  is the total inelastic cross section of (A-A) collision at the same incident energy. And the average number of collisions based to N-A is defined as,

$$v_{\rm NA} = A_{\rm p} \frac{\sigma_{\rm hA_{\rm r}}}{\sigma_{\rm AA}}$$
(3)

 $\sigma_{hAt}$  is total inelastic cross section of (h-A<sub>t</sub>) collision. The scaling function is then obtained by dividing the rapidity distribution by the average number of collisions  $v_{NN}$ . Following now a geometric approach which assumes that the nuclear radius is linearly proportional

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$$\nu = A_{t}^{\frac{1}{3}}, \nu_{NN} = \frac{A_{p}A_{t}}{A_{t}^{\frac{2}{3}} + A_{p}^{\frac{2}{3}} + 2A_{t}^{\frac{1}{3}}A_{p}^{\frac{1}{3}}}$$

$$\nu_{NA} = \frac{A_{p}A_{t}^{\frac{2}{3}}}{A_{t}^{\frac{2}{3}} + A_{p}^{\frac{2}{3}} + 2A_{t}^{\frac{1}{3}}A_{p}^{\frac{1}{3}}}$$
(4)

Let  $\overline{m}$  be the average multiplicity produced in N-N collision and  $\overline{m}_{NA}$  is the corresponding figure for N-A collision at the same incident energy. So that the average multiplicity produced in A-A collisions may be calculated in base of N-N

$$\overline{\mathbf{m}}_{\mathrm{AA}} = \mathbf{v}_{\mathrm{NN}} \overline{\mathbf{m}} \tag{5}$$

and in base of N-A as,

$$\overline{\mathbf{m}}_{\mathrm{AA}} = \mathbf{v}_{\mathrm{NA}} \overline{\mathbf{m}}_{\mathrm{NA}} \tag{6}$$

The comparison with the experimental data shows that the A-A collision behaves as a base of N-N not N-A collision.

# 3. The Multi-Peripheral Model

#### 3.1. Hadron-Nucleon Collision

The many body-system is expanded into subsystems, each concerns a two body collision. It is assumed that each hadron in the final state is produced at a specific peripheral surface that is characterized by a peripheral parameter. The phase space integral  $I_n(s)$  of the produced hadrons is a measure of the probability of producing *n* particle in the final state at center of mass energy  $\sqrt{s}$ . It depends mainly on the volume in phase space and the transition matrix element *T*, defined as,

$$I_{n}(s) = \int .... \int \prod_{i}^{n} \frac{d^{3}p_{i}}{2E_{i}} \delta^{4}(s - \sum_{j} p_{j}) |T|^{2}$$
(7)

This may be simplified as if expressed as a sequence of two particle decay,

$$I_{n}(s) = \int_{\mu_{n-1}^{2}}^{(M_{n}-m_{n})^{2}} dM_{n-1}^{2} I_{2}(k_{n}^{2}, k_{n-1}^{2}, p_{n}^{2}) I_{n-1}(M_{n-1}^{2}) |T|^{2} =$$

$$\int_{\mu_{n-1}^{2}}^{(M_{n}-m_{n})^{2}} dM_{n-1}^{2} \int d\Omega_{n-1} \frac{\lambda^{\frac{1}{2}}(M_{n}^{2}, M_{n-1}^{2}, m_{n}^{2})}{8M_{n}^{2}} I_{n-1}(M_{n-1}^{2}) |T|^{2}$$
(8)

Doing the integration over all possible values of  $M_i$ , so that,

$$I_{n}(s) = \int_{\mu_{n-1}}^{(M_{n}-m_{n})} dM_{n-1} d\Omega_{n-1} \frac{1}{2} P_{n} |T(p_{n-1})|^{2} \cdots$$

$$\int_{\mu_{2}}^{(M_{3}-m_{3})} dM_{2} d\Omega_{2} \frac{1}{2} P_{3} |T(p_{2})|^{2} \cdots \int d\Omega_{1} \frac{1}{2} P_{2} |T(p_{1})|^{2}$$
(9)

Where  $P_i = \lambda^{\frac{1}{2}} (M_i^2, M_{i-1}^2, m_i^2) / 2M_i$  is the three-

vector momentum of the  $i^{\text{th}}$  particle. The multiple integration in Eq. (9) may be solved by the Monte Carlo technique [11]. At extremely high energy, Eq. (9) has an asymptotic limit in the form;

$$I_{n}(s) = \frac{(\pi/2)^{n-1}}{(n-1)!(n-2)!} s^{n-2} |T|^{2(n-1)}$$
(10)

For the case of strong interactions  $T(p_i)$  has a parametric form [8] as,

$$T(p_i) = \exp(-\alpha_i p_i) \tag{11}$$

## 3-2 Hadron-Nucleus Collisions

On extending the model to the hadron-nucleus or nucleus-nucleus collisions, we follow the NN-base super position as expected from the features of the experimental data. The incident hadron makes successive collisions inside the target  $A_t$ . The energy of the incident hadron (leading particle) slows down after each collision, producing number of created hadrons each time that depends on the available energy. The phase space integral  $I_n^{NA}$  in this case has the form,

$$I_{n}^{NA}(s) = \sum_{\nu}^{At} I_{n_{\nu}}(s_{\nu}) P(\nu, A_{t}) \delta(n - \sum_{i}^{At} n_{i})$$
(12)

