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On the internal boundary condition s problem for the Parker's transport equation of galactic cosmic rays

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ABSTRACT

Parker's 2-dimentional transport equation has been numerically solved to investigate the influence of the different radial internal boundary conditions on the expected distributions of the density of galactic cosmic rays. The radial internal boundary conditions, obtained from the physical assumptions, $\left.\frac{\partial n}{\partial r}\right|_{r=0}=0$ and $\left.\frac{\partial n}{\partial r}\right|_{r=r1}=0$ (where *n* is relative density of galactic cosmic rays and r and r_1 are the relative radial distances from the Sun) and $\left.\frac{\partial n}{\partial r}\right|_{r=0}\neq 0$, obtained from the Parker's transport equation (with the singular point , r = 0) have been considered. The numerical solutions of the transport equation with drift for the different radial internal boundary conditions and for the various ratios of the perpendicular and parallel diffusion coefficients of galactic cosmic rays have been compared. It is concluded that for the solving of the Parker's transport diffusion equation (possessing a singular point, r = 0) one must use the boundary condition, $\left.\frac{\partial n}{\partial r}\right|_{r=0}\neq 0$.

INTRODUCTION

Problems of initial and boundary conditions for the differential equations up to day remain as a vital subject of investigation [1]. The Parker's anisotropy diffusion equation [2] describing a propagation of galactic cosmic rays (GCR) in interplanetary space is not an exception one in this direction. In fact, a choice of the internal boundary condition for Parker's transport equation at the point r = 0 or near this point is more or less clear [3, 6] based on the physical assumption about the distribution of the intensity of galactic cosmic rays (GCR). Nevertheless, from the point of view of the mathematical exactness the internal boundary condition obtained from the physical assumption is not justified with respect to the singular point (as is the point corresponding to the Sun's location in the interplanetary space) [5, 6]. It is true that the internal boundary condition at the point r = 0 or near it could not sufficiently influence on the distribution of the GCR intensity in the interplanetary space (with the dimension of the modulation region of the tens of astronomical units). However, to choose the internal boundary condition without a rigorous method is not an acceptable argument from the point of view of the mathematical accuracy; the more so, that there exists the method [7] how to find the boundary conditions for the differential equations containing a singular boundary (or singular point).

DETERMINATION OF THE INTERNAL BOUNDARY CONDITION

Parker's transport equation [2] has a form:

$$\frac{\partial N}{\partial t}=\nabla_i\left(k_{i,j}\nabla_k N\right)-\nabla_i\left(U_i N\right)+\frac{1}{3}\frac{\partial}{\partial R}(NR)\nabla_i U_i \qquad (1)$$

where N, and R are density (in interplanetary space) and rigidity of GCR particles, respectively; K_{ij} is diffusion tensor consisting from the symmetric and anti symmetric parts; U_i is the solar wind velocity and t – time. The equation (1) in the spherical coordinate system ρ, θ, ϕ, for 2-dimentional ($\frac{\partial n}{\partial \phi}=\frac{\partial^2 n}{\partial \phi^2}$=0), steady-state case (neglecting the term ∂N/∂t), can be written:

$$A_1\frac{\partial^2 n}{\partial r^2}+A_2\frac{\partial^2 n}{\partial \theta^2}+A_3\frac{\partial n}{\partial r}+A_4\frac{\partial n}{\partial \theta}+A_5 f+A_6\frac{\partial n}{\partial R}=0 \qquad (2)$$

The dimensionless density, n = N/N_0 , where N_0 is density of GCR in the interstellar medium accepted as,

$N_0 \propto R^{-2.5}$ for the rigidities R to which neutron monitors are sensitive; the dimensionless distance $r = \rho/r_0$, where r_0 is the size of the modulation region and ρ is the distance from the Sun; The $A_1, A_2, \ldots, A_6$ are the function of r, θ, and R. The parallel diffusion coefficient, $K_{||}$ is represented as ,

