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## Types of particle oscillations and their realizations in $K^o$ and $\nu$ oscillations

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**Abstract.** Two particle vacuum transitions (oscillations) were studied in the general case. We found that: 1) a nondiagonal mass term characterizing oscillations is the width of two particle transitions into each other (this width can be computed by the standard method); 2) two types of oscillations take place: real and virtual. Solution of the problem of origin of mixing angle in the theory of vacuum oscillations was given. It is shown that  $K^o$ -meson and neutrino oscillations must proceed via two stages.

### 1 Introduction

In the old theory of neutrino oscillations [1, 2], constructed in analogy with the theory of  $K^o$ ,  $\bar{K}^o$  oscillation, it is supposed that mass eigenstates are  $\nu_1, \nu_2, \nu_3$  neutrino states but not physical neutrino states  $\nu_e, \nu_\mu, \nu_\tau$ , and that the neutrinos  $\nu_e, \nu_\mu, \nu_\tau$  are created as superpositions of  $\nu_1, \nu_2, \nu_3$  states. This means that the  $\nu_e, \nu_\mu, \nu_\tau$  neutrinos have no definite mass, i.e. their masses may vary depending on the  $\nu_1, \nu_2, \nu_3$ admixture in the  $\nu_e, \nu_\mu, \nu_\tau$  states (naturally, in this case the low of conservation the energy and the momentum of the neutrinos is not fulfilled). Besides, every particle has mass shell and it will be left on its mass shell at passing through vacuum. Probably, this picture is incorrect one.

### 2 Probability of $a \xrightarrow{B} b$ Vacuum Transitions (Oscillations)

Let us consider two particles (states) a, b having numbers (it can be  $K^o, \bar{K}^o; K_1^o, K_2^o$  or  $\nu_e, \nu_\mu$ ) which can transit each into the other. We can use the mass matrix of a, b particles for consideration of transitions between these particles in the framework of the quantum theory (or particle physics) since the mass matrix is eigenstate of a type of interaction which creates these particles (see below).

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The mass matrix of a and b particles has diagonal form. Due to the presence of a interaction violating their numbers, a nondiagonal term appears in this matrix and then this mass matrix is transformed in the following nondiagonal matrix (CP-is conserved):

$$\left(\begin{array}{cc} m_a & m_{ab} \\ m_{ab} & m_b \end{array}\right),\tag{1}$$

which is diagonalized by turning through the angle  $\beta$  and [2]

$$tg2\beta = 2m_{ab}/|m_a - m_b|,$$
  

$$sin2\beta = 2m_{ab}/\sqrt{(m_a - m_b)^2 + (2m_{ab})^2}.$$
 (2)  

$$\begin{pmatrix} m_{a'} & 0\\ 0 & m_{b'} \end{pmatrix},$$
  

$$m_{a',b'} = \frac{1}{2} \left[ (m_a + m_b) \pm \left( (m_a - m_b)^2 + 4m_{ab}^2 \right)^{1/2} \right].$$
 (3)

It is interesting to remark that expression (3) can be obtained from the Breit-Wigner distribution [3]

$$P \sim (\Gamma/2)^2 / [(E - E_0)^2 + (\Gamma/2)^2]$$
 (4)

by using the following substitutions:

$$E = m_b, \ E_0 = m_a, \ \Gamma/2 = 2m_{ab},$$
 (5)

where  $\Gamma/2 \equiv W(...)$  is width of  $a \rightarrow b$  transition, then we can use standard method [4] for computing this value.

We can see that here take place two cases of a, b transitions (oscillations): real and virtual oscillations.

1. If we consider the real transition of a into b particle then

$$\sin^2 2\beta \cong 4m_{ab}^2 / [(m_a - m_b)^2 + 4m_{ab}^2], \qquad (6)$$

if the probability of the real transition of a particles into b particles through a interaction (i.e.  $m_{ab}$ ) is very small then

$$\sin^2 2\beta \cong 4m_{ab}^2/(m_a - m_b)^2 \cong 0.$$

How can we understand this real  $a \rightarrow b$  transition?

If  $2m_{ab} = \frac{\Gamma}{2}$  is not zero, then it means that the mean mass of *a* particle is  $m_a$  and this mass is distributed by  $sin^2 2\beta$  (or by the Breit-Wigner formula) and the probability of the  $a \rightarrow b$  transition differs from zero and it is defined by masses of *a* and *b* particles and  $m_{ab}$ , which is computed in the framework of the standard method as it is pointed out above. So, this is a solution of the problem of origin of mixing angle in the theory of vacuum oscillations.

