Proceedings of ICRC 2001: 3185 © Copernicus Gesellschaft 2001

ICRC 2001

Spectra of solar energetic protons derived from statistical analysis of experimental data on a large set of events.

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Abstract. The presented analysis of the energy spectra of solar energetic protons differs from the earlier analyses in that not only a broader energy range is analyzed, but also the method of simultaneous statistical analysis of a large data set is used. The SEP events supported by the proton peak flux data in a broad energy range (METEOR, IMP8, GOES, balloons, and ground-based neutron monitors) are analyzed. The proton energy for the analyzed events turns out to range from 4 MeV to 11 GeV. The analysis has shown that the ≥30 MeV proton energy spectra in SEP events can most adequately be described by power-law functions of rigidity.

1 Introduction

The solar physics is posed with a basic question as to how the high-energy particles are generated. The different mechanisms of particle acceleration and subsequent propagation through interplanetary space seem to result in the energy spectra that are described by different analytical forms, which are still open to discussion. The main associate difficulty lies in the peculiar experimental evidence for the SEP fluxes measured within different energy ranges by different experimental techniques. The proton fluxes, which are measured in the same SEP events on different satellites, turn out often to differ from each other by three-four times or even more. At the same time the difference in the indexes of their respective SEP event spectra can exceeded unity. The methodological uncertainties for relativistic proton flux sizes observed in a ground level events (GLE) are sometimes estimated to reach some 50%. The balloon-measured SEP fluxes are not always the peak fluxes because, on ascending to the stratosphere, the balloons are often late to fix the particle flux maximum.

Analyzing the experimental SEP flux data to find the functional form of the particle energy spectra has yielded incompatible results. For example, the analysis of the energy spectra of protons and alpha particles in separate SEP events (Freier and Webber, 1963) has led the authors to conclude that rigidity exponent is the best to describe the particle energy spectra. At the same time, the authors of the catalogues (Bazilevskaya et al., 1986, 1990; Sladkova et al., 1998) used to describe the SEP event energy spectra by power-law function of particle energy. Having analyzed the particle fluxes in a set of SEP events, Nymmik (1993) demonstrated that the power-law rigidity function was the best to describe the ≥30 MeV solar proton energy spectra.

The reliability of determining the energy spectrum form depends essentially on the energy range selected in an analysis to distinguish between the spectral forms. It is obvious that a rigidity exponent cannot be used to describe the energy spectra at >100 MeV (see Figure 1). In an attempt to describe the energy spectra by a power-law function of energy, any sufficiently broad energy range has to be divided into arbitrary intervals of different spectral indices. The reliability seems also to depend on the analysis methods, which must permit eliminating the effect of the methodological errors on the eventual results. One of the possible techniques for reducing the ambiguity in analyzing the SEP energy spectra is to use all the available data simultaneously. In the present analysis, not only the solar proton energy spectra are treated in a broader energy range, but also the aggregate set of SEP events is analyzed.

2 Method of analysis

The Sun-accelerated proton energy spectra are usually approximated by three functions, namely, the exponential rigidity function and the power-law rigidity and energy functions.

The exponental function of rigidity used was:

$$F(E)dE = F(R)\frac{dR}{dE}dE = C\cdot\exp\left(-\frac{R}{R_o}\right)\cdot\frac{dE}{\beta} \qquad (1)$$

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where $R_o = 239$ MV/nucleon is a selected parameter,

$R = \sqrt{E\left(E + \left(2mc^2\right)^2\right)}$ is a proton rigidity,

$\beta = R\Big/\sqrt{R^2 + \left(2mc^2\right)^2}$ is a relative particle velocity.

The power-law function of the particle energy E used is:

$$F(E)dE = C \cdot \left(\frac{E}{E_o}\right)^{-\gamma} dE \qquad (2)$$

where E_o=30 MeV.

The power-law function of particle rigidity used is:

$$F(E)dE = F(R)\frac{dR}{dE}dE = C \cdot \left(\frac{R}{R_o}\right)^{-\gamma} \cdot \frac{dE}{\beta}, \qquad (3)$$

We have selected the best approximation for each of the SEP events and for each of the functions by least-squares calculating the spectral parameters. However, any comparison between experimental data and rigidity exponent seems to be senseless (Fig. 1).

We assumed the ≥30 MeV proton spectrum to be a power-law rigidity function and found the particle flux sizes (at the logarithmically-equidistant energies). After that, we approximated the resultant sequence by a power-law energy function for the 30-1000; 30-30000, 30-10000, and 400-10000 MeV particle energy ranges.

The simulation results are displayed in Fig. 2, whose ordinate is the logarithm of the ratio of the approximating function (power-low function of energy) to the initial function (power-law function of rigidity).