$K_{||}=K_0 K(r) K(R)$, where (3)

$K(r) = 1 + \alpha_0 r$, $K(R) = R$

K_0 is equal to the 2×10^{22} cm^{2}s^{-1} for the energy of 10 GeV. the radius r_0 of the modulation region is 100AU and the solar wind velocity U equals 4×10^{7} cm/s. All functions $A_1,\ldots, A_6$ contain the expression r^β with the power, $\beta\geq1$, i.e. the equation (2) has the singular point, at r = 0. The functions A_1, A_2, A_4 and some addends of the A_3 contain the expression r^β with the $\beta >1$; at the same time one addend of function A3, and functions, A_5 and A_6 contain only the expression r^β with the power, $\beta =1$. According to the [7] there can be found an internal boundary condition from the differential equation with the singular boundary by the way, as is has been done in [5, 6]. All terms of the equation (2) must be cancelled by the expression r^β with the possibly maximum value of the power β. In the case of the equation (2), β maximum equals 1. After dividing all terms by the r and then tend to zero r in the all rest terms where r still exists, one can obtain:

$A_1 = 0$, $A_2 = 0$, $A_3 = 2$, $A_4 = 0$, $A_5 = -3$ S, $A_6 = 2$ S R/3

$$2\frac{\partial n}{\partial r} + A_5 n + A_6\frac{\partial n}{\partial R} = 0\,, \qquad (4)$$

From the equation (4) for the point r = 0, can be written:

$$\left.\frac{\partial n}{\partial r}\right|_{r=0} = \alpha\, n + \beta\frac{\partial n}{\partial R}, \qquad (5)$$

where, $\alpha = -3$ S/2, $\beta =$ S R/3 , and $S= Ur_0/K_0$.

The expression (5) is the internal boundary condition at the point r = 0 for the equation (2), which is using to solve the equation (2), e.g. in [8]. An internal boundary condition,

$$\left.\frac{\partial n}{\partial r}\right|_{r=0} = 0, \qquad (6)$$

can be obtained owing to the parabolic approximation using the average values of the densities around the centre (r = 0) of the coordinate system [5]. The more widely used popular boundary condition,

$$\left.\frac{\partial n}{\partial r}\right|_{r=r1} = 0, \qquad (7)$$

can be obtained based on the absorbing or reflecting conditions near the Sun or to assume that the radial gradient of the intensity of GCR just outside the boundary $r \geq r_1$ is equal to the gradient just inside $0\leq r \leq r_1$ e.g. in [3, 4, 9].

Before to discuss the influence of the different internal boundary conditions on the solution of the equation (2) there must be considered the case when the ratio α of the perpendicular and parallel diffusion coefficients ($\alpha = K_\perp/ K_{||}$) is assumed to be constant, e.g. as, in [10]. In this case, e.g. the function A_2 in the equation (2) does not contain the expression r^β (with the, $\beta \geq1$) and it is not possible to use the above-mentioned method for the finding of the internal boundary condition. In this case the determination of the internal boundary condition can be considered in two stages. First of all one can underline that near the region, $r\to0$ the IMF is so much strong that (in the equation (2) the strength H of the IMF is proportional to the distance r according to the Parker's spiral rule as, $H \propto 1/r^2$) a perpendicular diffusion of GCR can be neglected. So, the ratio, $\alpha = K_\perp/ K_{||}$ tends to zero at the point r = 0. The ratio α_1 of the drift K_d and parallel diffusion coefficients ($\alpha_1 = K_d/ K_{||}$) is proportional to Sinψ, which equals zero at the point, r =0. Thus, in the equation (2) remain only terms proportional to the expression of the r^β and the internal boundary condition can be found as above. Now, having different internal boundary conditions,