In this case probability of  $a \rightarrow b$  transition (oscillation) is described by the following expression:

$$P(a \to b, t) = \sin^2 2\beta \sin^2 \left[ \pi t | m_{b'}^2 - m_{a'}^2 | / 2p_a \right], \quad (7)$$

where  $p_a$  is momentum of *a* particle.

2. If we consider the virtual transition of a into b particle then, since  $m_a = m_b$ ,

$$tg2\beta = \infty, \tag{8}$$

i.e.  $\beta = \pi/4$ , then  $sin^2 2\beta = 1$ .

In this case probability of  $a \rightarrow b$  transition (oscillation) is described by the following expression:

$$P(a \to b, t) = \sin^2 \left[ \pi t 4m_{ab}^2/2p_a \right], \qquad (9)$$

To make these virtual oscillations real their participant in quasielastic interactions is necessary for their transitions to own mass shells [5].

It is clear that the process of  $a \rightarrow b$  transition is a dynamical process and at the beginning (i.e. at t = 0) here is no superposition of a', b' particles (states).

Let us pass to consideration of concrete transitions (oscillations) between different type particles (states).

### **3** $K^o, \bar{K}^o$ - oscillations

1) The  $K^o$ ,  $\bar{K}^o$ -mesons, which consist of the  $s, \bar{s}, d, \bar{d}$  quarks, are created in the strong interactions (the typical time of strong interactions are  $t_{\rm Str} \cong 10^{-23}$  s.) and are, accordingly, eigenstates of these interactions, i.e. the mass matrix of the  $K^o, \bar{K}^o$  mesons is diagonal.

2) If we take into account the weak interaction (typical times of weak interactions are  $t_{\text{weak}} \cong 10^{-8}$  s.) which violates strangeness, then the mass matrix of  $K^o$ -mesons will become nondiagonal. If we diagonalize this matrix, then we will come to the  $K_1^o, K_2^o$  states, which are eigenstates of the weak interaction [1]. So we can see that, if  $K^0$ -mesons are created in strong interactions, then  $K^o, \bar{K}^o$  mesons are produced, and if  $K^o$  mesons are created in weak interactions then  $K_1^o, K_2^o$  mesons are created. In second case no oscillations of  $K^o$  mesons will occur.

Now let us to give a phenomenological description of  $K^o, \bar{K}^o$  meson creation and oscillation processes. We will consider the creation of  $K^o, \bar{K}^o$ -mesons as a quasistationary process with a typical time  $t_{\rm str}$ . Within of this typical time  $-t_{\rm str}$ , weak interactions will violate strangeness and result

in the mass matrix of the  $K^{o}$ -mesons becoming nondiagonal. The probability for this process to occur in  $t = \pi t_{str}$  is:

$$W(t = \pi \Delta t_{str}) = (1 - e^{-\frac{t}{\Delta t_{str}}})/(1 - e^{-\frac{t}{\Delta t_{str}}}) \simeq \pi \Delta t_{str}/\Delta t_{weak} \simeq \pi \cdot 10^{-15},$$
(10)

where  $(1 - \exp(-t/t_{str,weak}))$  - is the decay probability of the quasistationary state during the time -t. The mass matrix of the  $K^o$ -mesons will become nondiagonal in  $t = \pi 10^{-23}$ s. with a probability of  $W \cong \pi 10^{-15}$ . And then the  $K_1^o, K_2^o$ mesons–eigenstates of weak interactions will be created. So we can see that in this case mainly  $K^o, \bar{K}^o$  mesons will be produced but not the  $K_1^o, K_2^o$ -mesons.

3) Then, when the  $K^o$ ,  $\bar{K}^o$  mesons, that were created in strong interactions, pass through vacuum, the mass matrix of the  $K^o$  mesons will become nondiagonal, owing to the presence of weak interactions violating strangeness. Diagonalizing it, we get  $K_1^o, K_2^o$ -meson states which are eigenstates of weak interactions. Obviously, the  $K^o, \bar{K}^o$  mesons are, then, converted in to superpositions of  $K_1^o, K_2^o$ -mesons

$$K^{o} = (K_{1}^{o} + K_{2}^{o})/\sqrt{2}, \qquad \bar{K}^{o} = (K_{1}^{o} - K_{2}^{o})/\sqrt{2}.$$
 (11)

Then, oscillations of the  $K^o$ ,  $\bar{K}^o$  mesons will take place on a background of  $K_1^o$ ,  $K_2^o$  decays. The length of these oscillations is [1, 6]:

$$L_{osc} = 2.48 p_{K^o}(MeV) / |m_{K_1^o} - m_{K_2^o}|^2 (eV)^2 \quad (12)$$

 $p_{K^o}$  is the momentum of  $K^o$ .

