

## Search for optimum technique of measuring high energy ( $10^{12}$ – $10^{16}$ eV) GCR which can provide maximum efficiency of the instrument at its minimum mass.

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**Abstract .** Different techniques of measuring the energy of the primary particle, by recording the total energy of gamma-quants, generated in a target with small atomic weight. It is shown, that these techniques give instrument efficiency per unit mass, which is  $\sim 2$  times smaller, than that of thick ionization calorimeter with spherically symmetric characteristics.

### 1 Introduction

Any instrument, intended for measuring GCR particles, should satisfy two major requirements: determine the kind of the particle (measure its charge  $Z$ ) and its energy  $E$ .

There are many techniques for measuring  $Z$ . Here there are many options, and we will not discuss them.

The situation with measuring  $E$  is much more complicated. Practically only two methods are used: a thick ionization calorimeter (IC) and a technique which combines a target made of light material with a detector for recording the energy of  $\gamma$ -quants, generated in the target by the primary particle. There is now a stable opinion, that in order to achieve high efficiency of an instrument (which is necessary for recording high energy particles the fluxes of which are small) the technique employing a light target +  $\sum E_\gamma$  detector has obvious advantages over a thick (and, therefore, heavy) ionisation calorimeter (Burnett et al., 1983; Seo et al., 1997; Aleksandrov et al., 1998). Below, we will show that this opinion is erroneous if a certain type of IC is used. The typical configuration for the target +  $\sum E_\gamma$  detector is the following: a target layer with the thickness of  $X_t$  g/cm<sup>2</sup> made of a material with small atomic weight and a relatively small range for inelastic interactions of primary particles  $\lambda_t$  and a  $\sum E_\gamma$  detector located under the target and made from a

material with large  $Z$ . We will consider a detector, made of lead and the target made from graphite.

The minimum thickness of the lead layer in the  $\sum E_\gamma$  detector should satisfy the condition

$X_{pb} = t_{\max} + (5 \div 6)$  cascade lengths;  $t_{\max}$  is the depth (in cascade lengths), where the number of particles in the cascade reaches maximum. The additional  $5 \div 6$  cascade lengths are necessary to assert with confidence, that the measured maximum number of particles corresponds to the maximum number of particles in the cascade, since only this determines the cascade energy. If the intention is to measure  $\sum E_\gamma = 10^{13} - 10^{15}$  eV, then for lead

$X_{pb} = 120 \div 150$  g/cm<sup>2</sup>. In order for a particle to be recorded, it should interact inside the target. The probability of the is  $P_{rec} = 1 - \exp(-\langle X \rangle / \lambda_t)$ . In this approximation (which differs from the exact expression by 10%)  $\langle X \rangle$  is the mean amount of matter, travelled by a particle in the target. We will call the expression  $\Gamma P_{rec}$  the efficiency of the instrument, where  $\Gamma$  is the geometry factor. For a given instrument mass, there is only one parameter, which can be changed arbitrarily, - it is the thickness of the target  $X_t$ . Decreasing  $X_t$ , we increase  $\Gamma$ , but decrease  $P_{rec}$ . Therefore the instrument efficiency has a maximum at a certain value of  $X_t$ .

We will introduce the parameter  $\kappa = \Gamma P_{rec} / M$ , which describes the efficiency of using the mass, for achieving maximum efficiency. Let us determine the maximum values of the  $\kappa$  parameter for the different configurations of the instrument, employing the target +  $\sum E_\gamma$  detector technique.

## 2 Flat structures (Fig 1a, 1b).

For a flat structure and isotropic distribution of primary particles :  $\kappa = \frac{\pi}{X_t + X_{pb}} [1 - \exp(-2X_t / \lambda_t)]$

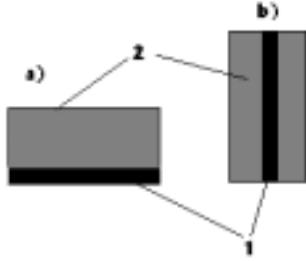


Fig. 1. The  $\sum E_\gamma$  detector -1, and the target -2.