(1) $\left.\frac{\partial n}{\partial r}\right|_{r=0} = 0$ and (2) $\left.\frac{\partial n}{\partial r}\right|_{r=0} = \alpha n + \beta\frac{\partial n}{\partial R}$

there is possible to investigate the role of each one in the solution of the equation (2).The equation (2) numerically was solved using the difference grid scheme for $\alpha= (1 + \omega^2\tau^2)^{-1}$, where $\omega\tau =300H\lambda R^{-1}$; H is the strength of the IMF and λ - the transport free path of GCR particles. At the Earth's orbit H = 5 nT, $\lambda = 2\times10^{12}$ cm, and $\omega\tau =3$, for the energy of 10GeV and then it changes depending on the spatial coordinates according to the Parker's spiral magnetic field [2]. At the boundary of the modulation region (Z =100AU) α tends to 1. Solutions of the equation (2) for the $\alpha_0 = 100$ are presented in Figures 1 and 2 for the different directions (qA >0 and qA <0) of the Sun's global magnetic field. It is seen from these figures that at the point r = 0, there are differences between the values of the densities of cosmic rays for different boundary conditions, but for the distances more than 0.5AU these differences are vanishing. The differences between the values of the densities of cosmic rays at the point r = 0 for different boundary conditions have the similar character (in Fig.1, 2) for the different ratios α of the perpendicular and parallel diffusion coefficients. Thus, as far the internal boundary condition (5) is obtained from the differential equation (with the singular point, r = 0) without any additional assumption, the (5) can be considered as the mathematically justified and acceptable radial internal boundary condition for the solving the Parker's transport equation (2).

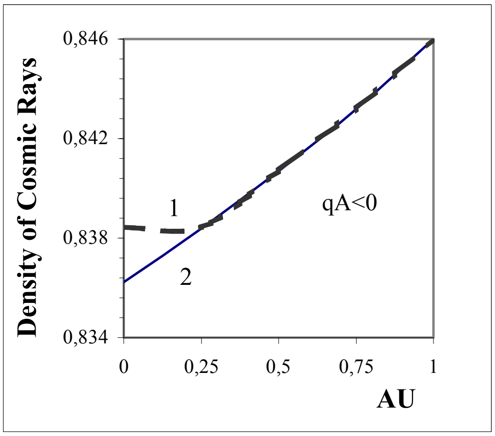

Fig.1. Changes of the expected density of cosmic rays for the internal boundary conditions

1) $\left.\frac{\partial n}{\partial r}\right|_{r=0} = 0$, (dashed line) and

2) $\left.\frac{\partial n}{\partial r}\right|_{r=0} = \alpha n + \beta \frac{\partial n}{\partial R}$ (solid line) in qA<0 .

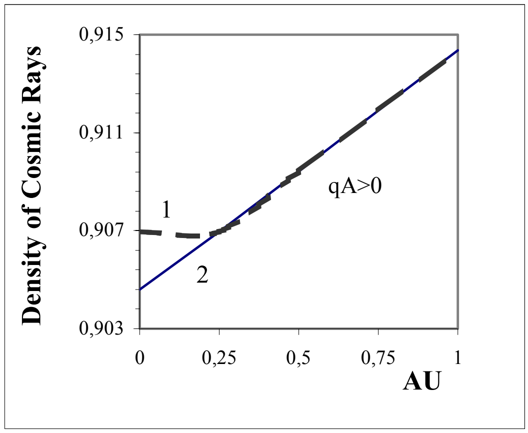

Fig.2. Changes of the expected density of cosmic rays for the internal boundary conditions

1) $\left.\frac{\partial n}{\partial r}\right|_{r=0} = 0$, (dashed line) and

2) $\left.\frac{\partial n}{\partial r}\right|_{r=0} = \alpha n + \beta \frac{\partial n}{\partial R}$ (solid line) in qA>0.

CONCLUSION

For the solving of the Parker's transport diffusion equation (possessing a singular point, r = 0) one must use the boundary condition $\left.\frac{\partial n}{\partial r}\right|_{r=0} = \alpha n + \beta \frac{\partial n}{\partial R}$, which is obtained from the differential equation (2) without any additional physical assumption.

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