The dashed lines in Fig. 2 confine the domain of the standard errors that characterize the accuracy of measuring the solar proton fluxes. The data were obtained by analyzing the deviations of the measured flux sizes from the approximation functions (see, for example, Fig. 3). From Fig. 2 it follows that there is no sense to try and distinguish between the two approximation functions in the analysis of the experimental data for a single SEP event in the 30-1000 MeV or 400 MeV – 10 GeV ranges. In other words, if for a given event there are not enough experimental points covering a wide energy range one cannot made the clear difference between the two approximation curves.

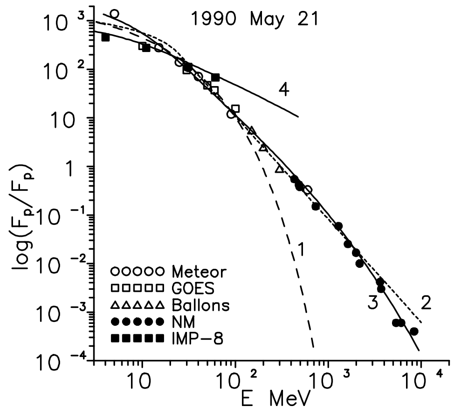

Fig.1. The proton peak flux data of the May 21, 1990 SEP event. The dots are the experimental data (displayed in the Figure). The curves are the best functional approximations of the experimental data by the exponent of rigidity for the E<100 MeV (1), by power law function of energy (2), and by power law function of rigidity (3). Curve 4 approximates the IMP-8 measurement data by the power-law function.

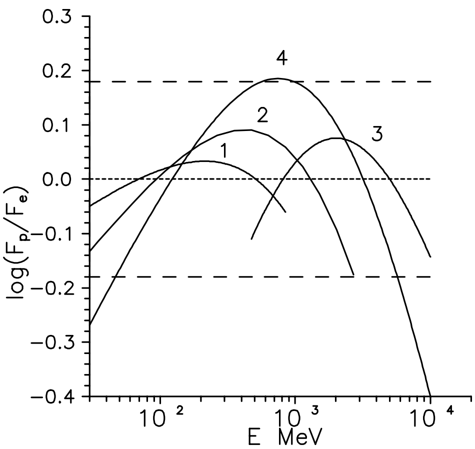

Fig. 2. Logarithm of the ratio of the model SEP spectrum (taken to be a power-law function of rigidity) to the spectrum approximated by a power-law function of energy in the 30 MeV -1 GeV (Curve 1), 30 MeV-3 GeV (2), 400 MeV-10 GeV (3), and 30 MeV-10 GeV (4) energy ranges. The dashed lines confine the domain of the standard errors in measuring the SEP fluxes. The dotted line is a zero deviation level.

In our opinion, the main problem is to choose between the power-law rigidity and energy functions.

Since the difference between the two functions is relatively small, while the particle flux measurement errors (both statistical and methodological) are fairly high, the tentative conditions must be defined to make the analysis feasible. For that purpose, the energy spectra and the techniques for approximating them were computer-simulated.

Quite a reliable result may be obtained using the total energy range (30 MeV – 10 GeV) for a great number of SEP events. In case a large set of SEP events are analyzed, copious experimental data are covered by the energy ranges to be used in defining the approximation function, so the aggregate statistical error gets reduced.

Experimental data

We used data on SEP events catalogued in Bazilevskaya et al.(1986), and Sladkova et al., (1998) to select proton energy spectra that overlap the range from 30 MeV to ≈10 GeV. They were measured in 13 ground level events (GLE) observed Jan. 24, 1971; July 25, Aug. 16, Sep. 29, Oct. 19, 22, 24, and Nov. 15, 1989; May 21, 24, 26, 28, 1990; June 15, 1991.The data on proton peak flux energy spectra are from measurements in space onboard METEOR (5-600 MeV), IMP-8 (4-60 MeV), GOES (10-100 MeV), from balloone-born measurements (100-500 MeV and are inferred from ground-based world-wide network neutron monitors (0.4-11 GeV.

4 Data analysis

The experimental data of proton peak fluxes were fitted twice for each of 13 GLE events, by power-low functions of energy and rigidity. The comparison among the standard deviations of the experimental data from the power-law functions of energy and rigidity has indicated that the power-law proton rigidity function for 11 events is best.

To compare a total set of different spectra in the all 13 events we show in Fig. 3 the ratio of experimental flux (F_{exp}) to calculated power-law energy calculated approximation (F_{E**G}).