The main question which arises now is: which type of oscillations real (implying actual transitions between the particles) or virtual (implying virtual transitions between particles without transition to mass shells) take place between the  $K^o$ ,  $\bar{K}^o$ -mesons? Since the masses of  $K^o$  and  $\bar{K}^o$  mesons are equal, oscillations between these mesons are real. But, if the masses of  $K^o$  and  $\bar{K}^o$  mesons were not equal, then the oscillations would be virtual (the case of  $K_1^o$ ,  $K_2^o$  transitions was considered in [7]).

So, the mixings (oscillations) appear since at creating of  $K^o$  mesons are realized eigenstates of the strong interaction (i.e.  $K^o, \bar{K}^o$  mesons) but not eigenstates of the weak interaction violating strangeness (i.e.  $K_1^o, K_2^o$  mesons) and then, when they pass through vacuum they are converted into superpositions of  $K_1^o, K_2^o$  mesons. If  $K_1^o, K_2^o$  mesons were originally created then mixings (oscillations) would not take place since the strong interaction conserves strangeness and isospin.

#### 4 *ν*-oscillations

We can now pass to the analysis of three neutrino oscillations, taking advantage of the example of  $K^o$ ,  $\bar{K}^o$ -meson oscillations. 1) The physical states of the  $\nu_e$ ,  $\nu_\mu$ ,  $\nu_\tau$  neutrinos are eigenstates of the weak interaction and, naturally, the mass matrix of  $\nu_e$ ,  $\nu_\mu$ ,  $\nu_\tau$  neutrinos is diagonal. All the available, experimental results indicate that the lepton numbers  $l_e$ ,  $l_\mu$ ,  $l_\tau$  are well conserved i.e. the standard weak interactions do not violate the lepton numbers.

2) Then, to violate the lepton numbers, it is necessary to introduce an interaction violating these numbers. It is equivalent to introducing nondiagonal mass terms in the mass matrix of  $\nu_e, \nu_\mu, \nu_\tau$ . Diagonalizing this matrix we go to the  $\nu_1, \nu_2, \nu_3$  neutrino states. Exactly like the case of  $K^o$  mesons creating in strong interactions, when mainly  $K^o, \bar{K}^o$  mesons are produced, in the considered case  $\nu_e, \nu_\mu, \nu_\tau$ , but not  $\nu_1, \nu_2, \nu_3$ , neutrino states are mainly created in the weak interactions (this is so, because the contribution of the lepton numbers violating interactions in this process is too small). And in the case 2) no oscillations take place.

3) Then, when the  $\nu_e, \nu_\mu, \nu_\tau$  neutrinos pass through vacuum, they will be converted into superpositions of the  $\nu_1, \nu_2, \nu_3$ owing to presence of the interactions violating the lepton number of neutrinos and will be left on their mass shells. And, then, oscillations of the  $\nu_e, \nu_\mu, \nu_\tau$  neutrinos will take place according to the standard scheme [1]. Whether these oscillations are real or virtual will be determined by the masses of the physical neutrinos  $\nu_e, \nu_\mu, \nu_\tau$ .

i) If the masses of the  $\nu_e, \nu_\mu, \nu_\tau$  neutrinos are equal, then real oscillation of the neutrinos will take place.

ii) If the masses of the  $\nu_e, \nu_\mu, \nu_\tau$  are not equal, then virtual oscillation of the neutrinos will take place. To make these oscillations real, these neutrinos must participate in the quasielastic interactions, in order to undergo transition to the mass shell of the other appropriate neutrinos by analogue with  $\gamma - \rho^o$  transition in the vector meson dominance

model. In case ii) enhancement of neutrino oscillations will take place if the neutrinos pass through a bulk of matter [8].

So, the mixings (oscillations) appear since at neutrinos creating are realized eigenstates of the weak interaction (i.e.  $\nu_e, \nu_\mu, \nu_\tau$  neutrinos) but not eigenstates of the weak interaction violating lepton numbers (i.e. (i.e.  $\nu_1, \nu_2, \nu_3$  neutrinos) and then, when they pass through vacuum they are converted into superpositions of  $\nu_1, \nu_2, \nu_3$  neutrinos. If  $\nu_1, \nu_2, \nu_3$  neutrinos were originally created then mixings (oscillations) would not take place since the weak interaction conserves lepton numbers.

The above considered approach for consideration of mass mixings is the mass mixings approach besides of this approach is an another approach, the charge mixings one, which is used in the vector dominance model [9].

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