If we introduce the variable  $X_t / \lambda_t = y$  and note, that,  $X_{pb} / \lambda_t = \frac{\lambda_{pb}}{\lambda_t} X_{pb} / \lambda_{pb} = 2.44B$  where  $B = 0.62 \div 0.77$

(we used the following values  $\lambda_{pb} = 195 \text{ g/cm}^2$ ,  $\lambda_t = 80 \text{ g/cm}^2$ ,  $X_{pb} = 120 \div 150 \text{ g/cm}^2$ ). Keeping this in mind, we obtain  $\kappa = 3.93 \cdot 10^{-2} (y + 2.44B)^{-1} (1 - e^{-2y})$ . The maximum values are:  $\kappa_{\max} = 1.01 \cdot 10^{-2} \text{ cm}^2 \text{ sr/g} = 1.01 \text{ m}^2 \text{ sr/ton}$  at  $y = 0.675$  ( $B = 0.62$ )  $\kappa_{\max} = 0.90 \cdot 10^{-2} \text{ cm}^2 \text{ sr/g} = 0.9 \text{ m}^2 \text{ sr/ton}$  at  $y = 0.735$  ( $B = 0.77$ ).

The 'b' version is also flat, but the  $\sum E_\gamma$  detector is surrounded by the target from both sides, and the detector plane is located vertically. Due to this the instrument records the particles, coming from the lower hemisphere, not shaded by the Earth, increasing  $\Gamma$  by a factor of 1.5.

The expression for  $\kappa$  has the form:  $\kappa = 5.9 \cdot 10^{-2} (y + 2.44B)^{-1} (1 - e^{-y})$ , where  $y = 2X_t / \lambda_t$ . Sometimes, an opinion is expressed, that in a flat structure the thickness of the lead  $\sum E_\gamma$  detector and the target can be decreased, and due to this the detector area  $S$ , and correspondingly the geometry factor can be increased.

Since the primary particles fall on the instrument isotropically, there is always a fraction of particles, which will travel through a sufficient amount of matter in the target and  $\sum E_\gamma$  detector, in order to be recorded. Though this approach undoubtedly increases  $S$ , does it increase the efficiency?

Let us assume, that we decrease the thickness of the  $\sum E_\gamma$  detector to the value of  $X_{pb}^* < X_{pb}$ . In this case the particles cascades arriving at the  $\sum E_\gamma$  detector at the angle of  $\theta < \theta_0$ , where  $\cos \theta_0 = X_{pb}^* / X_{pb}$ , will not develop to their maximum number of particles, and therefore, will not be used in the measurements. Therefore, the particles will be recorded only when their falling angles on the detector are within the range from  $\theta_0$  to  $90^\circ$ . For these particles the geometry factor is equal to  $\Gamma = \pi S \cos^2 \theta_0$  and  $S = M / (X_t + X_{pb}^*)$ . The mean amount of matter, traveled in the target will be  $2X_t / \cos \theta_0$  and  $P_{rec} = 1 - \exp(-2X_t \lambda_t \cos \theta_0)$ . Introducing all these values in to the expression for the  $\mathbf{K}$  parameter and substituting  $X_t / \lambda_t \cos \theta_0 = y$ , we obtain  $\kappa = \frac{\pi}{\lambda_t} (y + 2.44B)^{-1} (1 - e^{-2y}) \cos \theta_0 = \kappa_0 \cos \theta_0$ , where

$\mathbf{K}_0$  is the value at  $\theta_0 = 0^\circ$ . As we can see a decrease of the target and  $\sum E_\gamma$  detector thickness only decreases the instrument efficiency.

## 3 Spherical structures (Fig 2).

Let us consider the same particle recording technique : target and  $\sum E_\gamma$  detector but in a 'spherical' configuration, i.e. when particles are recorded within  $4\pi$  (see Fig.2).