Where  $P(v, A_t)$  is the probability that v nucleons out of  $A_t$  will interact with the leading particle and  $I_{n_v}(s_v)$  is the phase space integral of NN collision that produces  $n_v$  hadrons at energy  $s_v$ . The delta function in Eq. (12) is to conserve the number of particle in the final state. Treating all nucleons identically, and that  $\chi_{NN}$  is the N-N phase shift function, then, according to the eikonal approximation,

$$P(l, A_t) = - \begin{pmatrix} A_t \\ l \end{pmatrix}_{j=0}^{l} (-1)^{j} \begin{pmatrix} l \\ j \end{pmatrix}$$

$$[1 - \exp(2\operatorname{Rei}(A_t - l + j)\chi_{NN})]$$
(13)

The working out of this approach is to put the multidimension integration of Eq. (9) and the generated kinematical variables into a Monte Carlo subroutine. This in turn is restored  $\overline{v}$  times, where  $\overline{v}$  is the number of collisions inside the target nucleus. In the first collision, the incident hadron has its own incident energy E<sub>0</sub> and moves parallel to the collision axis (zaxis)  $\theta_0=0$ . The output of the subroutine determines the number of created hadrons  $n_1$  as well as the energy  $E_1$  (<  $E_0$ ) and the direction  $\theta_1$  of the leading particle. The leading particle leads the reaction in its second round with the energy  $E_1$  and  $\theta_1$  as input parameters and creates new number of particles n<sub>2</sub> and so on. The number n<sub>i</sub> is determined according to a multiplicity generator, which depends on the square of the center of mass energy. Fig.(1) demonstrates the particle rapidity distribution produced in the first three- collisions as predicted by the model for the p- <sup>32</sup>S at 200 GeV. The yield (as measured by the area under the curve) decreases with the order of the collision because of the

appreciable drop in the energy after the successive collisions.

On the other hand, the family of curves concerning the different number of collisions acquires gradually decreased rapidity range. The overall distribution is compared with the experimental data CERN-NA-035 in Fig. (2). The comparison shows good agreement.

# 3-3 Nucleus-Nucleus Collisions

The extension of the multi peripheral model to the nucleus- nucleus case is more complicated. The number of available collisions is multi-folded due to the contribution of the projectile nucleons.

By analogy to the N-A collision, it is possible to define the phase space integral  $I_n^{AA}$  in A-A collisions as,

$$I_{n}^{AA}(s) = \sum_{j}^{A_{p}} \sum_{k}^{A_{T}} I_{n_{j,k}}(s_{j,k}) P_{AA}(j, A_{p}, k, A_{t})$$

$$\delta(n - \sum_{j,k} n_{j,k})$$
(14)

Where  $I_{n_{j,k}}(s_{j,k})$  is the phase space integral due to the knocked on nucleon number *j* from the projectile and that, number *k* from the target. The probability that the A-A collision encounters  $V_P$  collisions from the projectile and  $V_T$  collisions from the target is treated as independent events. So that,

$$P_{AA}(\nu_p, A_p, \nu_T, A_t) = P(\nu_p, A_p) \cdot P(\nu_T, A_t) \quad (15)$$

Another modification is carried out on this calculation that is to consider the screening effect of the projectile nucleons upon the interaction. This effect is summarized as follows. The first projectile nucleon will face the target nucleus as a whole, i.e. it will see the complete  $A_t$  nucleons and interacts with only  $V_{T1}$  of them. The next projectile nucleon will see that target as partially screened by the first. The target size in this case is  $A_t - V_{T1}$  and it interacts with only  $V_{T2}$  nucleons according to a probability function,  $P(v_{T2}, A_t - v_{T1})$ , by simple iteration, the i<sup>th</sup> projectile nucleon will see a target as  $A_t - \sum_{k=1}^{i-1} v_{T_k}$  and so on. The prediction of the model is applied to the <sup>32</sup>S-<sup>32</sup>S collisions at 200 A GeV incident energy.



Fig. (1) The particle rapidity distribution produced in p-S at multiple order collisions as predicted by the MPM.



Fig. (2) The particle rapidity distribution produced in p-S collision at 200 GeV incident proton energy as predicted by the MPM and compared with the experimental data CERN-NA-035.

The rapidity distribution of the produced particles in the first 3- projectile nock on nucleons is demonstrated in Fig. (3). While the overall rapidity distribution is compared with the data of the experiment CERN-NA-035 in Fig. (4). The fair agreement obtained in N-A Fig. (2) and A-A collisions Fig. (4) shows that the model can reproduce the basic features of the dn/dy.

# **Summary and Conclusive Remarks**

- 1- A scaling rapidity function is obtained for particles produced in N-N, N-A and A-A collisions which assumes that the reaction is built on N-N base.
- 2- The multi-peripheral model is extended to the nucleon-nucleus and the nucleus-nucleus interactions on bases of nucleon-nucleon collisions.
- 3- The phase space integral of the nucleon-nucleon collision is folded several times according to the number of encountered nucleons from the target.
- 4- The probability that v nucleons from the target are encountered by a projectile nucleon is calculated in terms of the nucleon-nucleon phase shift according to the eikonal approximation.
- 5- The number of created particles in each collision is summed over to get the production in the nucleonnucleus case, where the conservation of number of particles in the final state is taken into consideration.
- 6- In nucleus-nucleus collisions, we followed the statistics of independent events. The screening effect among the interacting projectile nucleons is also considered.

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Fig. (4) The particle rapidity distribution produced in S-S collision at 200 GeV incident proton energy as predicted by the MPM and compared with the experimental data CERN-NA-035.