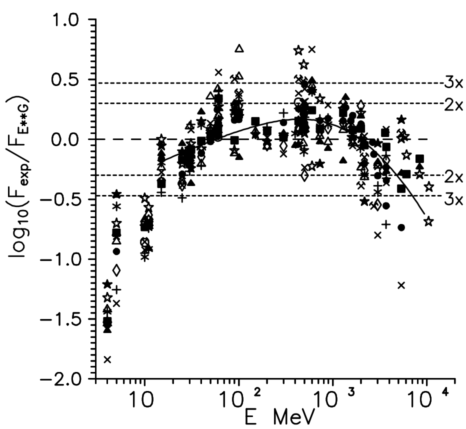

Fig. 3. The calculated logarithm of the ratio of the proton peak flux to the size defined by the approximating power law energy function versus energy. The markers indicate different SEP events. The 2x and 3x dotted lines designate two- and three-fold deviations of experiment from approximation. .Solid line is the power-law rigidity approximation

From Figure 3 it follows that the difference between the logarithms of the ratio is often more that two- and three-fold and can even reach ±1 (i.e., an order) for some of the experimental dots. It s also seen that the deviations of the experimental data from the power-law energy spectra are peaking at 100-1000 MeV and reach their minimum at both higher and lower energies.

Had the experimental data corresponded to the power-law energy function, the dots should have been distributed uniformly around the zero level of the ratio logarithm. Instead, we see that the deviation of the experimental data from the zero level corresponds to the deviation of the power-law rigidity function from the power-law energy function (solid line in Fig.3).

In the further analysis, we averaged the Figure 3 data in the equal-value ranges of the energy logarithm (4.7-10; 10-21.3, 21.3-46.8; 46.8-100 MeV, etc.). Figure 4 shows the results of the averaging. The error bars are the statistical errors due to the scatter of the Figure 3 dots over the respective energy ranges.

To conclude, we present Figure 5, which plots the measured proton peak flux-to-approximating power-law function logarithm ratio versus rigidity

The experimental dots above 240 MV (30 MeV) are distributed chaotically around the zero level, thereby confirming that the experimental proton peak flux data of the up-to 10 GV proton energy can quite properly be described by power-law rigidity spectra, with a spectral droop being never observed at relativistic energies.

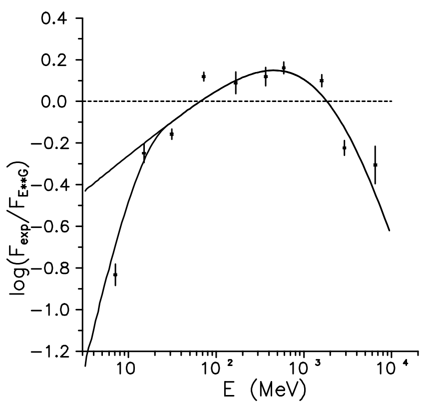

Fig. 4. The measured proton peak flux-to-approximating power-law function logarithm ratio versus energy. The upper solid line is the ratio of the power-law rigidity function to the power-low energy function The lower curve is the same ratio allowing for the droop of the power-law rigidity spectrum at E<30 MeV. The dotted line is the zero level of the ratio logarithm.

The <30 MeV experimental dots gets depressed abruptly as compared with the pure power-law spectrum, quite in agreement with (Nymmik, 1993). At the same time, the experimental data are much more scattered compared with the statistical deviations. This relates, first of all, to the experimental dot in the 46.8-100 MeV range of satellite measurements. We believe this fact to be due to the methodological defects of the satellite measurements that overestimate the particle fluxes. For example we notice the experimental data of IMP-8, which integral flux data are calculated from the measured differential fluxes in the 0.29-440 MeV energy range.

For the energy range of 30-440 MeV, the calculated IMP-8 power-law rigidity spectral index (3.2) for all the analyzed SEP events exceeds unity and is smaller than the spectral indices calculated for the full data set (4.5). This is clearly seen from the Fig. 1 data for the 21 May 1990 SEP event (curve 4).

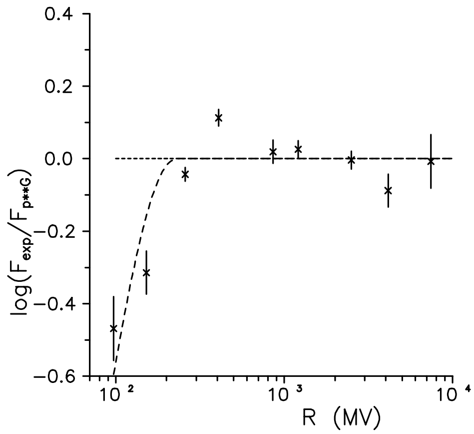

Fig. 5. The measured proton peak flux-to-approximating power-law function logarithm ratio versus rigidity. The dotted line is the zero level of the ratio logarithm. The descending line is the experimental data approximated by the power-law function of rigidity allowing for spectral droop (Nymmik, 1993).

5 Conclusion

The above functional and statistical analysis of the experimental SEP event proton peak flux data has shown that, within the energy range from 30 MeV to 10 GeV, the energy spectra are actually power-law functions without any marked droop at extremely high energies. This indicates that the acceleration by bow shock in a turbulent medium is the most probable mechanism that actually accelerates the particles. At the same time, the particle flux measurements by different experimental techniques have been shown to involve significant methodological errors, which cannot be disregarded when analyzing separate SEP measurement data obtained by a unified method.

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