In this configuration the  $\sum E_\gamma$  detector with thickness of  $X_{pb} / 2$  is a surface with spherical or cubic form. The interaction of the primary particle occurs in the target, and the generated gamma-quants create an electromagnetic cascade in two parts of the  $\sum E_\gamma$  detector, i.e. the cascade travels the total thickness of the lead  $X_{pb}$ . The geometry factor of such an instrument is determined by the geometry factor of the detector  $\sum E_\gamma$ , and the recording probability  $P_{rec}$  is determined by the mean amount of matter in the target over the unit area of the  $\sum E_\gamma$  detector.

### 3.1 Spherical target + spherical $\sum E_\gamma$ detector. (Fig.2a).

Let the  $\sum E_\gamma$  detector radius be  $r$  cm. Then the geometry factor with account for shading by the Earth

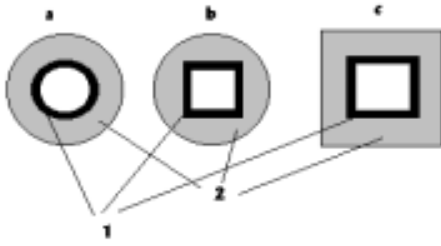


Fig.2. The  $\sum E_\gamma$  detector -1, and the target -2.

( $H = 500$  km) is equal to  $\Gamma = 9\pi r^2$ . The  $\sum E_\gamma$  detector mass is equal to  $4\pi r^2 \cdot X_{pb} / 2$  g and the mean amount of target matter per unit detector surface  $\langle X \rangle = (M - 2\pi r^2 X_{pb}) / 4\pi r^2 = M / 4\pi r^2 - 0.5 X_{pb}$ .

Thus,  $\Gamma P_{rec} = 9\pi r^2 [1 - \exp(X_{pb} / 2\lambda_t) \cdot \exp(-M / 4\pi r^2 \lambda_t)]$ . If, as before, we write down  $X_{pb} / \lambda_t = 2.44B$  (where  $B = X_{pb} / \lambda_{pb} = 0.62 \div 0.77$ ), and denote  $y = M / 4\pi r^2 \lambda_t$ , then we obtain:

$$\kappa = \frac{\Gamma P_{rec}}{M} = (2.25 / \lambda_t) (1 - e^{1.22B} e^{-y}) y^{-1} = 2.81 \cdot 10^{-2} (1 - e^{1.22B} e^{-y}) y^{-1}$$

This expression has a maximum value  $\kappa_{max} = 0.01$  cm<sup>2</sup> sr/g = 1.0 m<sup>2</sup> sr/ton at  $y = 1.78$  ( $B = 0.62$ ),  $\kappa_{max} = 0.92$  m<sup>2</sup> sr/ton at  $y = 2.06$  ( $B = 0.77$ ).

3.2. a cubic  $\sum E_\gamma$  detector and a spherical (b) or cubic (c) target.

If the side of the  $\sum E_\gamma$  detector cube is  $a$  cm, then the geometry factor is  $\Gamma = 4\pi a^2$ . The mass of the target is  $M - 6a^2 X_{pb} / 2 = M - 3a^2 X_{pb}$  and the mean amount of matter, per unit  $\sum E_\gamma$  detector area is equal to

$$\langle X \rangle = M / 6a^2 - 0.5 X_{pb}. \text{ Thus,}$$

$$\Gamma P_{rec} / M = 2\pi / 3\lambda_t (1 - e^{1.22B} e^{-y}) y^{-1} = 2.62 \cdot 10^{-2} (1 - e^{1.22B} e^{-y}) y^{-1}$$

As in the previous case, the maximum values are reached at  $y = 1.78$  ( $B = 0.62$ ) and  $y = 2.06$  ( $B = 0.77$ ).

These values give  $\kappa_{max} = 0.94$  m<sup>2</sup> sr/ton and 0.86 m<sup>2</sup> sr/ton, respectively.

4. Cubic  $\sum E_\gamma$  detector with spherical symmetry

Finally we will consider a configuration, where the  $\sum E_\gamma$  detector is a cube with side  $a$  made of a material

with the density of  $\rho$  g/cm<sup>3</sup>, with spherical symmetry of characteristics. The upper side of the cube and the four side surfaces are surrounded by a light target with thickness  $X_t$ , g/cm<sup>2</sup> (Fig.3.)

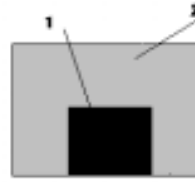


Fig.3 The  $\sum E_\gamma$  detector -1, and the target -2.

The sum of the target mass  $M_t$  and the  $\sum E_\gamma$  detector mass  $M_d$  is fixed, i.e.  $M_t + M_d = M = const$ . In this case only the target thickness  $X_t$  and the size of the  $\sum E_\gamma$  detector are varied, i.e. the  $a$  value, in such a way, so as to achieve the maximum value of the instrument efficiency. As before,  $P_{rec} = 1 - \exp(-M_t / 6a^2 \lambda_t)$ . The results of this calculation, made for  $\rho = 6$  and  $7$  g/cm<sup>3</sup> for three different values of the instrument mass  $M = 3, 5,$  and  $9$  tons are shown by dashed lines in Fig.4.

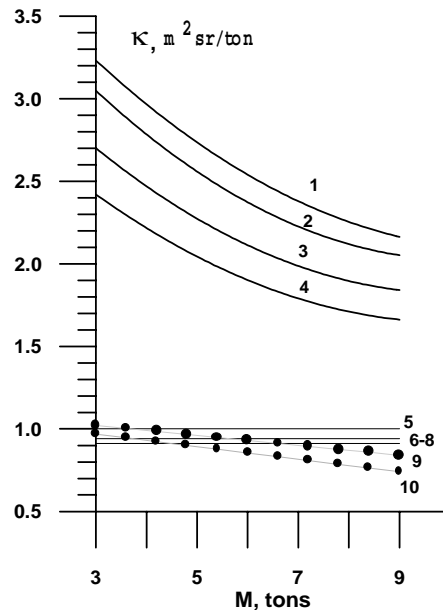


Fig.4. The  $\kappa$ -parameter versus mass.

## 5 Discussion

The final result is the following: in all the configurations (except one) instrument, employing the concept of a light target +  $\sum E_\gamma$  detector does not permit to achieve efficiency, exceeding 1 m<sup>2</sup>sr per 1 ton of the instrument mass. The exception is the flat version, where the detector plane is located vertically.

Finally, we will consider the capabilities of a thick ionization calorimeter of cubic form with spherically symmetrical characteristics (Grigorov et al., 1997). If the side of the cube is  $a$  cm and the absorber in the IC  $\rho$  g/cm<sup>3</sup>, then the instrument mass is  $M = \rho a^3$ , and the geometry factor  $\Gamma = 4\pi a^2$ . Since in this case  $P_{rec} \cong 1$ , then the efficiency  $\Gamma P_{rec} = \Gamma$  and  $\kappa = \Gamma / M = 4\pi / M^{1/3} \rho^{2/3}$ .

The dependence of  $\kappa$  was calculated for different  $\rho = 4.5; 5; 6; 7$  g/cm<sup>3</sup> and  $M = 3.5$  and 9 tons. The results are shown in Fig.4. (curves 1-4) The same figure shows the previous results (curve 5 corresponds to Fig.1; curves 6-8 to Fig.2.; curves 9 and 10 to Fig.3.). It can be seen from Fig.4., that the IC has efficiency per unit mass, which exceeds by a factor of 2-3 that of the target +  $\sum E_\gamma$  detector configuration.

In order to avoid misunderstandings, it should be mentioned, that the advantage in  $\kappa$  for the ionization calorimeter is only achieved when the IC has spherical symmetry of the parameters, i.e. records with equal efficiency particles, arriving from arbitrary directions within  $4\pi$ . For regular IC with restricted angular aperture the  $\kappa$  parameter is smaller than 0.1 m<sup>2</sup>sr/ton.

## References

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